Self-Tuning PID Controller Based on Fuzzy Wavelet Neural Network Model of Nonlinear System

M. Davanipour, R. Dadkhah, F. Shabani-nia
Department of Power and Control Engineering
Shiraz University
Shiraz, Iran
m.davanipour@shirazu.ac.ir
r.dadkhah@shirazu.ac.ir
shabani@shirazu.ac.ir

Abstract—**In this paper two objectives are followed; one is to illustrate the capability of Fuzzy Wavelet Neural Network (FWNN) in modeling; and the other is to propose a self-tuning PID controller based on this model. The controller is constructed by a FWNN structure combined with a PID controller. Gradient descent algorithm is used to accomplish tuning rules. The proposed method is applied for a nonlinear system, which is a liquid level system. The capability of FWNN in the modeling of nonlinear system by a few rules and susceptibility of the proposed controller will be shown by simulation. The convergence analysis of the algorithm will be done using Lyapunov theorem. It will be shown that the proposed method would increase the speed of tracking while having very little steady state error which is due to the high accuracy of fuzzy wavelet neural network.**

Keywords- fuzzy wavelet neural network; gradient descent algorithm; self-tuning PID controller; liquid level system

I. Introduction

Recently, soft computing and Fuzzy Wavelet Neural Networks (FWNNs) have been widely used in many application areas [1-8]. The FWNN which is a combined structure based on fuzzy rules includes wavelet functions in its consequent parts. This combined structure can avoid inherent limitations of each isolated methods. Wavelet transform can describe local details of the signals. Also neural networks with self-learning capability cause to increase the accuracy of the model. On the other hand fuzzy logic can decrease the complexity of the data and is suitable to be used in systems with uncertainty. Reference [1] proposed the first fuzzy wavelet network that was based on multi resolution analysis of wavelet transforms and fuzzy concepts used to approximate nonlinear functions. In [1] using Orthogonal Least Square (OLS) algorithm to select important wavelets has decreased the difficulties of selecting wavelets. Reference [2] has been suggested another FWNN structure to identify and control of nonlinear dynamic plants. In that work, the number of fuzzy rules and initial parameters of wavelets are determined by clustering algorithm. A fuzzy neural network model which is dynamic and uses wavelet functions in its processing units has been proposed in [8]. In that research, IF part of the fuzzy rules are comprised of Mexican Hat wavelet membership functions and THEN part of the rules are differential equations of linear functions. Also, in order to find optimal model parameters for nonlinear system modeling and/or control applications, a gradient based algorithm Broyden-Fletcher-Goldfarb-Shanno (BFGS) method has been used.

Using soft computing methods has directed to implement several types of adaptive controllers [9-13]. A Radial Basis Function (RBF) network was applied to find the optimal values of PID parameters in [9]. Random parameters and corresponding amounts of cost function earned from closed loop system were used as training data. Then RBF set the parameters fits to minimum cost function. A PID control based on wavelet neural networks identification and tuning was described in [10]. In that control scheme, two wavelet neural networks were used for identification and tuning online. The parameters of PID controller were the output of the second employed wavelet network. Therefore, it is needed to obtain suitable amounts of parameters as training data. In [11] self-tuning adaptive PID controller was proposed using a dynamic wavelet network. The control scheme was tested with a second order system with input saturation. The controller finally has tracked the desired signal; however, it seems from simulation results to take long time. In spite of good performance with noise contamination, the cost function minimized for control purpose was the difference between desired output and estimated output of wavelet network whereas in practice the error between desired output and actual output should be minimized. Reference [13] has been presented a multivariable self-tuning PID controller based on WFNNs for a class of nonlinear systems and the self-tuning PID controller is derived via a generalized predictive performance criterion.

In this paper, a nonlinear model which is a liquid level system is considered. At the first step, the system is modeled using FWNN. This network could identify the unknown model very well using few numbers of parameters. Then based on the obtained model a self-tuning PID controller is designed. The controller's parameters are optimized using simple gradient descent method. The tuning process could be begun without any trial and error unlike most of researches in this area. The control performance is well especially in steady state error and speed of tracking. These two stages are simulated. It could be seen that the combination of FWNN and PID controller would lead to a well performance and high speed controller. Indeed it allows us using the capability of FWNN in control of nonlinear dynamic systems.

This paper is organized as follows: Section II gives the structure of FWNN. Section III represents the liquid level system. In Section IV the self-tuning PID controller and the convergence analysis is proposed. In the next section, simulation results have been provided. Section VI includes conclusion remarks.

II. Fuzzy Wavelet Neural Network

The FWNN as depicted in Fig. 1 is constructed by some fuzzy rules [6]:

$$R^i: If\ x_1 is\ A_1^i, and\ x_2 is\ A_2^i, \dots, and\ x_q is\ A_q^i$$
$$Then\ y_i = w_i \sum_{j=1}^{q} \psi_{ij}(x_j) \tag{1}$$

in which x_j $(1 \le j \le q)$ is the j^{th} input and y_i $(1 \le i \le c)$ is the output of the local model for rule R^i, which is equal to the linear combination of a finite set of wavelets. The output signal of each WNN is calculated as

$$y_i = w_i \sum_{j=1}^{q} \psi_{ij}(x_j) \tag{2}$$

where w_i is the weight coefficient from the inputs and i^{th} output and ψ_{ij} is a wavelet family defined in the following form

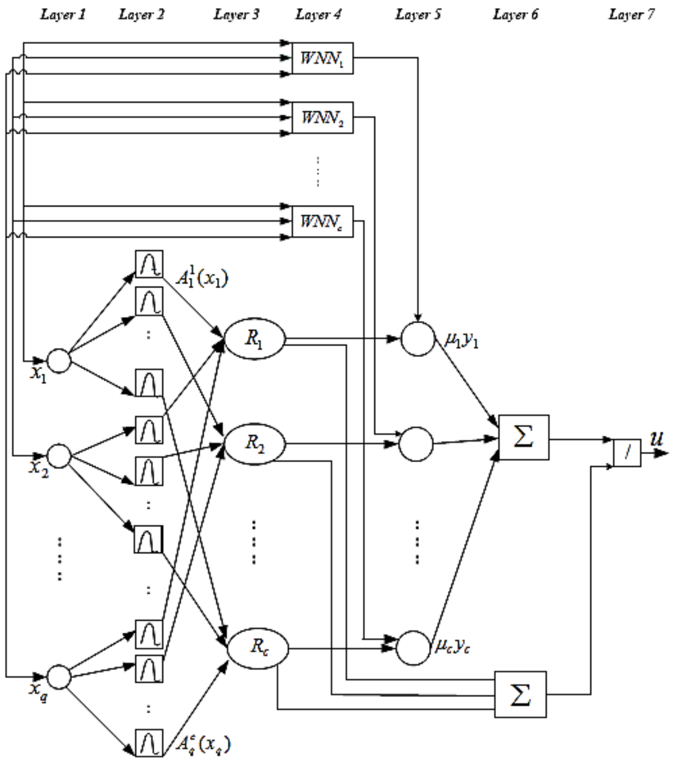

Figure 1. The structure of FWNN

$$\psi_{ij}(x_j) = \psi\left(\frac{x_j - b_{ij}}{a_{ij}}\right) \qquad a_{ij} \ne 0 \tag{3}$$

In (3), $\psi_{ij}(x)$ is obtained from a single mother wavelet function by dilations and translations (a, b).

According to above descriptions the output of FWNN is calculated as

$$\hat{y} = \frac{\sum_{i=1}^{c} \mu_i y_i}{\sum_{i=1}^{c} \mu_i}\ where\ \mu_i = \prod_{i=1}^{q} A_j^i(x_j) \tag{4}$$

for $i = 1, 2, \dots, c$ and $j = 1, 2, \dots, q$. Here c is the number of fuzzy rules and q is dimension of input vector.

Here the Gaussian membership functions have been selected to describe the linguistic terms. The reason for selecting Gaussian membership function is that it is able to approximate triangular and trapezoidal membership functions [6].

$$A_j^i(x_j) = \exp\left[-\frac{\left(x_j - c_j^i\right)^2}{\left(\sigma_j^i\right)^2}\right] \tag{5}$$

in which σ_j^i and c_j^i determine the center and the half-width of corresponding membership function. Mexican Hat wavelet function which is a symmetric function and commonly applied in FWNN is used in consequent parts of each fuzzy rule.

$$\psi(x) = \frac{1}{\sqrt{|a|}}(1 - 2x^2)\exp\left(-\frac{x^2}{2}\right) \tag{6}$$

III. Modeling of Nonlinear Liquid Level System with FWNN

A tank liquid level system is shown in Fig. 2 in which Q_{in} and Q_{out} are the maximum liquid flow rates in $\frac{m^3}{s}$ for input and outlet, respectively [12].

The controlled liquid flow rate is given as

$$q_{in} = Q_{in}\sin(\theta(t)) \qquad \theta(t) \epsilon \left[0, \frac{\pi}{2}\right] \tag{7}$$

The output liquid flow rate is such as

$$q_{out} = a_{out}\sqrt{2gh(t)} \tag{8}$$

The descriptive equation of the system is

$$h(t) = h(0) + \frac{1}{A}\int_0^t (q_{in}(\tau) - q_{out}(\tau))d\tau \tag{9}$$

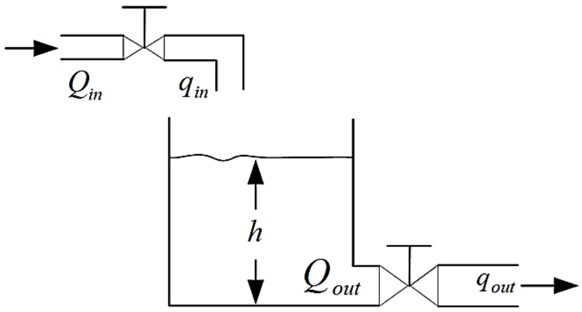

Figure 2. A simple nonlinear liquid level system

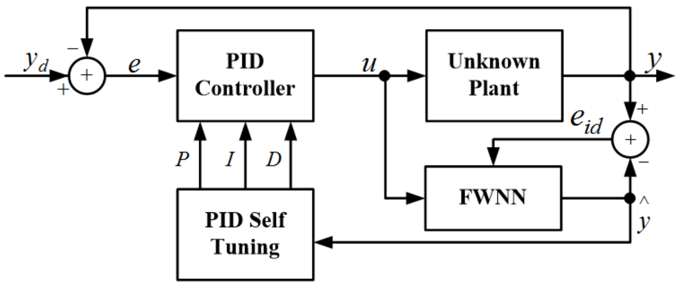

Figure 3. The structure of self-tuning PID controller

in which, h is the output variable in m which is the level of the liquid, $a_{out} = 0.01\,m^2$ is the surface area of the outlet, $A = 1\,m^2$ is the surface of the tank and $g = 9.81\frac{m}{s^2}$.

In this section, a FWNN has been used for modeling of this liquid level system. For this purpose a FWNN constructed by only two fuzzy rules could be able to model this nonlinear system perfectly. Fuzzy rules are constructed according to (1). It is a two inputs and one output network. In this case θ and q_{out} have been considered as inputs and h as output. WNN is also a two input one output structure. Initial wavelet parameters presented in (3) have been obtained by clustering algorithm. The others have been random selected. After that the learning process begins through which the optimum amounts of all parameters (a, b, c, σ, w) will be determined by gradient descent algorithm. Details of clustering and gradient descent methods can be found in many reports [14-17].

IV. Self-Tuning PID Controller based on FWNN

In the previous section modeling of a system by FWNN was discussed. In the next step, it is necessary to control that system using a PID controller based on the obtained FWNN model. To reach the final self-tuning controller, the tuning rules should be exploited. The following description tries to get these rules.

A. Controller Design

The control loop tuning is the adjustment of control parameters (P, I, D) to reach optimum values. The structure of closed loop system which is a combination of identifier network and self-tuning PID controller is shown in Fig. 3. In that figure, e_{id} is the identification error. The auto tuner computes the optimum values of control parameters by minimizing the bellow cost function:

$$E = \frac{1}{2}\sum_{k=1}^{T}\left(y_d(k) - \hat{y}(k)\right)^2 \qquad (10)$$

in which $y_d(k)$ is the desired output and $y(k)$ is the actual output of the system.

The PID controller is as follows:

$$\begin{aligned} u(k) &= u(k-1) + P[e(k) - e(k-1)] + Ie(k) \\ &\quad + D[e(k) - 2e(k-1) + e(k-2)] \end{aligned} \qquad (11)$$

In the above relation, $e(k)$ is the error produced by deviation of the actual output from the desired output. The cost function includes P, I and D which are selected as control parameters. So, the goal is finding these parameters to minimize the cost function in (10). Here, for this purpose gradient descent algorithm is applied.

The optimization formula is such as follows:

$$\begin{aligned} \Delta P(k) &= -\eta_P \frac{\partial E}{\partial P} \\ \Delta \mathrm{I}(k) &= -\eta_I \frac{\partial E}{\partial I} \\ \Delta D(k) &= -\eta_D \frac{\partial E}{\partial D} \end{aligned} \qquad (12)$$

In that for example η_P indicates the learning rate for P parameter and $\frac{\partial E}{\partial P}$ is

$$\frac{\partial E}{\partial P} = -\sum_{k=1}^{T} e(k)\frac{\partial \hat{y}}{\partial P} \qquad (13)$$

where e is the control error and $\hat{y}$ is the output of the model.

By substituting derivation we have

$$\frac{\partial E}{\partial P} = -\sum_{k=1}^{T} e\left(\frac{\partial \hat{y}}{\partial u}\frac{\partial u}{\partial P}\right) \qquad (14)$$

In that, $\frac{\partial \hat{y}}{\partial u}$ is computed using relations (1-4) and $\frac{\partial u}{\partial P}$ according to (8).

Then we would have

$$\hat{y} = \frac{\sum_{i=1}^{c}\left(\prod_{j=1}^{q} A_j^i(x_j)\right) w_i \sum_{j=1}^{q}\psi_{ij}(x_j)}{\sum_{i=1}^{c}\left(\prod_{j=1}^{q} A_j^i(x_j)\right)} \qquad (15)$$

Here x is the network's input (u). The derivation of (15) is such as

$$\frac{\partial \hat{y}}{\partial u} = \frac{\frac{\partial m}{\partial u}n - \frac{\partial n}{\partial u}m}{n^2} \tag{16}$$

where

$$m = \sum_{i=1}^{c}\left(\prod_{j=1}^{q} A_j^i(x_j)\right) w_i \sum_{j=1}^{q} \psi_{ij}(x_j)$$
$$n = \sum_{i=1}^{c}\left(\prod_{j=1}^{q} A_j^i(x_j)\right) \tag{17}$$

Then we have:

$$\frac{\partial u}{\partial P}(k) = \frac{\partial u}{\partial P}(k-1) + e(k) - e(k-1) \tag{18}$$

B. Convergence Analysis

The controller in (11) can stabilize the closed loop system. The proof is done by applying the Lyapunov stability theorem.

Consider the Lyapunov function such as bellow

$$y_p = E_p \tag{19}$$

where is the error between the desired output y^d and the output of the closed loop system y at epoch p .The deviation of Lyapunov function because of the learning procedure is

$$\Delta V_p = V_{p+1} - V_p = \left[E_{p+1}^2 - E_p^2\right] \tag{20}$$

Furthermore

$$E_{p+1} = E_p + \Delta E \;\Rightarrow\; E_{p+1}^2 = E_p^2 + \Delta^2 E + 2E_p \Delta E \tag{21}$$

Thus

$$\Delta V_p = \Delta E_p\left[E_p + \frac{1}{2}\Delta E_p\right] \tag{22}$$

The updating formula for parameter I (for example) in gradient descent algorithm can be expressed as

$$I_{p+1} = I_p - \eta\frac{\partial E_p}{\partial I} \tag{23}$$

Also, deviation of the error function because of changing in the parameter I, can be written as

$$\Delta E_p = E_{p+1} - E_p \approx \frac{\partial E_p}{\partial I}\Delta I \tag{24}$$

Therefore

$$\Delta V_p = \frac{\partial E_p}{\partial I}E_p + \eta\left(\frac{\partial E_p}{\partial I}\right)^2 \tag{25}$$

It is known from the Lyapunov stability theorem that if V_p is positive and ΔV_p is negative, the stability will be guaranteed. Therefore, the convergence condition is obtained such as following

$$\Delta V_p < 0 \;\Rightarrow\; \eta\frac{\partial E_p}{\partial I} < E_p \Rightarrow \eta < \frac{E_p}{\frac{\partial E_p}{\partial I}} \tag{26}$$

This is the condition for convergence.

V. Simulation Results

In this section the results of applying FWNN for modeling and control of the liquid level system have been proposed.

A. Modeling Section

The input data set is gathered in Fig. 4 which is the changes of control valve's flap angle given in (7). Fig. 5 shows the output of FWNN model and compares it with the output of mathematical model. As it is seen this FWNN using only two fuzzy rules has modeled this nonlinear system very well. Fig. 6 depicts the differences between the mathematical and FWNN model. The fuzzy rules used for modeling are such as (1) in which x_1 and x_2 are q_{in} and q_{out}, respectively. A is according to (5) and ψ is according to (6).

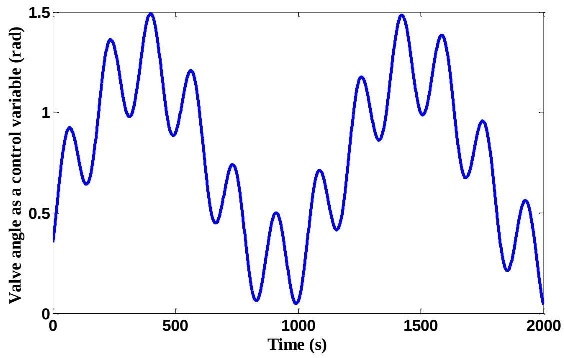

Figure 4. The input signal representing the changes of the valve angle as the control variable

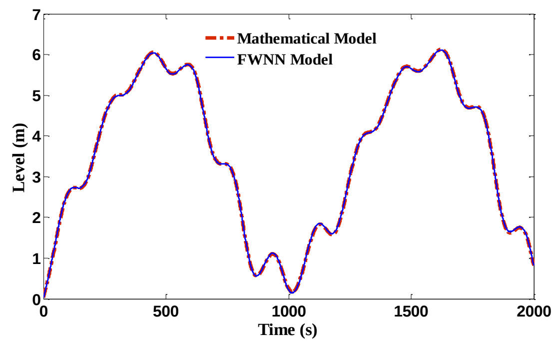

Figure 5. Agreement comprasion of mathematical model output (dashedline) and FWNN model output (solid line).

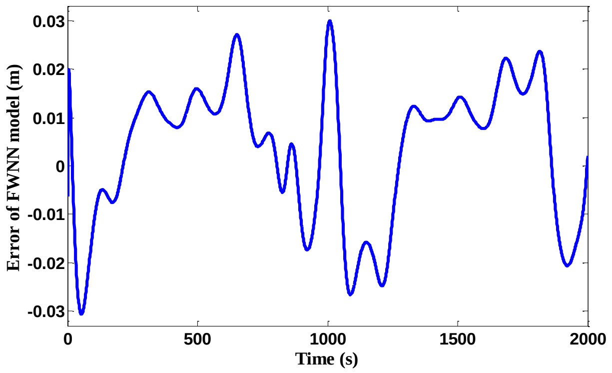

Figure 6. Difference between the output of mathematical and FWNN models

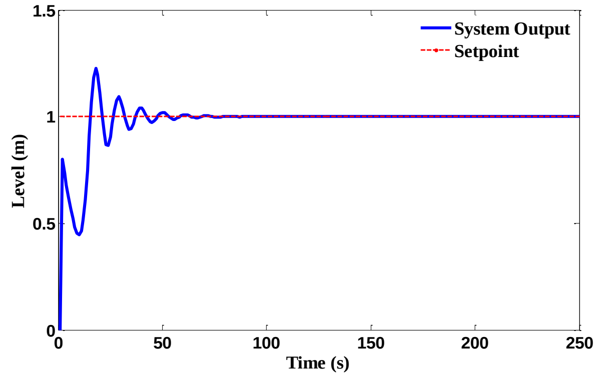

Figure 7. Performance of the controller in the response to the unit step setpoint

B. Control Section

After modeling, the self-tuning PID controller based the FWNN model proposed in the previous section has been used to force the system for tracking of unit step signal. The initial values of PID controller's parameters have been set to zero. Therefore, it needs no previous guess to start. The updating process is completed just for 100 epochs. The result of this section is shown in Fig. 7. The performance of the controller is well especially in steady state manner. In addition, the speed of tracking is acceptable. Fig. 8 depicts the controller's parameters tuning curves. The figure shows that the parameter's setting to suitable values quickly happens then variations are almost insensible. For more investigations on the performance of the controller, a staircase step input is applied to the system. The result is illustrated in Fig. 9. Also, the performance of the controller is checked with an input disturbance. A half of the unit step is considered as an input disturbance. It has been applied to the plant entrance at $t = 100\ s$. Fig 10 shows the response of system in the present of the disturbance. It can be seen that the controller is still good performing and the system output tracks the setpoint. The comparison of proposed method with some other methods is given in Table I. In that table the PID based FWNN is the proposed method here. The fuzzy based ANFIS model is the one studied in [12]. The PID based NN (Neural Network) is a PID controller based on the neural network model using 19 nodes and MLP (Multi-Layer Perceptron). The PID based ANFIS model is also a PID controller based the ANFIS model which is almost similar to the one in [12]. Based on Table I, the proposed method yields to less settling time, despite the considerable lower number of rules and parameters. Although, using of proposed method yields to more overshoot in comparison with fuzzy based ANFIS [12], but the reduction of overshoot in fuzzy based ANFIS [12] has been achieved in the cost of much more parameters and rules.

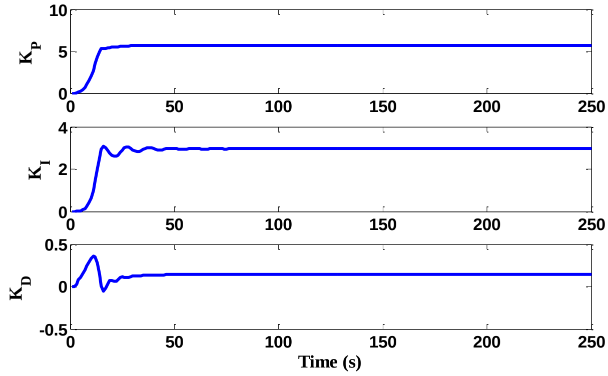

Figure 8. Controller parameters tuning curves

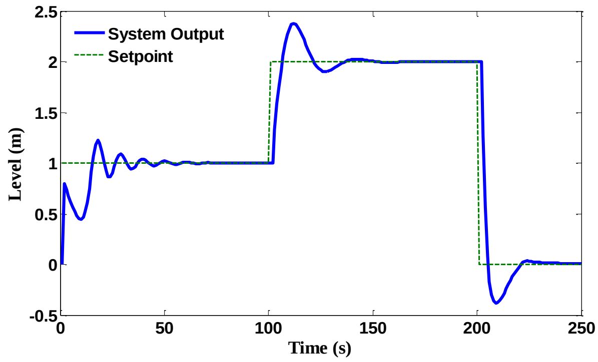

Figure 9. Performance of the controller in the response to the staircase unit step setpoint

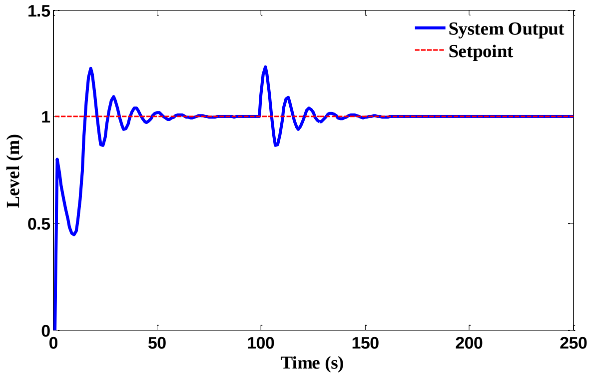

Figure 10. Performance of the controller in the response to an input disturbance. The amplitude of the applied disturbance to the entrance of the system at $t = 100\ s$ is 0.5 .

TABLE I. COMPARISON OF PROPOSED METHOD WITH OTHER METHODS

Method	***Number of rules***	***Number of parameters***	***Settling Time (s)***	***Overshoot (percentage)***
PID based FWNN	2	18	60	22
Fuzzy based ANFIS [12]	25	175	80	10
PID based NN	----	73	100	40
PID based ANFIS	25	175	90-110	38

VI. CONCLUSION

Due to the capability of FWNN in modeling of nonlinear unknown dynamic system, it was applied for modeling of a nonlinear liquid level system. Only two fuzzy rules could be able to model this nonlinear system. By using the obtained model, a self-tuning PID controller was proposed. The auto tuner enables the controller to usage the accuracy of FWN modeling which in turn increase the speed of tracking. It was seen that using a gradient descent algorithm with low number of iterations could result in the optimum values of the control parameters. The tuning rule began without any trial and error unlike most of researches in this area. The simulation part illustrated these results.

REFERENCES

[1] D. W. C. Ho, Z. Ping-Au; and X. Jinhua, "Fuzzy wavelet networks for function learning," in Fuzzy Systems, IEEE Transactions on, vol. 9, no. 1, pp. 200-211, Feb 2001.

[2] R. H. Abiyev, and O. Kaynak, "Fuzzy wavelet neural networks for identification and control of dynamic plants—a novel structure and a comparative study," in Industrial Electronics, IEEE Transactions on, vol. 55, no. 8, pp. 3133-3140, Aug. 2008.

[3] Y. C. Hung, F. J. Lin, J. C. Hwang, J. K. Chang, and K. C. Ruan, "Wavelet fuzzy neural network with asymmetric membership function controller for electric power steering system via improved differential evolution," in Power Electronics, IEEE Transactions on, vol. 30, no. 4, pp. 2350-2362, April 2015.

[4] F. J. Lin, Y. C. Hung, and K. C. Ruan, "An intelligent second-order sliding-mode control for an electric power steering system using a wavelet fuzzy neural network," in Fuzzy Systems, IEEE Transactions on, vol. 22, no. 6, pp. 1598-1611, Dec. 2014.

[5] L. L. S. Linhares, J. Medeiros de Araujo, and F. Meneghetti U.Araujo, "Nonlinear system identification of a lower limb model by fuzzy wavelet neural networks," in Industrial Electronics Society, IECON 2013 - 39th Annual Conference of the IEEE , pp. 3628-3633, 10-13 Nov. 2013.

[6] M. Davanipoor, M. Zekri, and F. Sheikholeslam, "Fuzzy wavelet neural network with an accelerated hybrid learning algorithm," in Fuzzy Systems, IEEE Transactions on, vol. 20, no. 3, pp. 463-470, June 2012.

[7] H. A. Bazoobandi, and M. Eftekhari, "Identification of dynamic plants using fuzzy wavelet network: A multimodal memetic approach," in Intelligent Systems (ICIS), 2014 Iranian Conference on, pp. 1-5, 4-6 Feb. 2014.

[8] Y. Sevcan, and O. Yusuf, "Nonlinear system modeling and control with dynamic fuzzy wavelet neural network," in Innovations in Intelligent SysTems and Applications (INISTA), 2015 International Symposium on, pp. 1-7, 2-4 Sept. 2015.

[9] M. S. M. Ali, "Optimization of controller parameters for a couple tank system using metamodeling technique", PhD Thesis, Malaysia University Technology, Malaysia, 2009.

[10] H. Li, H. Jin, and C. Guo, "PID control based on wavelet neural network identification and tuning and its application to fin stabilizer," in *Mechatronics and Automation, 2005 IEEE International Conference*, vol. 4, pp. 1907-1911, 29 July-1 Aug. 2005.

[11] R. Dhaouadi, Y. Al-Assaf, and W. Hassouneh, "A self tuning PID controller using wavelet networks," in Power Electronics Specialists Conference, 2008. PESC 2008. IEEE, pp. 773-777, 15-19 June 2008.

[12] S. N. Engin, J. Kuvulmaz, and V. E. Omurlu, "Fuzzy control of an ANFIS model representing a nonlinear liquid level system," Neural Computing and Application, vol. 13, no. 3, pp. 202-210, Sepember 2004.

[13] C. H. Lu, P. Y. Liao, Y. H. Charng, C.M. Liu, and J. Y. Guo, "Multivariable self-tuning PID controller based on wavelet fuzzy neural networks," in Machine Learning and Cybernetics (ICMLC), 2014 International Conference on, vol. 2, pp. 755-759, 13-16 July 2014.

[14] O. A. Abbas, "Comparisons between data clustering algorithms,". Int. Arab J. Inf. Tech., vol. 5, no. 3, pp. 320-325, July 2008.

[15] S. Na, L. Xumin, and G. Yong, "Research on k-means clustering algorithm: An improved k-means clustering algorithm," in Intelligent Information Technology and Security Informatics (IITSI), 2010 Third International Symposium on, pp. 63-67, 2-4 April 2010.

[16] O. Nelles, Nonlinear system identification: form classical approaches to neural networks and fuzzy models, Springer-Verlag, Berlin Heidelberg, 2001.

[17] J. S. R. Jang, C. T. Sun, E. Mizutani, Neuro- Fuzzy and soft computing: A computational Aapproach to Llearning and machine intelligence, 1st ed., Prentice-Hall , New Jersey, 1997.