Performance Analysis of MIMO-FSO Communication Systems in Gamma-Gamma Turbulence Channels with Pointing Errors

Fatemeh Akhavan Mahdavi
MSc in Telecommunications Engineering
Department of Electrical and computer Engineering
Islamic Azad University, Science and Research Branch.
Tehran, Iran
Email: Fatemeh.akhavan@srbiau.ac.ir

Hossein Samimi
Assistant Professor
Iran Telecom Research Center
Tehran, Iran
Email: hossein-samimi@itrc.ac.ir

Abstract—**In this paper, we analyze the performance of multiple-input multiple-output (MIMO) free-space optical (FSO) communication systems over Gamma-Gamma (GG) turbulence channels with pointing errors. We have assumed repetition coding across lasers at the transmitter and equal gain combining (EGC), at the receiver. Performance analysis of such system needs the statistics of the sum of GG-distributed random variables (RVs), which, here, is approximated by the α-μ distribution. Numerical results show the accuracy of the developed approximation method. Moreover, the proposed method can be applied efficiently for different channels characterized by GG turbulence, pointing error and attenuation. Based on the derived results, we have analyzed the performance of MIMO-FSO system employing EGC diversity technique considering the combined effects of turbulence-induced fading, pointing error and path loss, and investigated the effects of the different design parameters on the performance of the system.**

Keywords- multiple-input multiple-output(MIMO); free-space optic (FSO); Gamma-Gamma(GG); pointing error; turbulence; equal gain combining (EGC); α-μ distribution; path loss;

I. INTRODUCTION

In the recent few years, the exponential grows in the demand of high throughput and low latency application has led to congestion in conventionally used radio frequency (RF) spectrum and arises a need to shift from RF carrier to optical carrier. Wireless optical communication (WOC), also known as free-space optical (FSO) communication, has the attractive features such as license-free operation, high security, extremely high bandwidth, ease of deployment and inexpensive setup. Moreover, FSO is an excellent candidate for the last-mile connectivity. However, the performance of FSO communication systems is highly vulnerable to adverse atmospheric condition, pointing error and atmospheric turbulence [2-4].

Turbulence-induced fading, also known as scintillation, causes fluctuations in both the intensity and the phase of received signal due to variations in the refractive index along propagation path. Over the last two decades, many statistical models have been proposed to describe this fluctuation in all fading regimes. Among them the Gamma-Gamma (GG) distribution has received considerable interest as it provides an excellent fit with measured data under a wide range of turbulence conditions [5,6]. So we consider the GG model for intensity fluctuation through FSO channel.

On the other hand, the building sway which is caused by thermal expansion, weak earthquakes, and dynamic wind loads, induces the vibration of the transmitter beam and arises a misalignment between transmitter and receiver. This phenomenon, known as pointing error, limit the performance of FSO links significantly [7].

In addition, the attenuation of laser power through the atmosphere is another factor which degrades the performance of FSO link [8].

Recently, it has been shown that the effect of fading in FSO, such as RF communication, can be reduced by using a multiple-input multiple-output (MIMO) FSO system with multiple lasers at the transmitter and multiple apertures at the receiver employing diversity technique [9]. Among exiting diversity combining scheme, equal gain combining (EGC) is of a practical interest; because it provides the efficiency which is close to the optimal maximal ratio combining (MRC) technique while having relatively less implementation complexity [10]. The main difficulty in studying EGC receivers is require the closed-form distribution of the sum of received irradiance from the individual received branches. This problem has been addressed in several previous work. For example, in [11] it has been shown that the probability density function (PDF) and the cumulative density function (CDF) of the sum of K-distributed random variables (RV) can be efficiently approximated by using the exponential distribution. In addition, approximated distribution of the sum of independent but not necessarily identically distributed (i.n.i.d) GG RVs based on the Beaulieu series proposed in [12]. Moreover in [13] the PDF of the sum of independent identically distributed (i.i.d) GG variates has been approximated by a single GG variates, and for the case of i.n.i.d GG RVs approximated by a finite weighted sum of GG variates. Finally in [14], convergent infinite series representations for the error performance of FSO links with L-branches EGC receivers operating over GG fading channel. Notice that in [11], [13] and [14], only the effect of turbulence is considered.

In this paper, we analyze the performance of MIMO FSO channels suffering from GG fading with pointing error and path loss. We assume repetition coding across lasers at the transmitter, and EGC diversity technique at the receiver. Performance analysis of MIMO FSO systems in this case needs the statistics of the sum of the received irradiance considering the distribution of the turbulence-induced fading, pointing error and path loss attenuation. To the best of author's knowledge, this problem has not been addressed in the open technical literature. We use the α-µ distribution for deriving the distribution of sum of received irradiance, which can be applied for different channel characterized by GG strong turbulence, pointing error and attenuation.

II. CHANNEL MODEL

The channel state (h) models the random attenuation of the propagation channel. In our model, h is considered to be a product of three factors: path loss h_l, pointing error h_p and atmospheric turbulence h_a. Therefor the channel state can be formulated as [15]

$$h = h_l h_p h_a \tag{1}$$

Note that h_l is deterministic and h_p and h_a are random.

A. Atmospheric attnuation

Atmospheric attenuation is one of the most important factor that must be considered in evaluation of FSO systems. It shows that how signal power attenuated due to various weather condition. The attenuation of the laser power through the atmosphere is described by the exponential Beers-Lambert Law [16]

$$h_l(L) = \frac{P(L)}{P(0)} = \exp(-\sigma L) \tag{2}$$

Where $P(L)$ is the laser power at distance L, $P(0)$ is laser power at the source, and σ is the attenuation coefficient which is contributed from the absorption and scattering of laser photons by different aerosols and gaseous molecule in the atmosphere. Therefore, the attenuation $h_l(L)$ is constant during a long period of time, and no randomness exists in its behavior.

B. Atmospheric statistical model

As previously noted, to consider a wide range of turbulence condition (weak to strong), the GG turbulence model is assumed here, which its PDF is given by [5,6]:

$$f_{h_a}(h_a) = \frac{2(\alpha\beta)^{\frac{(\alpha+\beta)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} (h_a)^{\frac{(\alpha+\beta)}{2}-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right) \tag{3}$$

Where Γ(.) is the well-known Gamma function [17, eq.8.310/1] and $K_v(.)$ is the vth-order modified Bessel function of the second kind [17, eq.8.407/1]. The parameter α and β can be adjusted to achieve a good agreement between (3) and measurement data. Alternatively, assuming spherical wave propagation, α and β can be directly linked to physical parameter through the following expressions [5]:

$$\alpha = \left[\exp\left(\frac{0.49\chi^2}{\left(1+0.18d^2+0.56\chi^{12/5}\right)^{7/6}}\right)-1\right]^{-1} \tag{4}$$

$$\beta = \left[\exp\left(\frac{0.51\chi^2\left(1+0.69\chi^{12/5}\right)^{-5/6}}{\left(1+0.9d^2+0.62\chi^{12/5}\right)^{5/6}}\right)-1\right]^{-1} \tag{5}$$

Where $\chi^2 = 0.5C_n^2 k^{7/6} L^{11/6}$ is the Rytov variance and $d=(kD^2/4L)$. Here, $k=2\pi/\lambda$ is the optical wave number, λ is the wavelength, D is the diameter of the aperture diameter of the receiver, L is the link distance and C_n^2 is the strength of atmospheric turbulence.

The nth moment of h_a defined as $E[h_a^n] = \int_0^\infty h_a^n f_{h_a}(h_a) dh_a$ which can be obtained by using [17, eq.6.561/16] as:

$$E[h_a^n] = (\alpha\beta)^{-n} \frac{\Gamma(\alpha+n)\Gamma(\beta+n)}{\Gamma(\alpha)\Gamma(\beta)} \tag{6}$$

C. Pointing error

The pointing error is the overall displacement between the beam weight center and the receiver aperture center. The pdf of h_p can be obtained by using the assumption and procedure described in [15]. Based on this model, if we consider a circular detection aperture of radius a, and a Gaussian spatial intensity profile of beam waist w_l, on the receiver plane at distance L from the transmitter, the attenuation due to geometric spread with radial displacement r from the origin of the detector can be approximated as:

$$h_p(r;L) = A_0 \exp\left(-\frac{2r^2}{w_{l_{eq}}^2}\right) \tag{7}$$

Where $h_p(.)$ represents the fraction of the power collected by the detector, $v = \sqrt{\pi}r/\sqrt{2}w_l$, $w_{l_{eq}}^2 = w_l^2\sqrt{\pi}\,\mathrm{erf}(v)/2v\exp\left(-v^2\right)$ is the equivalent beam width, $A_0 = [\mathrm{erf}(v)]^2$ is the fraction of the collected power at $r=0$ and erf(.) is the error function [17, eq.8.250/1]. On the other hand, the radial displacement r follows a Rayleigh distribution. Then, the pdf of h_p is given by:

$$f_{h_p}(h_p) = \frac{\gamma^2}{A_0^{\gamma^2}} h_p^{\gamma^2-1} \qquad 0 \le h_p \le A_0 \tag{8}$$

Where $\gamma = w_{zeq}/2\sigma_s$ is the ratio between the equivalent beam radius and the pointing error displacement standard deviation at the receiver.

The nth moment of h_a defined as $E[h_p^n] = \int_0^{A_0} h_p^n f_{h_p}\left(h_p\right) dh_p$ which can be obtained using [17, eq. 8.445] as:

$$E[h_p^n] = \frac{A_0^n \gamma^2}{n+\gamma^2} \tag{9}$$

D. Statistices of the combined attenuation

Using the previous pdfs for turbulence and pointing error, a closed-form expression for the pdf of $h = h_l h_p h_a$ can be calculated as:

$$f_h\left(h\right) = \int f_{h|h_a}\left(h|h_a\right) f_{h_a}\left(h_a\right) dh_a \tag{10}$$

Where $f_{h|ha}(h|h_a)$ is the conditional probability given h_a. Recall that h_l is the deterministic and act as a scaling factor. Then the conditional distribution can be expressed as:

$$f_{h|h_a}\left(h|h_a\right) = \frac{\gamma^2}{A_0^{\gamma^2} h_a h_l}\left(\frac{h}{h_a h}\right)^{\gamma^2 - 1} \quad 0 \le h \le A_0 h_a h_l \tag{11}$$

By substituting (11) and (3) into (10), and using [17, eq. 9.34/3] to express the $K_v(x)$ in terms of the Meijer G function [17, eq. 9.301], and also the integral identity [18, eq. 07.34.21.0085.01], the finished PDF of the received irradiance can be described as:

$$\begin{aligned} f_h\left(h\right) = {} & \frac{\alpha\beta\gamma^2}{A_0 h_l \Gamma\left(\alpha\right)\Gamma\left(\beta\right)}\left(\frac{\alpha\beta I}{A_0 h_l}\right)^{\frac{\alpha+\beta}{2} - 1} \\ & \times G_{1,3}^{3,0}\left(\frac{\alpha\beta h}{A_0 h_l}\left| \begin{matrix} 1+\gamma^2 - \frac{\alpha-\beta}{2} \\ \gamma^2 - \frac{\alpha+\beta}{2}, \frac{\alpha-\beta}{2}, -\frac{\alpha-\beta}{2} \end{matrix}\right.\right) \end{aligned} \tag{12}$$

Where $G_{p,q}^{m,n}(.)$ denotes the Meijer G function. Equation (12) can be further simplified using [18, eq. 07.34.16.0001.01] as:

$$f_h\left(h\right) = \frac{\alpha\beta\gamma^2}{A_0 h_l \Gamma\left(\alpha\right)\Gamma\left(\beta\right)} G_{1,3}^{3,0}\left(\frac{\alpha\beta h}{A_0 h_l}\left| \begin{matrix} \gamma^2 \\ \gamma^2 - 1, \alpha - 1, \beta - 1 \end{matrix}\right.\right) \tag{13}$$

Since h_l is constant, while ha and hp are independent random variables, the nth moment of h can be obtained as:

$$E\left[h^n\right] = h_l^n E\left[h_p^n\right] E\left[h_a^n\right] \tag{14}$$

With the derived moment for h_p and h_a in (6) and (9), we have:

$$E\left[h^n\right] = \frac{\left(A_0 h_l\right)^n \gamma^2 \Gamma(\alpha+n)\Gamma(\beta+n)}{\left(n+\gamma^2\right)\Gamma(\alpha)\Gamma(\beta)}\left(\alpha\beta\right)^{-n} \tag{15}$$

III. STATISTICS OF THE SUM OF THE RECEIVED IRRADIANCE

In this section, we use the moment matching method for approximate the sum of the received irradiance in GG distributed turbulence channel with pointing error and attenuation, by α-μ distribution. This distribution is a general fading distribution that can be used to represent the small-scale variation of the fading signal better. As its name implies, it is written in term of two physical parameter namely α and μ [19].

Let h_j, $j=1, 2, \ldots, N$ be N independent identically distributed (i.i.d) random variables, each with PDF $f_h(h)$. Denote the sum of the N random variables by $Z = \sum_{j=1}^{N} h_j$. It is shown that the PDF and the CDF of Z can be computed by the α-μ PDF and CDF given in [19].

$$f_Z\left(z\right) = \frac{\alpha\mu^\mu z^{\alpha\mu - 1}}{\hat{z}^{\alpha\mu}\Gamma\left(\mu\right)} \exp\left(-\mu\frac{z^\alpha}{\hat{z}^\alpha}\right) \tag{16}$$

$$F_Z\left(z\right) = 1 - \frac{\Gamma\left(\mu, \mu z^\alpha / \hat{z}^\alpha\right)}{\Gamma\left(\mu\right)} \tag{17}$$

Where $\hat{z} = \sqrt[\alpha]{E[Z^\alpha]}$, $\mu = \frac{E^2(Z^\alpha)}{E(Z^{2\alpha}) - E^2(Z^\alpha)}$ and $\Gamma(.,.)$ is the incomplete gamma function [17, eq. 8.350/2]. The kth moment $E(Z^k) = \int Z^k f_Z(z) dZ$ is obtained as:

$$E\left(Z^k\right) = \frac{\hat{z}^k \Gamma\left(\mu + k/\alpha\right)}{\mu^{k/\alpha}\Gamma\left(\mu\right)} \tag{18}$$

The parameter α and μ can be derived using the assumption and methodology described in [19]. Based on this model, we define a measurable parameter μ_a, where $a > 0$ is an arbitrary parameter and μ_a generalized the definition of μ and reduce to it for $a = \alpha$. Using the definition of μ in terms of the moments as given previously , we have:

$$\mu_a = \frac{\Gamma^2\left(\mu + a/\alpha\right)}{\Gamma\left(\mu + 2a/\alpha\right)\Gamma\left(\mu\right) - \Gamma^2\left(\mu + a/\alpha\right)} \tag{19}$$

By defining $\mu_{a=1}$ and $\mu_{a=2}$, moment-based estimator for α and μ is obtained as:

$$\frac{\Gamma^2\left(\mu + 1/\alpha\right)}{\Gamma\left(\mu + 2/\alpha\right)\Gamma\left(\mu\right) - \Gamma^2\left(\mu + 1/\alpha\right)} = \frac{E^2(Z)}{E(Z^2) - E^2(Z)} \tag{20}$$

$$\frac{\Gamma^2\left(\mu + 2/\alpha\right)}{\Gamma\left(\mu + 4/\alpha\right)\Gamma\left(\mu\right) - \Gamma^2\left(\mu + 2/\alpha\right)} = \frac{E^2(Z^2)}{E(Z^4) - E^2(Z^2)} \tag{21}$$

The required moment $E[Z]$, $E[Z^2]$ and $E[Z^4]$ can be obtained by using (15) and multinomial identity as:

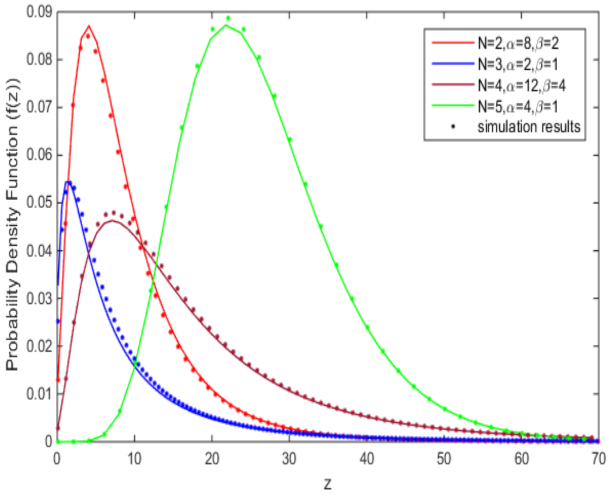

Figure 1. Exact and simulated PDF of N=2, N=3, N=5 and N=6 i.i.d GG variables with pointing error and attenuation.

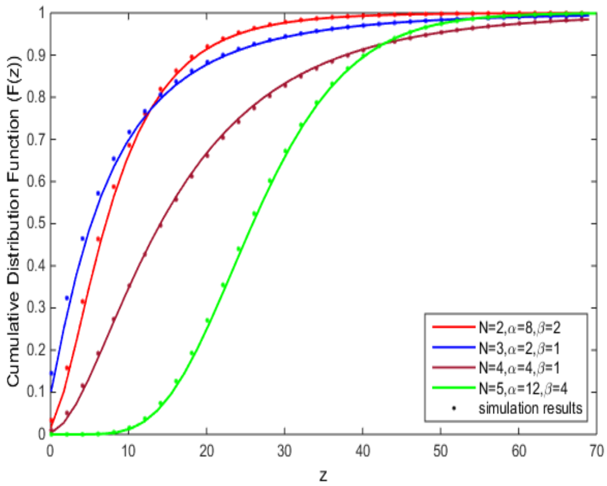

Figure 2. Exact and simulated CDF of N=2, N=3, N=5 and N=6 i.i.d GG variables with pointing error and attenuation.

$$E\left[z^n\right]=\sum_{j_1=0}^{n}\sum_{j_2=0}^{j_1}\cdots\sum_{j_{N-1}}^{j_{N-2}}\binom{n}{j_1}\binom{j_1}{j_2}\cdots\binom{j_{N-2}}{j_{N-1}}$$
$$\times E\left[X_1^{n-j_1}\right]E\left[X_2^{j_1-j_2}\right]\cdots E\left[X_N^{j_{N-1}}\right] \quad (22)$$

Fig. 1 and Fig. 2 show respectively the approximate PDF and CDF of sum of two, three, five and six i.i.d GG variables with pointing error and attenuation for different values of parameter α and β. Numerical results show that the proposed approximation method accurately approximate the exact PDF and CDF of the sum of GG variates. In the following, we use these method in performance analysis of MIMO-FSO communication system employing EGC diversity receiver.

IV. SYSTEM MODEL

We adopt a MIMO FSO system with M transmit and N receive apertures. It is assumed that the M lasers simultaneously illuminate the N photodetectors using repetition coding. By considering high signal-to-noise (SNR) ratio regime, we can use Gaussian noise model. Assuming on-off keying (OOK) modulation scheme, the received signal at the jth received aperture is given by [20]:

$$r_j=\eta s\sum_{m=1}^{M}I_{mn}+v_n \qquad n=1,\ldots,N \quad (23)$$

Where η is the optical-to-electrical conversion coefficient, $s\in\{0,1\}$ represent the information bit and v_n is the Additive White Gaussian Noise (AWGN) with zero mean and variance $\sigma_v^2=N_0/2$, i.e. $v_n\sim N(0,N_0/2)$. At the receiver, the received optical signals from the N aperture are combined using EGC scheme. Hence, the output of the receiver is:

$$r=\sum_{j=1}^{N}r_j=\frac{s\eta}{MN}\sum_{n=1}^{N}\sum_{m=1}^{M}I_{mn}+v \quad (24)$$

The factor M is assumed to ensure that the total power of diversity system is similar to the power of the benchmark single-input single-output (SISO) link. In contrast, the factor N ensures that the sum of the N receive aperture areas is the same as the area of the receive aperture of the SISO link [20]. The received electrical SNR of the FSO link between the mth transmit and nth receive aperture is defined as $\gamma_{mn}=\eta^2 I_{mn}^2/N_0$, whereas its average is given by $\overline{SNR}=\eta^2(E[I_{mn}])^2/N_0$ [21]. Considering the above definition, the electrical SNR of the combined signal at the output of the receiver is given by:

$$\gamma_{EGC}=\frac{\eta^2}{N^2M^2N_0}\left(\sum_{n=1}^{N}\sum_{m=1}^{M}I_{mn}\right)^2 \quad (25)$$

A. Outage Probability

In this section, the outage probability (OP), as a performance measure, is evaluated for different MIMO FSO configurations over GG distributed atmospheric turbulence channels with pointing errors and attenuation. It is defined as the probability that the instantaneous combined SNR, γ_{EGC}, falls below a certain specified threshold, γ_{th} [22]. Using (25) and (17), an accurate closed form approximation for the OP is obtained as:

$$P_{out}=P\left(\gamma_{EGC}<\gamma_{th}\right)=P\left(\frac{\eta^2}{N^2M^2N_0}\left(\sum_{n=1}^{N}\sum_{m=1}^{M}I_{mn}\right)^2<\gamma_{th}\right)$$
$$=P\left(I<\frac{NM}{\eta}\sqrt{N_0\gamma_{th}}\right)=F_Z\left(NM\sqrt{\gamma_{th}/\mu}\right) \quad (26)$$

Where $I=\sum_{n=1}^{N}\sum_{m=1}^{M}I_{mn}$.

In Fig.3, the OP of the considered system is plotted for fixed γ, different value of diversity order and $L=2\,km$. The value of GG parameters are selected based on (4) and (5). Specially, we selected $\lambda = 1550\,nm$, $D/L \rightarrow 0$ and $C_n^2 = 10^{-14}$ which correspond to strong turbulence condition.

There is a strong inverse correlation between the attenuation and turbulence intensity. Therefore, we assume $h_l = 0.9$ which is corresponded to clear weather. As it can be observed, in all considered cases, the analytically evaluation and computer simulation results are highly close together.

By fixing N and M, and by choosing arbitrary GG parameter, the effect of the pointing error is investigated. Fig.4 clearly shows that with the increasing of γ, the effect of the pointing error is decreased and the OP of the system is improved.

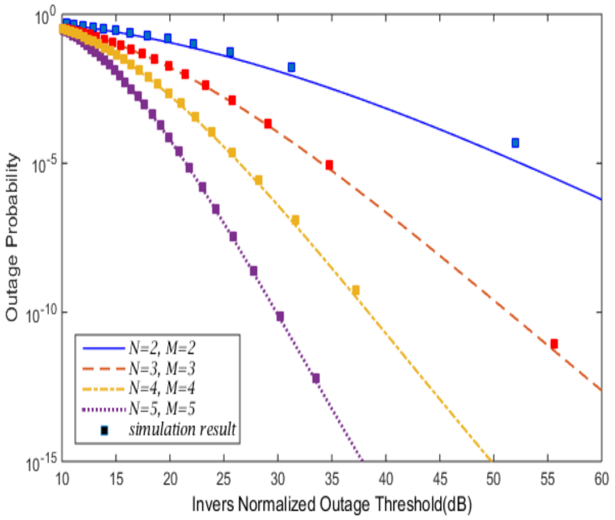

Figure 3. Outage performance of MIMO FSO system employ EGC and operate over GG turbulence channel with pointing error and attenuation

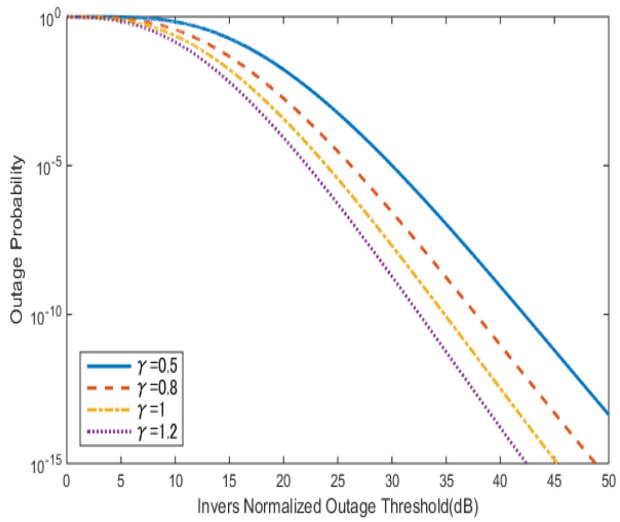

Figure 4. Outage performance of MIMO FSO system with employing EGC and operating over GG turbulence channel with varying pointing error effects. (M=3, N=3, α=4.3, β=4)

B. *Average Bit Error Probability*

The bit error rate of IM/DD with OOK is given by *Pb(e)=p(1)p(e|1)+p(0)p(e|0)*, where *p(1)* and *p(0)* are the probabilities of sending 1 and 0 bits, and *p(e|1)* and *p(e|0)* denote the conditional bit error probabilities when the transmitted bit is 1 and 0, respectively [20].

Assuming perfect Channel State Information (CSI), the ABEP of the considered FSO system is given by [23]:

$$\bar{P}_{be} = \frac{1}{2}\int_0^{\infty} f_Z(z)\,\mathrm{erfc}\left(\frac{\eta}{2MN\sqrt{N_0}} I\right) dI \tag{27}$$

ABEP obtained by substituting (16) to (27) and performing integration.

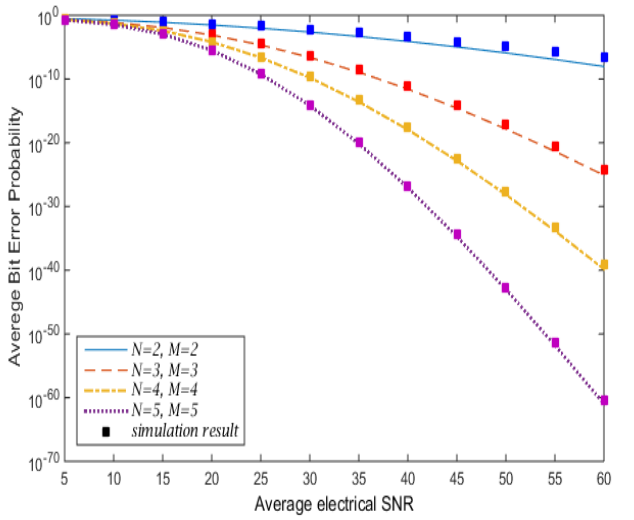

Figure 5. ABEP of MIMO FSO system employ EGC and operate over GG turbulence channel with pointing error and attenuation.

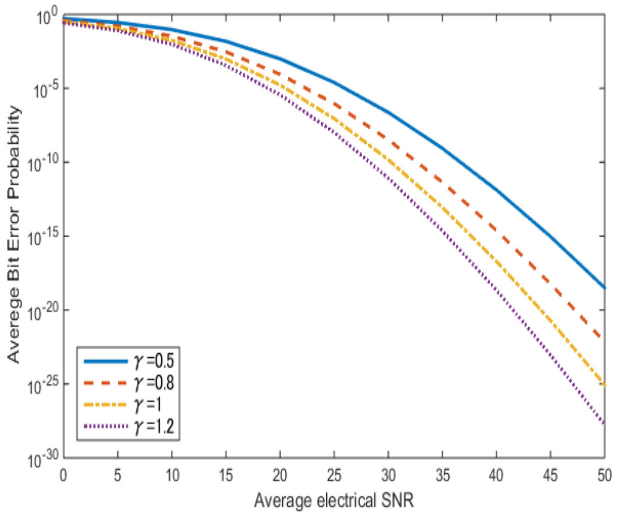

Figure 6. ABEP of MIMO FSO system with employing EGC and operating over GG turbulence channel with varying pointing error effects. (M=3, N=3, α=4.3, β=4)

In Fig.5, the ABEP of the considered MIMO FSO system is depicted as a function of the average electrical SNR by using the same parameters considered in the OP case. The accuracy of considered approximation is clearly illustrated. Fig.6 shows the effect of pointing error in ABEP.

V. CONCLUSION

The Gamma-Gamma distribution is popularly accepted to model the received intensity fluctuation in the FSO communication. In contrast, the turbulence, pointing error and attenuation are the main challenges in FSO systems. Hence, in this study we investigated the performance of MIMO FSO communication systems with EGC receiver and operating over GG turbulence channel with pointing error and path loss. It is shown that the PDF and CDF of sum of GG random variables are approximated by α-μ distribution. To give an example of the application of the derived analytical result, the average bit error rate and outage performance of the considered system is presented.

REFERENCES

[1] Z. Ghassemlooy and W. O. Popoola, Terrestrial free-space optical communications, INTECH Open Access Publisher, November. 1983, ch 17, pp.352-392.

[2] H. Samimi and M. Uysal. "End-to-end performance of mixed RF/FSO transmission systems", IEEE. J. Opt. Commun. & Net., vol. 5, no. 11, pp. 1139-1144, November 2013.

[3] M. Niu, J. Cheng, and J. F. Holzman, Terrestial coherent Free-Space optical communication systems, INTECH Open Access Publisher, ch 8, 2012.

[4] Q. Liu, C. Qiao, G. Mitchell, and S. Stanton, "Optical wireless communication networks for first and last-mile broadband access," IEEE. J. optical networking. vol. 4, no. 12, 2005, pp. 807-828.

[5] L. C. Andrews, R. L. Philip, and M. A. Alhabash, "Mathematical model for the irradiance probability density function of a laser beam propagation through turbulence media", Optical Engineering, vol. 40, pp.1554-1562, August 2001.

[6] A. J. Navas, J. M. G. Balsells, J. F. Paris, and A. P. Notario, A unifying statistical model for atmospheric optical scintillation, Spain. 2011. pp. 181-206.

[7] I. M .Ansari, F. Yilmaz, and M. S. Alouini, "Impact of pointing error on the performance of mixed RF/FSO dual-hop transmission systems" IEEE wireless communications letters, vol. 2, pp. 351-354 , February 2013.

[8] O. Fiser and V. Brazda, "Clear air attenuation on FSO links", IEEE. IMOC. pp. 1-4. August 2013.

[9] A. A. Farid and S. Hranilovic, "Diversity gain and outage probability for MIMO free space optical links with misalignment", IEEE. Trans. Commun., vol. 60, no. 2, pp. 479-487, February 2012.

[10] H. Samimi and P. Azmi, "An approximate analytical framework for performance analysis of equal gain combining technique over independent Nakagami, Rician and Weibull fading channels", Wirel. Pers. Commun. vol. 43, no. 4, pp. 1399-1408, June 2007.

[11] H. Samimi, "Distribution of the sum of K-distributed random variables and applications in free-space optical communications", IET. J. Optoelec., vol. 6, 2012, pp. 1-6.

[12] H. Samimi, "Equal-gain combining reception over Gamma-Gamma turbulence channels with pointing errors", IET . J. Optoele., vol. 8, no. 5, 2014, pp. 191-195.

[13] N. D. Chatzidiamantis and G. K. Karagiannidis, "On the distribution of the sum of gamma-gamma variates and applications in RF and optical wireless communications", IEEE Trans. Wirel. Commun, vol. 59, pp. 1298-1308, May 2011.

[14] H. Samimi and P. Azmi, "Performance analysis of equal-gain diversity receivers over generalized gamma fading channels", AEU-International. J. Elec. &. Commu., vol. 62, no. 7, pp. 496-505, August 2008.

[15] A. A. Farid and S. Hranilovic, "Outage capacity optimization for free-space optical links with pointing errors", IEEE. J. Lightwave Tech., vol. 25, pp.1702-1710, Jul. 2007.

[16] H. Willebrand and B. S. Ghuman, free space optics: enabling optical connectivity in today's network, SAMS publishing, ch 3, 1st ed., December 2001

[17] I. S. Gradshteyn and I. M. Ryzhik, Table of integrals, series, and products. 7th ed., New York:Academic, 2000.

[18] Wolfram Research: 'Mathematica edition: version 8.0. champaign' (Wolfram Research, Inc., IL, 2010)

[19] M. D . Yacoub, "The α-μ distribution: a general fading distribution", IEEE. Int. Symp. PIMR. Commun., vol. 2, pp. 629-633, September 2002.

[20] S. M. Navidpour, M. Uysal, and M. Kavehrad, "BER performance of free-space optical transmission with spatial diversity", IEEE trans. Wirel. Commun., vol. 6, no. 8, pp. 2813-2819, August 2007.

[21] X. Zhu and j. M. Kahn, "Free-space optical communications through atmospheric turbulence channels", IEEE Tans. Wirel. Commun., vol. 50, no. 8, pp. 1293-1300. August 2002.

[22] A. G. Zambrana, C. C. Vázquez, and B. C. Vázquez, "Outage performance of MIMO FSO links over strong turbulence and misalignment fading channels", vol. 19, no. 14, pp.13480-13496, July 2011.

[23] H. G. Sandalidis, T. A. Tsiftsis, G. K. Karagiannidis and M. Uysal, "BER performance of FSO links over strong atmospheric turbulence channel with pointing errors", IEEE Commun. Letter, vol. 12, no. 1, pp. 44-46, June 2008.