A New Maximum Torque Per Ampere Operation Scheme for Interior Synchronous Permanent Magnet Motors Based on Extremum Seeking Control with a Modified Finite-Time Gradient Algorithm

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***Abstract*— In this paper, a Maximum Torque Per Ampere (MTPA) control approach based on an Extremum Seeking Control (ESC) algorithm is proposed. First, for the application of MTPA control, a finite-time gradient algorithm is introduced, analyzed, and then, employed in the gradient-based ESC algorithm to estimate the MTPA operating point of an Interior Permanent Magnet Synchronous Motor (IPMSM). A convergence analysis is carried out in the presence of a new adaptive Neural Network (NN)-based speed loop controller in the next step. Finally, simulation results are presented to investigate the performance of the proposed MTPA control algorithm.**

I. Introduction

Permanent Magnet Synchronous Motors (PMSMs) are used in a wide range of servo applications such as electric vehicles, elevator drives and electric power steering systems. This is due to their advantages of simple structure, high power/weight ratio and easy maintenance. Also, PMSMs have higher efficiency in comparison with induction motors because of the absence of rotor cooper losses [1]–[4].

Interior PMSMs (IPMSMs) are a special kind of PMSMs in which the permanent magnets are buried inside the body of the rotor. As a result, they provide a reluctance torque due to the difference between the q-axis and d-axis inductances. However, in order to exploit this reluctance torque efficiently, more involved control algorithms compared to the conventional vector controls are needed.

Especially in high power and low speed IPMSMs, that the iron losses are negligible compared to the copper losses, maximizing the ratio between the electromagnetic torque and the current amplitude results in a considerable increase in the efficiency of the drive [5], [6]. To control this ratio, which is usually known as the Maximum Torque Per Ampere (MTPA) control, it is required to force the current space vector to track the MTPA trajectory. Due to several reasons, the MTPA control is not so straightforward. The main reason is that the MTPA trajectory is dependent on the uncertain parameters of the system. That the MTPA trajectory is implicitly a function of the system states and disturbances, is the other reason which makes it difficult to analyze the stability of the designed controller. However, in most cases the current control loops are designed to be much faster than the mechanical loops, thus it can be assumed that during a sampling time period of current loops the current references, produced by MTPA trajectory, are constant and their derivatives are equal to zero. To date several solutions have been proposed for the first problem. In [7], required parameters are computed in an off-line manner and used to calculate the MTPA operating point. Because of the parameter variation during the operating time, such methods suffer from the lack of robustness. The other solution is to use adaptive parameters of adaptive control laws [8]. In this method, however, the convergence of the adaptive parameters to the true values cannot be guaranteed. Conventional gradient-based optimization methods are also widely used to deal with this problem. But the requirement of the uncertain gradient function is the main drawback of these methods. In [9], [10], without providing any stability analysis, the gradient function is estimated based on search algorithms which are not robust against variations in operating point. The Extremum Seeking Control (ESC) method is the other online gradient-based solution which has been used to estimate the MTPA point [5], [6]. Briefly, a gradient-based online optimization is the ESC algorithm which estimates the gradient function and feeds it into a conventional gradient algorithm to estimate the optimal point [11]. It is worth noting that other forms of ESC algorithm, in particular Newton-based algorithms, have also been developed in recent years and successfully applied in some power engineering applications [12].

The main drawback of the conventional gradient algorithm is the extreme drop in the convergence speed in the vicinity of the optimal point. To deal with this problem, inspiring the concepts of finite-time sliding mode control [13], [14] and also non-quadratic Lyapunov functions [15], we have proposed a modified finite-time gradient algorithm with application to MTPA control in section III. In section IV, this new algorithm is replaced with the conventional one in the ESC of the MTPA control scheme, which has a new structure. A convergence analysis in the presence of the adaptive Neural Network (NN)-based speed loop controller is performed and included in the same section. Simulation results are brought in section V, and finally, a conclusion is made in section VI.

II. MTPA trajectory

The instantaneous electromagnetic torque of an IPMSM in the rotor reference frame is expressed as follows:

$$T_e = \frac{3p}{2}\left(\lambda_m i_q + (L_d - L_q) i_d i_q\right) \tag{1}$$

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where T_e is the electromagnetic torque, i_j and L_j, $j = d, q$ are the j-axis stator current and the inductance, respectively, and λ_m is the maximum flux linkage due to the permanent magnets. It is clear that the above equation has an infinite solution set for i_d and i_q with a constant value of T_e. These degree of freedom can be exploited to gain a higher electrical efficiency. One of the most common optimization problems that is defined to meet this goal is the problem of minimum amplitude current subject to a constant torque, which can be formulated as a constrained optimization problem as follows:

$$\begin{cases} \min \quad I_s^{*2} = i_q^{*2} + i_d^{*2} \\ s.t. \quad \alpha i_q^* - \beta i_d^* i_q^* = T_e^* \end{cases} \tag{2}$$

where i_j^* is the reference for i_j, I_s is the current amplitude reference, $\alpha = \frac{3p}{2}\lambda_m$ and $\beta = -\frac{3p}{2}(L_d - L_q)$.

To solve this constrained optimization problem, according to the constraint, we have

$$i_q^* = \frac{T_e^*}{\alpha - \beta i_d^*} \tag{3}$$

Substituting i_q^* from (3) into the objective function of (2) results in an unconstrained optimization problem as follows:

$$J(i_d^*) = I_s^{*2} = i_d^{*2} + \frac{T_e^{*2}}{(\alpha - \beta i_d^*)^2} \tag{4}$$

Differentiating the $J(i_d^*)$ with respect to i_d^* and setting it to zero, we have:

$$i_d^*(\alpha - \beta i_d^*)^3 = -T_e^{*2}\beta \tag{5}$$

Since it is hard to solve the above forth-degree equation, we substitute the torque T_e^* from the constraint of the problem (2) into (5) to obtain an easier to solve and torque independent second-degree equation in terms of both i_d^* and i_q^*. Thus:

$$\begin{aligned} & i_d^*(\alpha - \beta i_d^*)^3 = -(\alpha i_q^* - \beta i_d^* i_q^*)^2 \beta \\ & \Rightarrow i_d^*(\alpha - \beta i_d^*) = -i_q^{*2}\beta \\ & \Rightarrow -\beta i_d^{*2} + \alpha i_d^* + \beta i_q^{*2} = 0 \end{aligned}$$

The feasible solution of this equation with respect to i_d^* is:

$$i_d^* = \frac{-\lambda_m}{2(L_d - L_q)} - \sqrt{\frac{\lambda_m^2}{4(L_d - L_q)^2} + i_q^{*2}} \tag{6}$$

Since computing i_d^* from the above equation results in the maximum ratio between I_s^* and T_e^*, it is usually referred to as the MTPA trajectory. In the remainder of this paper, from the 'optimal i_d^*' we mean the value of i_d^* that satisfied this equation for a given i_q^*.

III. Finite-time gradient algorithm with application to MTPA control

It is obvious that the exact values of L_d, L_q and λ_m are needed to calculate the optimal i_d^* from the MTPA equation (6). These parameters, however, are usually unknown, extremely uncertain and may vary during the operating time. Therefore, to use (6), one solution is to use the estimates rather than the actual values. The online identification of this system, however, requires complicated identification methods and also powerful microcontrollers to execute the identification plans in real-time. In some references, such as [8], the adaptive parameters obtained from the adaptive controller are used as the estimates of parameters in (6). But, this method may result in inaccurate results, because the convergence of adaptive parameters to accurate values can only be guaranteed under certain conditions of persistence of excitation of inputs. Another solution is to directly estimate the minimum point of (4) by means of the gradient-based optimization methods rather than using (6). In this section, we will propose a modified gradient-based method to estimate the optimal i_d^*. The gradient-based methods are useful for applications in which the system maintains in the steady-state for a long enough time. Elevator drives can be investigated as a potential application for such kind of methods, because the time period between the two consecutive stops is usually long enough and, besides, in these drives the energy efficient approaches are vital and highly required. Another potential application are home appliances such as washing machines.

The conventional gradient algorithm for this specific problem can be expressed as follows:

$$\dot{i}_q^* = \gamma_g \nabla J(i_d^*), \qquad \nabla J(i_d^*) = \frac{\partial J}{\partial i_q^*} \tag{7}$$

where γ_g is a positive parameter known as step size. The main drawback of the conventional gradient-based optimization as above equation is considerable drop in the convergence speed near the extremum point. Increasing the step size near this point, a common solution to overcome this problem that is, however, sometimes accompanied by instability problems. In one-dimensional gradient algorithms, the original gradient function at right hand side of (7) can be replaced with any other function which has the same sign as it. Exploiting this fact, to avoid the extreme drop in the converges speed, we modify the conventional gradient algorithm (7) as follows:

$$\dot{i}_d^* = -\gamma_g \left(\frac{\partial J}{\partial i_d^*}\right)^\kappa \tag{8}$$

where $\kappa \in (0, 1]$, and for the scalars z and η we have the following definition:

$$z^\eta = \begin{cases} |z|^\eta \mathrm{sgn}(z); & 0 < \eta \leq 1 \\ |z|^\eta \quad ; & 1 < \eta \leq 2 \end{cases}$$

Therefore, since the term $\left(\frac{\partial J}{\partial i_d^*}\right)^\kappa$ is an odd function of the gradient $\frac{\partial J}{\partial i_q^*}$, the expressed condition for one-dimensional gradient is met. The modified gradient algorithm provides us with not only a faster but also a finite convergence time. The finite-time convergence property of this algorithm is presented as the following theorem:

Theorem 1: The gradient algorithm (8) with the objective function (4) converges to the minimum point in finite time if its initial condition $i_d^*(0)$ is close enough to the this point.

Proof: According to (4), we have:

$$\frac{\partial J}{\partial i_q^*}=2\left(i_d^*+\frac{\beta T_e^2}{\left(\alpha-\beta i_d^*\right)^3}\right) \tag{9}$$

Let define the error state e_d as:

$$e_d=i_d^*+\frac{\beta T_e^2}{\left(\alpha-\beta i_d^*\right)^3}$$

Thus, the value of i_d^* for which $e_d(i_d^*)=0$ is the optimal solution of (4). Therefore, to prove the theorem it is enough to show that $e_d(t)$ converges to zero in finite time.

The time derivative of e_d along (8) is:

$$\dot{e}_d=-\bar{\gamma}_g\left(1+\frac{3T_e^2\beta^2}{\left(\alpha-\beta i_d^*\right)^4}\right)e_d^\kappa \tag{10}$$

where $\bar{\gamma}_g=2\gamma_g$. It is obvious that in the feasible operating region $h(i_d^*)=\frac{3T_e^2\beta^2}{(\alpha-\beta i_d^*)^4}>0$. Now, considering the Lyapunov function $V=0.5e_d^2$, its time derivative is:

$$\dot{V}=-\bar{\gamma}_g\left(1+\frac{3T_e^2\beta^2}{\left(\alpha-\beta i_d^*\right)^4}\right)e_d^{1+\kappa}<0$$

Therefore, e_d, at least, exponentially converges to the origin, and it can be concluded that if the initial condition of $i_d^*(t)$ is close enough to the optimal point then $h(i_d^*)$ has a bounded value. According to (10) and without loss of generality for the initial condition $e_d(0)>0$, we have:

$$\int_0^{e_d(0)}\frac{de_d}{e_d^\kappa}=\bar{\gamma}_g\left(t_s+\int_0^{t_s}h(i_d^*)dt\right) \tag{11}$$

where $e_d(t_s)=0$. Now, we define $f(t_s)$ as follows:

$$f(t_s)=t_s+\int_0^{t_s}h(i_d^*)dt$$

Since during the operating time $h(i_d^*)>0$, it can be shown

$$\frac{\partial f}{\partial t_s}=1+h(i_d^*)\geq\bar{f}>0$$

Also, due to the roundedness of h, it can be easily concluded that $f(0)=0$. To continue, first we state implicit function theorem [16]:

Lemma 1: Assume that $h(x,u)$ is continuously differentiable $\forall(x,u)\in\mathbb{R}^n\times\mathbb{R}\to\mathbb{R}$, and there exists a positive constant $\bar{r}$ such that $\frac{\partial h}{\partial u}>\bar{r}>0$. Then there exists a smooth and continuous function $u=u^*(x)$ for which $h(x,u^*)=0$.

According the to the lemma (1), it can be shown that for the positive constant c, the following equation:

$$t_s+\int_0^{t_s}h(i_d^*)dt=c$$

will be met for a bounded $t_s>0$, and according to (11),

$$c=\frac{1}{\bar{\gamma}_g}\int_0^{e_d(0)}\frac{de_d}{e_d^\kappa}=\frac{\left|e_d(0)\right|^{1-\kappa}}{\bar{\gamma}_g\left(1-\alpha\right)}$$

■

As it can be seen from (9), the main problem for implementation of the both finite-time and conventional gradient methods is that the gradient function is dependent on the unknown parameters of the system. In fact, by using the gradient-based optimization algorithm rather than the MTPA equation, the problem of the estimation of three parameters is reduced to a problem of one unknown parameter which is the gradient function (9). One of the most common approaches to deal with such problems is the ESC. In many of control applications the mapping from the reference to the system output has an extremum point, and the control objective is to regulate the system output close to this point. For example, consider the following system:

$$\dot{x}=f(x,u),\qquad y=g(x)$$

and assume that there is a x^* for which $y^*=g(x^*)$ is an extremum for $g(.)$. In many cases because of the system uncertainties neither x^* nor $g(.)$ are exactly known. The main goal of ESC is to estimate x^* and to force the closed-loop system output to converge to it [17], and works based on the gradient algorithm. Briefly and simply, first the gradient function is estimated and then it is used in the gradient algorithm to estimate x^* in a closed-loop system.

IV. Finite-time gradient-based ESC algorithm

ESC is a very old method and its emergence dates as far back as the early 1920s [17], and gained a high interest since the year 2000 when the first comprehensive convergence analysis was presented by Krstic and Wang [11]. This method has also been used in IPMSM drives to estimate the optimal i_d^*, and to the best of our knowledge, until now, in all works the ESC algorithm was accompanied by PI controllers in the both current and speed control loops, and mostly the speed controller generates the current magnitude reference I_s^* [5], [6], [18]. In this section, we investigate the performance of the ESC algorithm based on the modified gradient algorithm in the presence of a new adaptive NN-based speed loop controller which generate i_q^* rather than I_s^*. The block diagram of the proposed modified gradient algorithm-based ESC for the IPMSM drive is shown in Fig. 1. In this figure, $a>0$ and ω_d are the amplitude and the frequency of the injected excitation signal Δi_d^*, respectively, ω_l and ω_h are the band passes of the low pass and high pass filters, respectively, the block MTPA calculates the nominal value of optimal i_d^* according to (6), f_{ts} is a signal which determines whether the system is in the transient or steady-state operation. According to this block diagram, during the transient operation, when $f_{ts}=0$, the output of the MTPA block which calculates i_d^* from the nominal MTPA equation (6) is used. Although the obtain $i_d^*<0$ by MTPA block is not optimal, injecting a negative i_d during transient operation for increasing the margins of saturation voltages is inevitable. On the contrary, when f_{ts} switches from zero to 1, which shows that the system has nearly entered the steady-state operation, the ESC algorithm starts working and estimating the error between the optimal i_d^* and the output of the MTPA block. Then, this

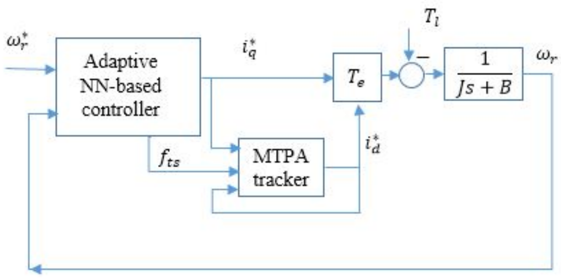

Fig. 2. Simplified model of the closed-loop system.

error is added to the output of MTPA block. To know about the mechanism of generating f_{ts}, you can refer to [19].

Remark 1: When the ESC starts working, the initial value of the integrator is set to zero.

A. Convergence analysis of the proposed ESC structure

In this section we investigate the convergence of the proposed ESC structure in presence of adaptive NN-based speed loop controller. Considering that current control loops are significantly faster than the speed control loop and ESC dynamics, from the speed dynamic point of view, it can be assumed that $i_q = i_q^*$, $i_d = i_d^*$. Therefore, the closed-loop system can be simplified as the block diagram shown in Fig. 2. In this figure, J is the moment of inertia, B is the viscous friction coefficient, ω_r is the mechanical speed of the motor, ω_r^* is the reference for ω_r, T_l is load torque, the block called "MTPA tracker" represents the proposed block diagram in Fig. 1, and the adaptive NN-based control law is as follows:

$$i_q^* = \hat{W}^T S(\omega_r) - k_1 e_r \tag{12}$$

where ω_r is the rotor speed, k_1 is a positive control parameter, $\hat{W} = [\hat{w}_1, \ldots, \hat{w}_n]^T$ is an estimate of the RBF NN ideal weight vector W^*, $S = [s_1, \ldots, s_n]^T$ is a vector of Gaussian RBFs with the following form $s_i(\omega_r) = \exp(-(\omega_r - \mu_i)^T(\omega_r - \mu_i)/\eta^2)$ with $\mu_i = [\mu_{i1}, \ldots, \mu_{ir}]^T, i = 1, ..., n$ is the center of the receptive field, $\mu = [\mu_1^T, ..., \mu_n^T]$ and η is the width of the Gaussian function, and $e_r = \omega_r - x_m$ where $x_m \in \mathbb{R}$ is the state of the following first-order reference system:

$$\dot{x}_m = -a_m x_m + a_m \omega_r^*(t) \tag{13}$$

where ω_r^* is the reference speed. The positive constant a_m describes the desirable speed response of the closed-loop dynamic. The σ-modified robust adaptive law for $\hat{W}$ is given by [16]

$$\dot{\tilde{W}} = \dot{\hat{W}} = \Gamma_a \left(-S(\omega_r)e_r - \sigma\hat{W}\right) \tag{14}$$

where $\tilde{W} = \hat{W} - W^*$, $\Gamma_a = \Gamma_a^T > 0$ is the adaptive gain and the small value $\sigma > 0$ is used to guarantee the robustness of the adaptive law.

Due to the nonlinear controller and also the nonlinear torque equation, the overall speed control loop has a nonlinear dynamic which makes the convergence analysis difficult. Therefore, to have an easier analysis we use small-signal linearization of the control loop around its steady-state operating point. A small-signal linear approximation is really fairly reasonable, because the amplitude of Δi_d^* is taken to be sufficiently small. For a fixed value of i_d^*, which is generated by MTPA tracker, the set P^* denotes the steady-state operating point as follows:

$$P^* = \left\{\omega_r = e_r^* + \omega_r^*, i_{d,q} = i_{d,q}^*, T_e = T_e^*(i_{d,q}^*), \hat{W} = \hat{W}^*\right\}$$

where e_r^* is a small tracking error due to the σ-modification. Linearizing the speed dynamic around P^* gives [5]:

$$J\frac{d}{dt}\Delta\omega_r = -B\Delta\omega_r + \Delta T_e$$

Where:

$$\Delta T_e = \left.\frac{\partial T_e}{\partial i_q}\right|^* \Delta i_q^* + \left.\frac{\partial T_e}{\partial i_d}\right|^* \Delta i_d^* \tag{15}$$

Therefore, in the frequency domain we have:

$$\Delta\omega_r = P(s)\Delta T_e, \qquad P = \frac{1}{J_m s + B_m} \tag{16}$$

And according to (12), the linearized model of the control input is:

$$\Delta i_q^* = -D_n\Delta\omega_r + C_n\Delta\hat{W} \tag{17}$$

where:

$$D_n = \left.\frac{\partial i_q^*}{\partial\omega_r}\right|^* = k_1 - \hat{W}^{*T}\left.\frac{\partial S}{\partial\omega_r}\right|^*, \qquad C_n = \left.\frac{\partial i_q^*}{\partial\hat{W}}\right|^* = S^{*T}$$

Also the linearized dynamics of the adaptive law is ($\Delta e_r = \Delta\omega_r$):

$$\Delta\dot{\hat{W}} = A_n\Delta\hat{W} - B_n\Delta\omega_r \tag{18}$$

where:

$$A_n = -\Gamma_a\sigma, \qquad B_n = \Gamma_a\left(\left.\frac{\partial S}{\partial\omega_r}\right|^* e_r^* + S^*\right)$$

Since e_r^* has a small value, it can be assumed that all the entries of matrix B_n are positive. According to (18,17), for Δi_q^* in the frequency domain we have:

$$\Delta i_q^* = -C(s)\Delta\omega_r \tag{19}$$

where

$$C = C_n(sI - A_n)^{-1}B_n + D_n = D_n\frac{s + \sigma\gamma_a + \frac{S^{*T}B_n}{D_n}}{s + \sigma\gamma_a}$$

Therefore, according to the linearized models (15,16 and 19), the small-signal block diagram of Fig. 3 can be drawn.

Considering this block diagram, we have:

$$\Delta i_q^* = k_t G(s)\Delta i_d^* \tag{20}$$

where

$$G(s) = \frac{bP(s)C(s)}{1 + bP(s)C(s)}, b = \left.\frac{\partial T_e}{\partial i_q}\right|^*$$

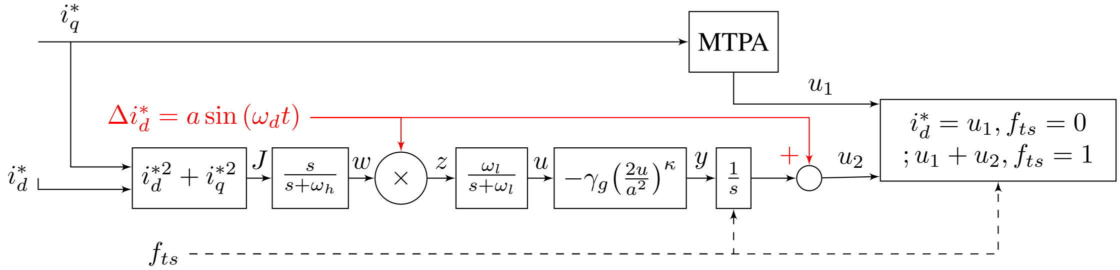

Fig. 1. Block diagram of the proposed MTPA control scheme.

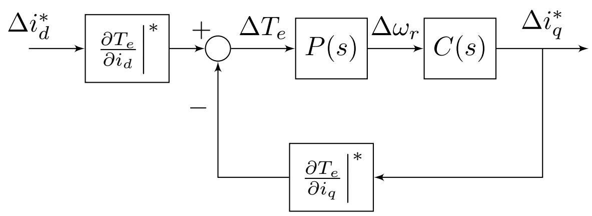

Fig. 3. Small-signal model of the simplified closed-loop system in Fig. 2

and $k_t = \left.\frac{\partial i_q}{\partial i_d}\right|^*$. Since the speed control loop is stable [19], it can be concluded that the transfer function $G(s)$ is also stable. According to Fig. 1, Since Δi_d^* is a sinusoidal signal and the closed-loop system is stable, it can be shown that in the steady-state:

$$\Delta i_q^* = k_t k_s a \sin(\omega_d + \varphi_s) \tag{21}$$

where $k_s = |G(j\omega_d)|$ and $\varphi_s = \angle G(j\omega_d)$. Considering the objective function as J from (2), one can write:

$$\begin{aligned}\Delta J &= \frac{\partial J}{\partial i_d^*}\Delta i_d^* + \frac{\partial J}{\partial i_q^*}\Delta i_q^* = 2i_d^*\Delta i_d^* + 2i_q^*\Delta i_q^* \\ &= 2ai_d^*\sin(\omega_d t) + 2ak_t k_2 i_q^*\sin(\omega_d t + \varphi_s)\end{aligned} \tag{22}$$

The signal ΔJ from (22) is, in fact, the output of the high pass filter [11]. Therefore, according to the block diagram of Fig. 1, during the steady-state operation we have $w \simeq \Delta J$. As a resul:

$$\begin{aligned}z &= wa\sin(\omega_d t) = a^2 i_d^* \sin(\omega_d t)^2 \\ &+ 2a^2 k_t k_2 i_q^* \sin(\omega_d t + \varphi_s)\sin(\omega_d t) \\ &= a^2(i_d^* + k_t k_s \cos\varphi_s i_q^*) - a^2 k_t k_s i_q^* \cos(2\omega_d t + \varphi_s) \\ &- a^2 i_d^* \cos 2\omega_d t\end{aligned} \tag{23}$$

As can be seen, the above equation has a dc part and an ac part with a frequency of $2\omega_d$. Thus, if the band pass of the law pass filter ω_l is low enough, then in the steady-state we will have:

$$u \simeq a^2(i_d^* + k_t k_s \cos\varphi_s i_q^*) \tag{24}$$

Therefore, setting $u = 0$, the following equation is achieved:

$$i_d^* + k_t k_s \cos\varphi_s i_q^* = 0 \tag{25}$$

Besides, if we would assume that the system is static, then the objective of ESC was to estimate the extremum point of the static function $T_e^* = T(i_d^*, i_q^*)$ and it could be easily shown that setting $u = 0$ would result in:

$$i_d^* + k_{ts} i_q^* = 0 \tag{26}$$

In fact the above equation gives the optimal value of i_d^* for a given i_q^*, because it is proved that in the case of static functions, the ESC algorithm converges to the extremum point [11], therefore the equation will hold in the steady-state. On the other hand, the closed loop dynamical system of Fig. 2 cannot be guaranteed to meet $k_s \cos\varphi = 1$. Therefore, considering that (26) gives the optimal solution, holding (25) means that the proposed ESC does not completely converge to the optimal point. Since (20) is a proper and stable transfer function, for a big value ω_d it can be concluded that $k_s \simeq 0$. Therefore, according to (25) it can be concluded that implementing the proposed ESC scheme with a very high ω_d cause i_d^* to converge to zero. On the other hand, since the transfer function (20) has no zeros or poles at the origin, for small values of ω_d it can be easily shown that $\varphi \simeq 0$. Thus, in low frequencies, that $k_s \cos\varphi \simeq 1$, the equation (25) is almost equivalent to (26), and the ESC algorithm converges to a small neighborhood of the optimal point.

V. Simulation Results

To investigate the performance of the proposed MTPA control method, some simulations are performed in MATLAB/Simulink. The nominal parameters of the IPMSM are: $R_s = 1.9\Omega$, $L_q = 31mH$, $L_d = 15.2mH$, $\lambda_m = 0.227\frac{V.s}{rad}$, $J = 0.0005kg.m^2$, $B = 0.005\frac{N.m.s}{rad}$. Also, the parameters of the speed controller and MTPA control are: $a_m = 30$, $\mu = [-30 : 10 : 200]$, $k_1 = 2, \Gamma_a = 20I_{26\times 26}$, $\sigma = 0.002$, $\eta = 10$, $a = 0.025$ and $\gamma_g = 0.37$. The proposed PI and adaptive control method in [19] is employed in current control loops. To model uncertainties, the actual L_d, L_q and λ_m are selected as 80%, 50% and 70% of their nominal values, respectively.

First, we study the performance of the MTPA control scheme with conventional gradient algorithm ($\kappa = 1$) for different values of ω_d. As shown in Fig. 5, for small values of ω_d the MTPA control converges to the optimal i_d^* with a small tracking error. As was expected, for very big ω_d, the algorithm converges to zero. It is clear that the ESC with $\kappa = 1$ is extremely slow. The performance of the finite-time gradient based-ESC for $\kappa = 0.6$ is shown in Fig. 4. While the steady-state responses of the both conventional and proposed

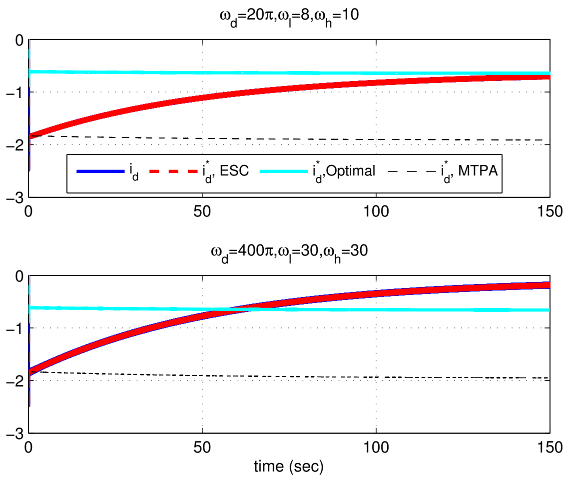

Fig. 4. The performance of MTPA control for different values of ω_d; $\kappa = 1$

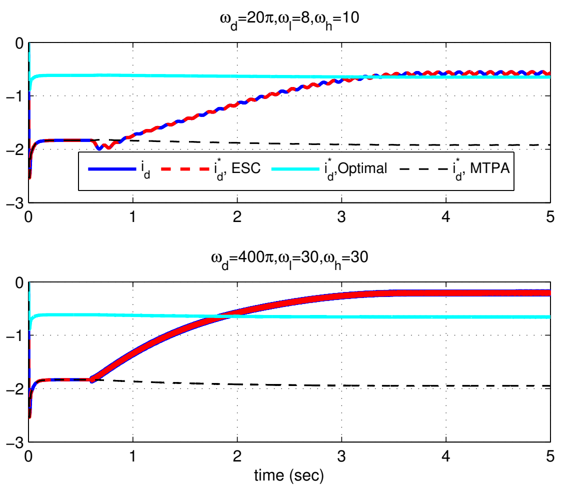

Fig. 5. The performance of MTPA control for different values of ω_d; $\kappa = 0.6$

algorithms are almost the same, the convergence speed is significantly improved by using the modified gradient algorithm.

VI. Conclusion

First, for the application of the MTPA control, a finite-time gradient algorithm was proposed, analyzed and, then, utilized in the gradient-based ESC algorithm to estimate the MTPA operating point of an IPMSM. A convergence analysis was also performed in the presence of a new adaptive NN-based speed loop controller. Simulation results were presented to study the performance of the proposed MTPA control algorithm. The results show the faster convergence of the proposed MTPA control than the conventional approaches. It was also verified that this method does not provide a complete convergence to the optimal point, and there is always a bounded tracking error which can be reduced by lowering the frequency of the injected excitation signal.

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