Analytical Calculation of the Resonant Frequencies for a Corner Cut Square SIW Cavity

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Abstract—**In this paper, the resonant frequencies of a dual mode square SIW cavity with a corner cut perturbation are calculated, analytically. Since introducing the perturbation makes small changes in the field pattern of this cavity, the perturbation theory can be used to calculate the resonant frequencies. Using this theory, the first degenerate modes of the dual mode cavity are calculated, approximately. Also, the mode matching technique is used to calculate these resonant frequencies as the eigenvalues of the structure. The analytically obtained results are verified by comparison with full wave simulation results. For the perturbation size less than the third of diameter of the square cavity, the error between resonant frequencies obtained by perturbation theory and HFSS is less than 1%. The mode matching results are also in a good agreement with full wave simulation results.**

Keywords- mode matching technique; perturbation theory; resonant frequency; SIW cavity

I. INTRODUCTION

Well-known Substrate Integrated Waveguide (SIW) technology, as a promising alternative for the classical waveguide and planar structures, has found many applications in the realization of microwave and millimeter wave devices [1-2]. The structure of SIW is consisting of two rows of metallic pins which implement electrical contact between the top and bottom metallic layers of the substrate. For small distance between pins, the SIW operates like as a dielectric-filled waveguide. So far, several passive and active components are realized by using of SIW technology such as antennas [3], filters [4-6], and etc.

Dual mode filters which have a low profile structure and high performance operation conventionally are realized by introducing a perturbation in a cavity which supports degenerate modes [7]. If the electromagnetic field variation in the cavity due to introducing the perturbation is negligible, the perturbation theory can be used to calculate the resonant frequency of the perturbed cavity [8]. So far, several dual mode SIW filters, by using of circular and square shape SIW cavities are proposed [9-11]. The perturbation of these cavities is in the form of some metallic pins located inside the cavity [10] or a slot etched on the metallic layer of the substrate [11].

Although the commercial full wave softwares, like as ANSYS HFSS and CST Microwave Studio, can be used easily to simulate the SIW components precisely, these general softwares are time-consuming especially in the optimization of sensitive components to frequency like as filters. Many attempts have been made to model the SIW structure based on the numerical methods such as Boundary Integral-Resonant Mode Extraction (BI-RME) [12] and Method of Moment (MoM) [13] for equivalent circuit extraction and also Mode Matching Technique (MMT) for dispersion analysis [14-16]. Until now, a number of works have been reported for simulating the SIW filters and SIW diplexers by using the numerical methods such as MMT [17-18]. The main focus of these papers is on obtaining the scattering parameters of the device.

A corner cut square SIW cavity is proposed by the authors in [19] and using this cavity a fourth order dual mode SIW filter is realized [20]. In this paper, the resonant frequencies of the first degenerate modes of this cavity are calculated analytically by the perturbation theory and also by MMT. Structure of the corner cut square SIW cavity is shown in Fig. 1(a) and the equivalent classical waveguide of this cavity is depicted in Fig. 1(b). Equivalent width of this cavity, a', is determined by the following formula [14]

$$a' = a - p\left(0.766\, e^{0.4482\, d/p} - 1.176\, e^{-1.214\, d/p}\right) \tag{1}$$

where a is the width of cavity and p and d are the center to center distance of pins and diameter of pins, respectively. The first degenerate modes of the square SIW cavity are TE_{102} and TE_{201} modes or any arbitrary linear combination of them, for instance sum and difference of these modes with equal coefficients. It should be noted that since the height of substrate is much smaller than wavelength, there is no field variation in y direction. Also, TM modes cannot propagate in the SIW due to leakage from gaps between pins [1]. After introducing the corner cut perturbation, the field pattern of the resonant modes are symmetric with respect to the diameter of A-A'. Therefore, as discussed in [20] the new resonant modes are odd and even modes which are similar to the sum and difference modes with equal coefficients, except around the corner cut perturbation. In the following, the resonant frequencies of the odd and even modes are calculated.

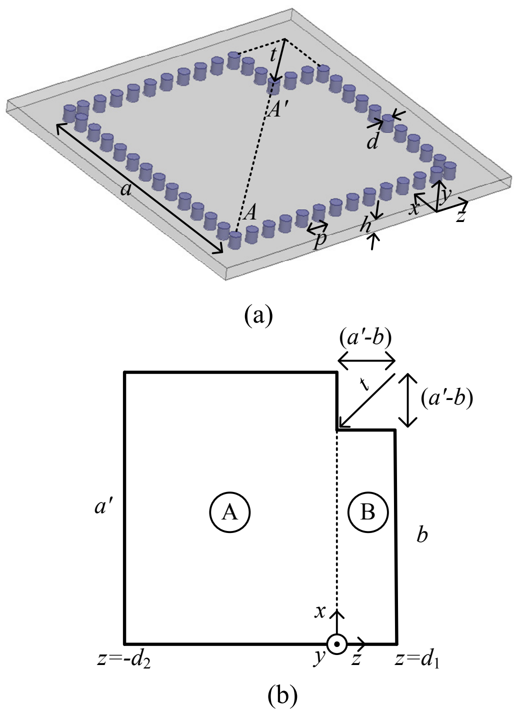

Figure. 1. (a) Corner cut square SIW cavity and (b) its equivalent waveguide.

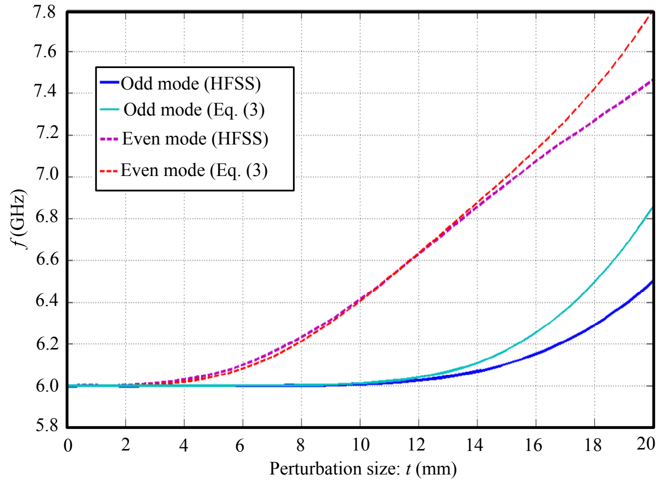

Figure. 2. Comparison of resonant frequencies of odd and even modes versus t obtained by HFSS and the perturbation theory (with a' = 29.88mm, h = 1.54mm, p = 2.4mm, d = 1.4mm, and ε_r = 3.5)

II. Calculation Of The Resonant Frequencies By Perturbation Theory

When a perturbation is introduced in a cavity, the resonant frequency of the perturbed cavity can be obtained by the following formula [8]

$$f - f_0 = \frac{j\oiint_{\Delta S} \left(\overline{H} \times \overline{E}_0^*\right)\cdot d\overline{s}}{2\pi \iiint_V \left(\varepsilon \overline{E} \cdot \overline{E}_0^* + \mu \overline{H} \cdot \overline{H}_0^*\right) dv} \tag{2}$$

where f_0, $\overline{E}_0$ and $\overline{H}_0$ are the resonant frequency, electric field and magnetic field of early cavity, respectively and f, $\overline{E}$ and $\overline{H}$ are for the perturbed cavity. Also, ΔS is the surface of the perturbation and V is the volume of the early cavity. Because $\overline{E}$ and $\overline{H}$ are unknown, the above relation cannot be used straightly. It is shown that if the electromagnetic field variation in the cavity due to introducing the perturbation is negligible, (2) can be rewritten as follows, approximately [8]

$$\frac{f - f_0}{f_0} \cong \frac{\iiint_{\Delta V} \left(\mu\left|\overline{H}_0\right|^2 - \varepsilon\left|\overline{E}_0\right|^2\right) dv}{\iiint_V \left(\mu\left|\overline{H}_0\right|^2 + \varepsilon\left|\overline{E}_0\right|^2\right) dv} \tag{3}$$

where ΔV is the volume of the perturbation. As shown in [19], particularly for small values of perturbation size, t, the perturbation causes a negligible change on the field pattern of the odd and even modes with respect to the sum and difference modes. Therefore, (3) can be used for the resonant frequency calculation of the odd and even modes. Fig. 2 illustrates a comparison between resonant frequencies obtained by HFSS and perturbation theory versus the perturbation size. A good agreement between results can be seen particularly for small t. For values of t less than the third of diameter of the square cavity, the error between resonant frequencies obtained by HFSS and perturbation theory is less than 1%.

III. Calculation Of The Resonant Frequencies By Mode Matching Technique

In order to calculate the resonant frequencies of the dual mode cavity, as is seen in Fig. 1(b), the cavity is divided to two regions A and B. Therefore, for region A, that is $-d_2 < z < 0$ and $0 < x < a'$, by considering of M modes, the electromagnetic fields are as follows [21]

$$\begin{cases} E_y = \sum_{n=1}^{M} \left(A_n^+ \sin(\frac{n\pi}{a'}x) e^{-\gamma_{an} z} + A_n^- \sin(\frac{n\pi}{a'}x) e^{+\gamma_{an} z} \right) \\ H_x = \sum_{n=1}^{M} \left(A_n^+ Y_{an} \sin(\frac{n\pi}{a'}x) e^{-\gamma_{an} z} - A_n^- Y_{an} \sin(\frac{n\pi}{a'}x) e^{+\gamma_{an} z} \right) \end{cases} \tag{4a}$$

where γ_{an} and Y_{an} are the propagation constant and admittance of nth mode in the region A, respectively which are determined as

$$Y_{an} = \frac{\gamma_{an}}{j\omega\mu} \;,\; \gamma_{an} = \sqrt{\omega^2 \mu\varepsilon - \left(\frac{n\pi}{a'}\right)^2} \tag{4b}$$

Similarly, for region B, that is $0 < z < d_1$ and $0 < x < b$, we have

$$\begin{cases} E_y = \sum_{n=1}^{M} \left(B_n^+ \sin(\frac{n\pi}{b}x) e^{-\gamma_{bn} z} + B_n^- \sin(\frac{n\pi}{b}x) e^{+\gamma_{bn} z} \right) \\ H_x = \sum_{n=1}^{M} \left(B_n^+ Y_{bn} \sin(\frac{n\pi}{b}x) e^{-\gamma_{bn} z} - B_n^- Y_{bn} \sin(\frac{n\pi}{b}x) e^{+\gamma_{bn} z} \right) \end{cases} \tag{5a}$$

where γ_{bn} and Y_{bn} corresponding to the region B, are given by

$$Y_{bn} = \frac{\gamma_{bn}}{j\omega\mu} \;,\; \gamma_{bn} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{n\pi}{b}\right)^2} \quad (5b)$$

By applying boundary condition of $E_y = 0$ in (4a) and (5a) at $z = -d_2$ and $z = d_1$, it can be written

$$A_n^+ = \rho_{an} A_n^- \quad , \quad \rho_{an} = -e^{-2\gamma_{an} d_2} \quad (6a)$$

$$B_n^- = \rho_{bn} B_n^+ \quad , \quad \rho_{bn} = -e^{-2\gamma_{bn} d_1} \quad (6b)$$

Also, based on continuity of tangential components at the boundary of regions A and B, that is z=0, from (4a) and (5a) we have

$$\sum_{n=1}^{M} \sin(\frac{n\pi}{a'}x)\left(A_n^+ + A_n^-\right) = \begin{cases} \sum_{n=1}^{M} \sin(\frac{n\pi}{b}x)\left(B_n^+ + B_n^-\right), & 0 < x < b \\ 0 & , \; b < x < a' \end{cases} \quad (7a)$$

$$\sum_{n=1}^{M} \sin(\frac{n\pi}{a'}x) Y_{an}\left(A_n^+ - A_n^-\right) = \sum_{n=1}^{M} \sin(\frac{n\pi}{b}x) Y_{bn}\left(B_n^+ - B_n^-\right), \; 0 < x < b \quad (7b)$$

Now by substituting of (6a) in (7a) and multiplying both sides of the equation by $\sin(\frac{m\pi}{a'}x)$ and then, integration from z = 0 to $z = a'$ we have

$$(\frac{a'}{2})(1+\rho_{am})A_m^- = \sum_{n=1}^{M} H_{mn}(1+\rho_{bn})B_n^+ \quad , \quad m = 1,2,..., M \quad (8a)$$

where H_{mn} is defined as

$$H_{mn} = \int_0^b \sin(\frac{m\pi}{a'}x)\sin(\frac{n\pi}{b}x)dx \quad (8b)$$

Similarly, by substituting of (6b) in (7b) and multiplying both sides of the equation by $\sin(\frac{m\pi}{b}x)$ and then, integration from z = 0 to $z = b$ it can be obtained

$$-\sum_{n=1}^{M} \tilde{H}_{mn} Y_{an}(1-\rho_{an})A_n^- = (\frac{b}{2})Y_{bm}(1-\rho_{bm})B_m^+ \quad , \quad m = 1,2,..., M \quad (9a)$$

where $\tilde{H}_{mn}$ is defined as

$$\tilde{H}_{mn} = \int_0^b \sin(\frac{m\pi}{b}x)\sin(\frac{n\pi}{a'}x)dx \quad (9b)$$

Equations (8a) and (9a) can be written in the matrix form as follows

$$\frac{a'}{2}\left[\mathbf{R}_{a,p}\right]_{M\times M}\left[\mathbf{a}^-\right]_{M\times 1} = \left[\mathbf{H}\right]_{M\times M}\left[\mathbf{R}_{b,p}\right]_{M\times M}\left[\mathbf{b}^+\right]_{M\times 1} \quad (10a)$$

$$-\left[\mathbf{H}\right]^t_{M\times M}\left[\mathbf{Y}_a\right]_{M\times M}\left[\mathbf{R}_{a,m}\right]_{M\times M}\left[\mathbf{a}^-\right]_{M\times 1} = \frac{b}{2}\left[\mathbf{Y}_b\right]_{M\times M}\left[\mathbf{R}_{b,m}\right]_{M\times M}\left[\mathbf{b}^+\right]_{M\times 1} \quad (10b)$$

where the above matrices are defined as follows

(11)

$$\left[\mathbf{a}^-\right] = \begin{bmatrix} A_1^- \\ A_2^- \\ \vdots \\ A_M^- \end{bmatrix} \quad , \quad \left[\mathbf{b}^+\right] = \begin{bmatrix} B_1^+ \\ B_2^+ \\ \vdots \\ B_M^+ \end{bmatrix}$$

$$\left[\mathbf{R}_{a,p}\right] = \begin{bmatrix} 1+\rho_{a1} & 0 & \cdots & 0 \\ 0 & 1+\rho_{a2} & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & \cdots & & 1+\rho_{aM} \end{bmatrix}, \quad \left[\mathbf{R}_{b,p}\right] = \begin{bmatrix} 1+\rho_{b1} & 0 & \cdots & 0 \\ 0 & 1+\rho_{b2} & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & \cdots & & 1+\rho_{bM} \end{bmatrix}$$

$$\left[\mathbf{R}_{a,m}\right] = \begin{bmatrix} 1-\rho_{a1} & 0 & \cdots & 0 \\ 0 & 1-\rho_{a2} & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & \cdots & & 1-\rho_{aM} \end{bmatrix}, \quad \left[\mathbf{R}_{b,m}\right] = \begin{bmatrix} 1-\rho_{b1} & 0 & \cdots & 0 \\ 0 & 1-\rho_{b2} & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & \cdots & & 1-\rho_{bM} \end{bmatrix}$$

$$\left[\mathbf{Y}_a\right] = \begin{bmatrix} Y_{a1} & 0 & \cdots & 0 \\ 0 & Y_{a2} & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & \cdots & & Y_{aM} \end{bmatrix} \quad , \quad \left[\mathbf{Y}_b\right] = \begin{bmatrix} Y_{b1} & 0 & \cdots & 0 \\ 0 & Y_{b2} & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & \cdots & & Y_{bM} \end{bmatrix}$$

Finally by eliminating of [$\mathbf{b}^+$] from (10a) and (10b), it can be written as

(12)

$$\left(\frac{a'b}{4}\left[\mathbf{Y}_b\right]\left[\mathbf{R}_{b,m}\right]\left[\mathbf{R}_{b,p}\right]^{-1}\left[\mathbf{H}\right]^{-1}\left[\mathbf{R}_{a,p}\right] + \left[\mathbf{H}\right]^t\left[\mathbf{Y}_a\right]\left[\mathbf{R}_{a,m}\right]\right)\left[\mathbf{a}^-\right] = 0$$

The resonant frequencies of the cavity are the frequencies that make the determinant of coefficients matrix in the left side of [$\mathbf{a}^-$] in the above relation equal to zero.

A MATLAB code is written to realize the above mentioned procedure based on MMT to calculate the resonant frequencies of the corner cut square cavity. Table I shows the resonant frequencies of the cavity obtained by MMT with assuming 10 modes and also obtained by using of HFSS for different values of t in which a good agreement between results can be seen. To study the effect of the number of considered modes in MMT, the calculated resonant frequencies are listed in Table II for different values of M with assuming t = 9.2mm. It can be concluded that the results almost do not change for M higher than 10 and therefore, 10 modes seems to be enough to calculate the resonant frequencies. In this case, using a normal

TABLE I. RESONANT FREQUENCIES OBTAINED BY HFSS AND MMT (WITH a' =29.88mm AND M = 10)

t (mm)	Odd mode (HFSS) (GHz)	Odd mode (MMT) (GHz)	Even mode (HFSS) (GHz)	Even mode (MMT) (GHz)
0	5.997	5.997	5.998	5.997
2.83	5.999	5.997	6.007	6.004
5.65	5.999	5.998	6.081	6.074
8.48	6.001	6.000	6.270	6.260
11.31	6.017	6.015	6.553	6.539
14.14	6.072	6.071	6.872	6.860

TABLE II. RESONANT FREQUENCIES OBTAINED BY MMT (WITH a' =29.88mm AND t = 9.2mm)

M	Odd mode (GHz)	Even mode (GHz)
1	6.101	---
2	6.003	6.271
3	6.002	6.307
6	6.002	6.317
10	6.002	6.322
20	6.002	6.323
HFSS	6.003	6.331

CPU, calculation by MMT takes around one minute which is one-sixth of the time elapsed by HFSS.

IV. CONCLUSION

The resonant frequencies of a corner cut square SIW cavity has been studied by using of perturbation theory and mode matching technique, in this paper. Perturbation theory is in acceptable agreement with full wave simulation results, particularly when the perturbation size is small and therefore, variations of field patterns are negligible. Mode matching technique is much faster than full wave simulation of cavity to obtain the resonant frequencies with the same accuracy. Assuming 10 modes in mode matching technique seems to be enough to calculate the resonant frequency of the cavity.

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