Directivity Optimization of Fractal Antenna Arrays Using PSO Algorithm

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Abstract—**Self-similarity property of fractal shapes allows fractal antennas to be compact, multiband, or even wideband. Applying fractals to antenna arrays also develops multiband or broadband arrays. In this paper, we have investigated the multiband behaviour of fractal antenna arrays based on cantor set linear generators, which is impossible for classical arrays to achieve. After that, we have optimized the designed structure for maximum directivity using Particle Swarm Optimization (PSO) algorithm, both for theoretical and full-wave cases. A comparison between two methods is also included. This comparison helps to figure out that analytical formulas for fractal arrays are not only super-fast, but also completely precise despite of their simple form and hence their results are completely reliable and accurate.**

Keywords-Array Factor (AF); Directivity; Fractal; Generator; Particle Swarm Optimization (PSO)

I. INTRODUCTION

Over the past decade, data consumption of users over mobile broadband has continued to explode due to powerful new networks, new handheld devices, and close to two million mobile applications [1]. This challenge has led to new opportunities for antenna industries to find novel ways of handling that vast amount of data rates. In fact, using both compact and multiband antennas is the last and definitely the best way to simultaneously see all the demands of users at different frequencies. Undoubtedly, fractal antennas are the key solution to this problem.

From the time that Mandelbrot used the term "fractal" to describe the shapes that are self-similar in their structures [2], a wide variety of applications for fractals has been successfully introduced in many branches of science and engineering. Our branch of intense interest is called "fractal electrodynamics", in which new problems of radiation, propagation, and scattering are solved by means of fractal geometries [3-6].

For years, there has been considerable interest in the possibility of developing new types of antennas that employ fractal concepts in their design, formally called "fractal antenna engineering". Fortunately, self-similarity of fractals allows for smaller and multiband/broadband antennas because different parts of the antennas are similar to each other at different scales [7]. Applying the concept to antenna arrays also introduces multiband/broadband arrays that can be optimized for their gain or directivity.

A rich class of fractal arrays exists that can be formed by repetition of a subarray, officially called generator, or initiator. In fact, a generating subarray is a small array at scale one (P = 1) which is used to construct larger arrays at higher scales. The final fractal array is then produced by copying and scaling of the generator. As a matter of fact, the fractal arrays are known to be arrays of arrays [8]. AF of these fractal arrays can be expressed in the general form

$$AF_P(\psi) = \prod_{p=1}^{P} GA(\delta^{p-1}\psi) \tag{1}$$

where $GA(\psi)$ represents the array factor associated with the generating subarray. The parameter δ is the scale factor that controls the grows of the array with each iteration [9-10]. The expression given in (1) is simply the product of scaled versions of a generating subarray factor and hence, is a formal statement of the *pattern multiplication theorem* for fractal arrays.

When $P \to \pm\infty$, then

$$\begin{aligned} AF\left(\delta^{\pm q}\psi\right) &= \prod_{p=-\infty}^{\infty} GA\left(\delta^{p\pm q-1}\psi\right) \\ &= \prod_{n=-\infty}^{\infty} GA\left(\delta^{n-1}\psi\right) = AF\left(\psi\right) \qquad q = 0, 1, 2, \ldots \end{aligned} \tag{2}$$

which means that the array repeats its radiation pattern over completely known discrete frequencies, called "frequency shift property" [11]. These frequencies can be expressed as

$$f_{\pm q} = \delta^{\pm q} f_0 \qquad q = 0, 1, \ldots \tag{3}$$

in which f_0 is the design frequency.

A simple but powerful design methodology is based on the generators that use cantor set linear arrays, in which we can control the behavior of the arrays by turning on (i.e. 1) or off (i.e. 0) the individual elements. The normalized array factor for these arrays may be expressed in the form

$$GA(\psi) = \left(\frac{2}{\delta+1}\right) \frac{\sin\left[0.5(\delta+1)\psi\right]}{\sin\psi} \tag{4}$$

where $\delta = 2n+1\ (n = 1,2,\ldots)$, $\psi = kd\left(\cos\theta - \cos\theta_0\right)$, d is the space between elements, and $k = 2\pi/\lambda$ is the wavenumber.

The fractal array factor for a particular stage of growth P may then be derived directly from (1). The resulting expression in normalized form is

$$AF_P(\psi) = \left(\frac{2}{\delta+1}\right)^P \prod_{p=1}^{P} \frac{\sin\left[0.5(\delta+1)\delta^{p-1}\psi\right]}{\sin\left[\delta^{p-1}\psi\right]} \tag{5}$$

II. PARTICLE SWARM OPTIMIZATION

Particle Swarm Optimization [12], commonly known as PSO, is a metaheuristic self-organizing algorithm developed by Kenedy and Eberhart in 1995 which is inspired by swarms in which a group of particles cooperate with each other, just by 3 simple rules, in order to find the best solution. The rules are

1. moving in the same direction of neighbors
2. being close to the neighbors
3. avoiding colliding with neighbors

The velocity and position of particles is updated with the equations

$$\begin{aligned} v_n &= (w \times v_n) + c_1 r_1 \left(g_{best,n} - x_n\right) \\ &\quad + c_2 r_2 \left(p_{best,n} - x_n\right) \end{aligned} \tag{6}$$

$$x_n = x_n + v_n \tag{7}$$

in which w is a weight parameter to control the tendency of particles to move, r_1 and r_2 are two random numbers in $[0,1]$ to show the randomness of movements of particles, $c_1 \simeq c_2 \simeq 2$ are used to control the amount of attraction of particles to each other, p_{best} is the best solution of each particle, and g_{best} is the best solution of the swarm found so far.

w is also modified to better manage the inertia of the particles. In the linear case,

$$w_n = 0.9 - 0.5\frac{n}{maxIt} \qquad n = 0, 1, \ldots, maxIt \tag{8}$$

in which $maxIt$ is the maximum number of required iterations. The flowchart of this algorithm is shown in Figure 1.

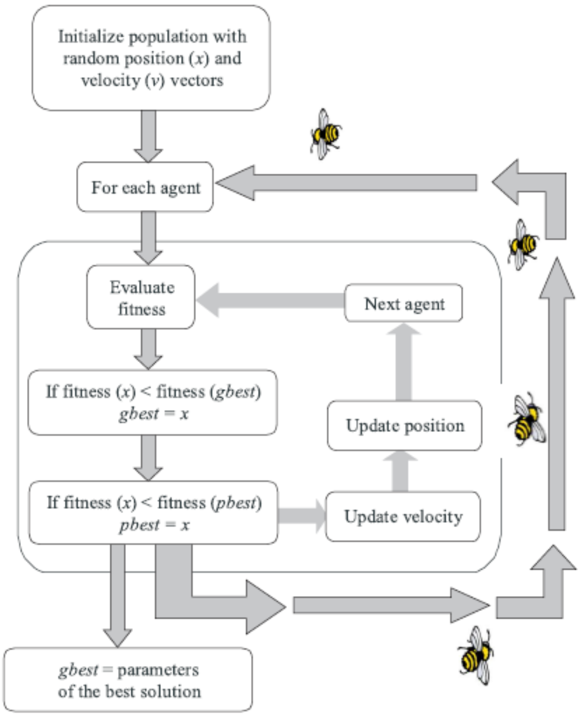
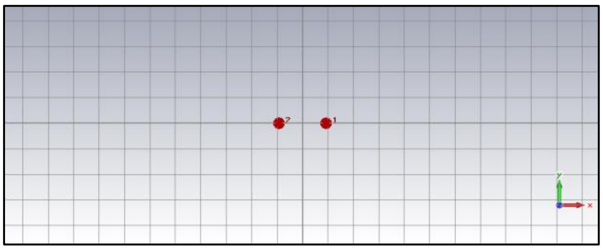


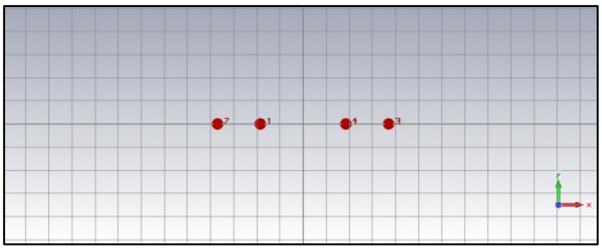
Figure 1. PSO flowchart

III. DESIGN & ANALYSIS PROCEDURES

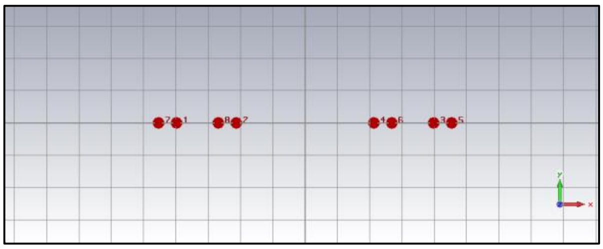
Consider a simple 3-element linear cantor set subarray in the form of 101, as shown in Figure 2(a). If we use this as the generator for our fractal array, then the next stage would be the configuration of 101000101, shown in Figure 2(b). As mentioned, this is achieved by replacing each of the elements of the subarray by an exact copy of original subarray. The next stage (101000101000000000101000101) is also shown in Figure 2(c).



(a)



(b)



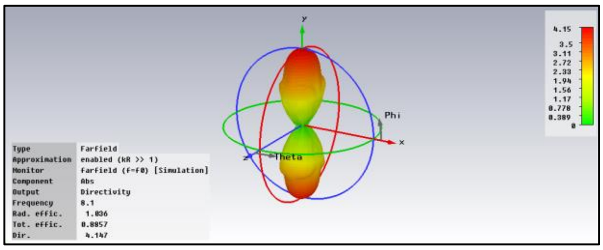
(c)

Figure 2. Fractal antenna array based on 101 cantor set subarray (a) Stage 1 (b) Stage 2 (c) Stage 3

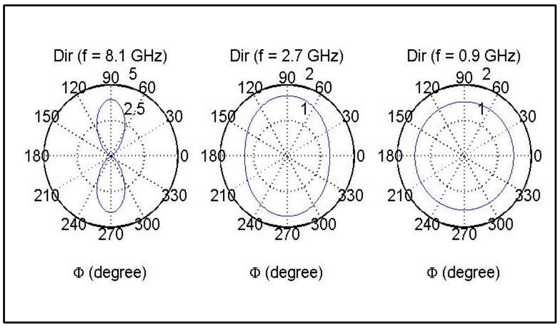
After investigating the multiband behavior of the final array, which is simply composed of 4 or 8 half-wave dipole antennas at center frequency of 8.1 GHz with scaling factor $\delta = 3$, it will be then optimized for maximum directivity using PSO algorithm.

IV. RESULTS

Figure 3 shows the radiation pattern of cantor set subarray. As it can be seen from Figure 4, the stage 2 has replicated the pattern of generator in a new frequency. In other words, by using stage 2, the radiation pattern of generator at design frequency is now shifted to a new frequency, which is δ times less than the design frequency, as predicted by (3). At the same time, the same pattern is also reproduced at the design frequency, except with some side-lobes.

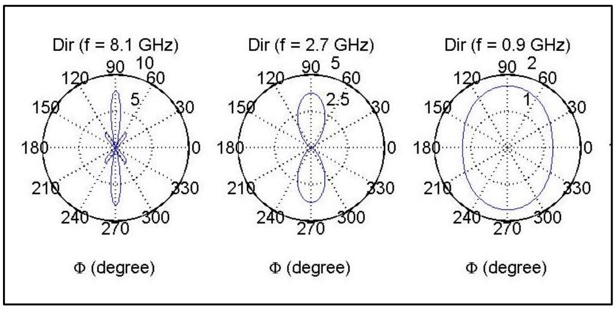


(a)

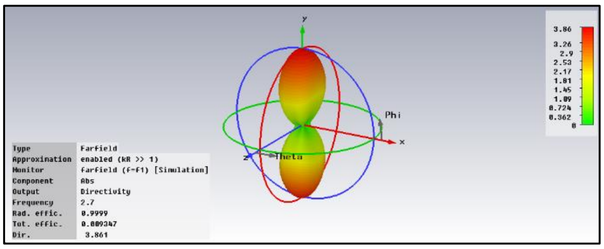


(b)

Figure 3. Radiation patterns of cantor set generator (a) Full-wave (b) Fractal concept



(a)



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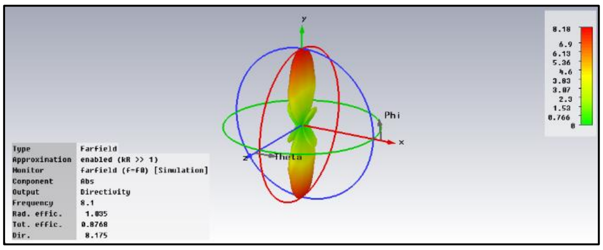


(c)

Figure 4. Radiation patterns of Stage 2
(a) Fractal concept
(b) Full-wave (New frequency: 2.7 GHz)
(c) Full-wave (Design frequency)

Figure 5 shows the radiation patterns of stage 3 fractal array. As seen, the stage 3 has now replicated the pattern of generator in another frequency. In other words, by using the stage 3, each of the radiation patterns of Figure 4(a) is now shifted to the next new frequency, which is δ times less than the previous frequency, as predicted by (3). At the same time, the same pattern is also reproduced at the original frequency, except with some side-lobes.

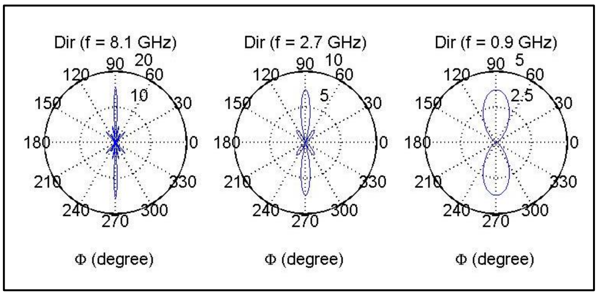
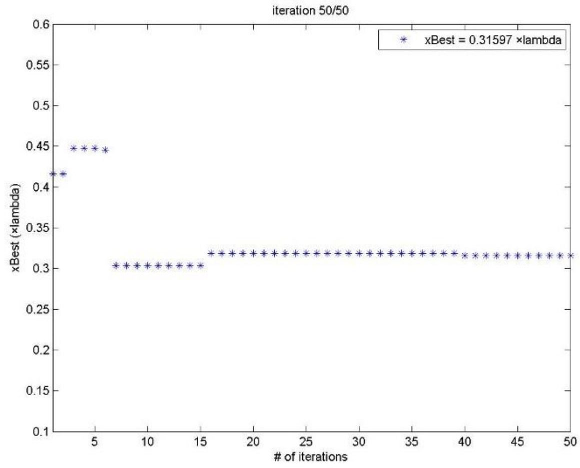
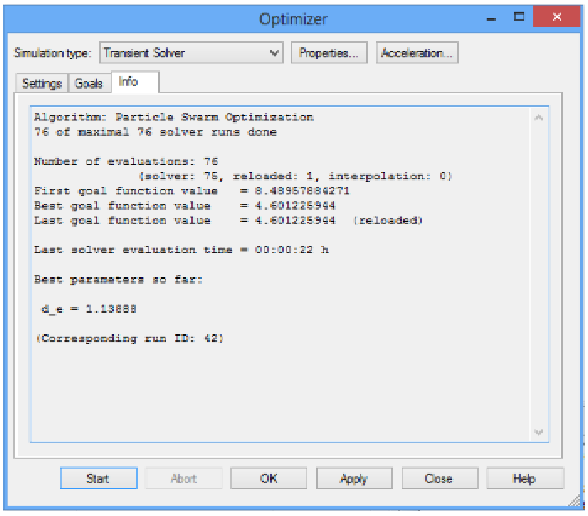


Figure 5. Radiation patterns of Stage 3

Figure 6(a) shows the convergence of PSO algorithm for 5 particles and 50 iterations of optimization. Figure 5(b) also shows the convergence of CST's PSO optimization engine with 5 particles and 15 iterations of optimization.



(a)

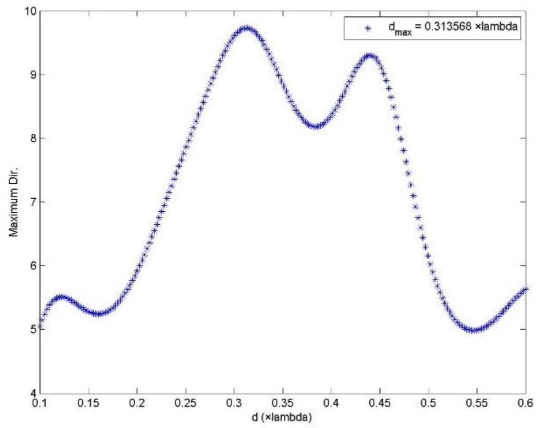


(b)

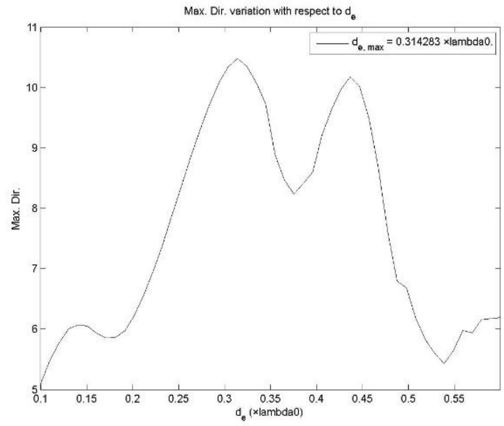
Figure 6. Convergence of PSO algorithm
(a) Theory (b) CST STUDIO SUITE

As it can be seen from Figure 6, theoretical PSO has converged to $d \simeq 0.316\lambda$, and CST has converged to $d \simeq 0.307\lambda$. This little difference is obvious because if we look at Figure 2(c), for example, we see that with each iteration, some elements become too close to each other which indeed increases the coupling. The most important thing is that both methods have nearly converged toward a similar distance.

Figure 7(a) shows the theoretical maximum directivity [13] of the array. Also Figure 7(b) shows the full-wave maximum directivity for different values of d. These two figures are another proof for the correct convergence of optimization processes.



(a)



(b)

Figure 7. Max. directivity for different values of d
(a) Fractal concept (b) Full-wave

As Figure 7 shows, both theoretical and full-wave methods have similar behaviours and both have converged to the correct values, and this shows that the optimization is completely successful. So, the designed array will have its maximum directivity for

$$d \simeq 31\lambda \tag{9}$$

V. CONCLUSION

In this work, we used a 3-element linear cantor set generator to produce a fractal antenna array at different stages, so that it can work in two or three bands of frequencies, which is nearly impossible for classic arrays to achieve. Then, PSO algorithm was successfully used to optimize the distance between the elements of the subarray to obtain the maximum directivity from the design. For this, we treated the structure with both analytical and full-wave analyses to get the best distance, which is $d \simeq 0.31\lambda$.

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