Two-Way MIMO Relay Beamforming Scheme for Sum-Rate Maximization Approach Based on Second-Order Statistics

Seyed Mohsen Rezvani, Hengameh Keshavarz
Faculty of Electrical and Computer Engineering, University of Sistan and Baluchestan, Zahedan, Iran
Email: s.mohsen_rezvani@pgs.usb.ac.ir, keshavarz@ece.usb.ac.ir

***Abstract*—In this paper, sum-rate maximization is investigated in a two-way MIMO relay network where a multi-antenna relay node assists two transceivers for exchanging their data. For sum-rate maximization, beamforming weights are designed whereas total transmit power constraint in the multi-antenna relay node is satisfied. Assuming that the second order statistics (SOS) of channel state information (CSI) are only available, we confront with a non-convex optimization problem. Using the semidefinite relaxation (SDR) technique and by help of the bisection search method, the beamforming weights are specified. Simulation results show, in a constant maximum allowable relay total transmit power, increasing the number of antennas of multi-antenna relay node increases the maximum sum-rate.**

***Index Terms*—two-way networks, multi-antenna relay, general-rank beamforming, second-order statistics, sum-rate maximization.**

I. Introduction

Beamforming design has absorbed attention as a candidate in cooperative methods where cooperation between nodes of network used in order that overcome to common challenges in wireless networks, such as path loss. In this area, various scenarios and assumptions are discussed in literature [1], [2], [3], [4], [5]. In [2], a cooperative network has been considered where a set of distributed relay nodes help a transmitter for sending data to a receiver. In this network, with imperfect CSI assumption, beamforming design has been followed in two different strategies, namely Signal-to-noise ratio (SNR) maximization under relay power budget constraint and total relay transmit power minimization under quality of service constraint.

Two-way relay assisted networks have been studied in literature. For avoiding interference in two-way relaying networks, different strategies are suggested. Traditionally, it is suggested that four time-slots used for exchanging data between to transceivers through relay node. This strategy works as a half-duplex scenario, because it is needed that the information of one transceiver reaches to another one and then in opposite way another transceiver sends its information. In [6], using network coding method leads to reduce the number of time-slots to three. Finally, by use of beamforming technique, the number of time-slots for exchanging data reduce to two [7], [8]. In [8], three different optimality criteria has been proposed, namely total transmit power minimization under quality of services (QoSs) constraints, total relay transmit power minimization under QoSs constraints and SNR balancing under a total power budget constraint with full CSI assumption.

The common assumption in two-way networks is that the perfect CSI is provided [7]. In some literature there is this presupposition that imperfect CSI is only included, and this assumption adds more complexity to the problems. In [9], assuming availability of the SOS of CSI, a two-way distributed relay assisted network has been investigated. In this paper, the authors has been determined the transmit beamforming vector for Signal-to-noise-plus-interference ratio (SINR) balancing under a total relays power budget constraint.

In this paper we consider a cooperative two-way network where two transceivers exchange their information via a helping multi-antenna relay node. By using MIMO relay, we benefit receive and transmit beamformig jointly and this leads to design of a general rank beamforming matrix. Assuming the SOS of CSI is available, in this paper, the achievable rate region is first determined and then the beamforming weights are designed for sum-rate maximization whereas a total transmit power constraint at the MIMO relay node are satisfied. Since we consider the imperfect CSI, in determining the achievable rate region and designing beamforming weights for the main goal of this paper, i.e. sum-rate maximization, we confront with non-convex optimization problems. We study the optimization problems by use of the semidefinite relaxation technique and the bisection search method.

Notation: Throughout this paper, we use the following standard notations: $(\cdot)^*$, $(\cdot)^H$, $\mathrm{tr}(\cdot)$ and $E\{\cdot\}$ represent the conjugate, the hermitian transpose, the trace and the statistical expectation, respectively. The notation $\mathrm{diag}(\mathbf{A})$ is a vector which contains the diagonal entries of the square matrix $\mathbf{A}$ and $\mathrm{diag}(\mathbf{a})$ is a diagonal matrix whose diagonal elements are different entries of the vector $\mathbf{a}$. $\mathrm{vec}(\mathbf{X})$ is the vector obtained from the matrix $\mathbf{X}$. The notation $\mathbf{I}$ denotes the identity matrix. $\otimes$ stands for Kronecker multiplication.

The rest of this paper is organized as follows. In the next section, based on second-order statistics of CSI, system model of the MIMO relay assisted network is described. In Section III, the achievable rate region is determined. In Section IV the sum-rate maximization approach under total transmit MIMO relay power constraint is developed. The simulation results are given in Section V. And finally, the main results are

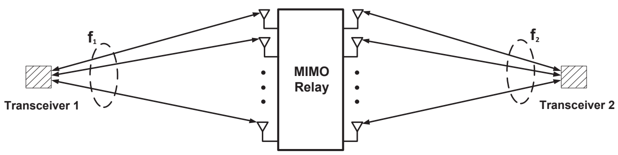

Fig. 1. A two-way multi-antenna relay assisted network

summarized in section VI.

II. System Model

As illustrated in Fig. 1, we consider a cooperative two-way network consists of two transceivers and a multi-antenna relay node. Each transceiver is equipped with single-antenna while N_r antennas are embedded in the relay node. We assume there is not efficient direct link between two transceivers due to path loss, and so exchanging the information is only possible through the relay node.

Assuming a flat fading scenario, we denote the channel vectors between the transceiver 1 and the relay node and between the relay node and the transceiver 2 by $\mathbf{f}_1 = [f_{11}\, f_{21} \dots f_{N_r1}]^T$ and $\mathbf{f}_2 = [f_{12}\, f_{22} \dots f_{N_r2}]^T$, respectively. It is assumed the instantaneous channel state information is not available, and we only access to SOS of CSI. The correlation matrices of the channel vectors $\mathbf{f}_1$ and $\mathbf{f}_2$ can be expressed, respectively, as

$$\mathbf{R}_{f_1} = E\{\mathbf{f}_1{\mathbf{f}_1}^H\} \quad \mathbf{R}_{f_2} = E\{\mathbf{f}_2{\mathbf{f}_2}^H\}$$

Let s_1 and s_2 denote the data symbols of transceivers 1 and 2, respectively whereas they have unit power, i.e. $E\{|s_1|^2\} = E\{|s_2|^2\} = 1$. We consider a 2-step scenario for exchanging data. In the first step, the multi-antenna relay receives data from two transceivers, as

$$\mathbf{r} = \sqrt{P_1}\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{f}_2 s_2 + \mathbf{v} \tag{1}$$

Where $\mathbf{r}$ is the $N_r \times 1$ received signal vector at the multi-antenna relay node. Also P_1 and P_2 denote the transmission powers of transceivers 1 and 2, respectively, and $\mathbf{v}$ is the $N_r \times 1$ additive white Gussian noise (AWGN) vector at the multi-antenna relay node with correlation matrix $\mathbf{R}_v = \sigma_v^2\mathbf{I}$.

The multi-antenna relay multiplies the received signal vector by the $N_r \times N_r$ specified general-rank beamforming matrix $\mathbf{W}$, and resulting is sent to two transceivers in the second step. It is worth to mention that the general-rank beamforming matrix has been actually obtained by multiplying the receive beamforming vector by the transmit one. As a result, the $N_r \times 1$ vector $\mathbf{t}$ transmitted by the multi-antenna relay node to two transceivers in the second step can be expressed as

$$\mathbf{t} = \mathbf{W}\mathbf{r} \tag{2}$$

The total power transmitted by the multi-antenna relay node can be written as

$$\begin{aligned} P_r &= \mathrm{tr}(\mathbf{W}E\{\mathbf{r}\mathbf{r}^H\}\mathbf{W}^H) = \mathrm{tr}(\mathbf{W}(P_1\mathbf{R}_{f_1} + P_2\mathbf{R}_{f_2} + \mathbf{R}_v)\mathbf{W}^H) \\ &= \mathbf{w}^H(\mathbf{I} \otimes (P_1\mathbf{R}_{f_1} + P_2\mathbf{R}_{f_2} + \mathbf{R}_v))\mathbf{w} \end{aligned} \tag{3}$$

Where $\mathbf{w} = \mathrm{vec}\{\mathbf{W}^H\}$. The received signals in the transceivers 1 and 2 can be represented, respectively, by

$$y_1 = \sqrt{P_1}{\mathbf{f}_1}^H\mathbf{W}\mathbf{f}_1 s_1 + \sqrt{P_2}{\mathbf{f}_1}^H\mathbf{W}\mathbf{f}_2 s_2 + {\mathbf{f}_1}^H\mathbf{W}\mathbf{v} + n_1 \tag{4}$$

$$y_2 = \sqrt{P_1}{\mathbf{f}_2}^H\mathbf{W}\mathbf{f}_1 s_1 + \sqrt{P_2}{\mathbf{f}_2}^H\mathbf{W}\mathbf{f}_2 s_2 + {\mathbf{f}_2}^H\mathbf{W}\mathbf{v} + n_2 \tag{5}$$

Where n_k is AWGN at the kth transceiver with variance $\sigma_{n_k}^2$, for $k = 1, 2$. Since it is assumed that the SOS of CSI is only available and the transceivers don't access to the instantaneous CSI, by accepting an error term, we consider two transceivers use the available mean of channel vectors, i.e. $\bar{\mathbf{f}}_1$ and $\bar{\mathbf{f}}_2$, for eliminating the interference signal as

$$\begin{aligned} \tilde{y}_1 = &\underbrace{\sqrt{P_2}{\mathbf{f}_1}^H\mathbf{W}\mathbf{f}_2 s_2}_{\text{desired signal}} + \underbrace{\sqrt{P_1}({\mathbf{f}_1}^H\mathbf{W}\mathbf{f}_1 - \bar{\mathbf{f}}_1\mathbf{W}\bar{\mathbf{f}}_1)s_1}_{\text{undesired signal}} + \\ &\underbrace{{\mathbf{f}_1}^H\mathbf{W}\mathbf{v} + n_1}_{\text{total noise}} \end{aligned} \tag{6}$$

$$\begin{aligned} \tilde{y}_2 = &\underbrace{\sqrt{P_1}{\mathbf{f}_2}^H\mathbf{W}\mathbf{f}_1 s_1}_{\text{desired signal}} + \underbrace{\sqrt{P_2}({\mathbf{f}_2}^H\mathbf{W}\mathbf{f}_2 - \bar{\mathbf{f}}_2\mathbf{W}\bar{\mathbf{f}}_2)s_2}_{\text{undesired signal}} + \\ &\underbrace{{\mathbf{f}_2}^H\mathbf{W}\mathbf{v} + n_2}_{\text{total noise}} \end{aligned} \tag{7}$$

The powers of desired signals received by the transceivers 1 and 2 are expressed, respectively, as

$$\begin{aligned} P_{s_1} &= P_2E\{|\mathbf{f}_1^H\mathbf{W}\mathbf{f}_2|^2\}\underbrace{E\{|s_2|^2\}}_{=1} = P_2E\{|\mathrm{tr}(\mathbf{W}\mathbf{f}_2\mathbf{f}_1^H)|^2\} \\ &= \ P_2\mathbf{w}^H\mathbf{R}_h\mathbf{w} \end{aligned} \tag{8}$$

$$\begin{aligned} P_{s_2} &= P_1E\{|\mathbf{f}_2^H\mathbf{W}\mathbf{f}_1|^2\}\underbrace{E\{|s_1|^2\}}_{=1} = P_1E\{|\mathrm{tr}(\mathbf{W}\mathbf{f}_1\mathbf{f}_2^H)|^2\} \\ &= \ P_1\mathbf{w}^H\mathbf{R}_h\mathbf{w} \end{aligned} \tag{9}$$

Where $\mathbf{R}_h = E\{\mathbf{h}\mathbf{h}^H\}$, while $\mathbf{h} = \mathrm{vec}\{\mathbf{f}_1\mathbf{f}_2^H\}$. The powers of undesired signals received by the transceivers 1 and 2 are expressed, respectively, as

$$\begin{aligned} P_{e_1} &= P_1E\{|\mathbf{f}_1^H\mathbf{W}\mathbf{f}_1|^2\}\underbrace{E\{|s_1|^2\}}_{=1} - P_1E\{|\bar{\mathbf{f}}_1^H\mathbf{W}\bar{\mathbf{f}}_1|^2\}\underbrace{E\{|s_1|^2\}}_{=1} \\ &= P_1\mathbf{w}^H\mathrm{E}_1\mathbf{w} \end{aligned} \tag{10}$$

$$\begin{aligned} P_{e_2} &= P_2E\{|\mathbf{f}_2^H\mathbf{W}\mathbf{f}_2|^2\}\underbrace{E\{|s_2|^2\}}_{=1} - P_2E\{|\bar{\mathbf{f}}_2^H\mathbf{W}\bar{\mathbf{f}}_2|^2\}\underbrace{E\{|s_2|^2\}}_{=1} \\ &= P_2\mathbf{w}^H\mathrm{E}_2\mathbf{w} \end{aligned} \tag{11}$$

In (10) and (11), $\mathbf{E}_1$ and $\mathbf{E}_2$ are defined, respectively, as

$$\mathbf{E}_1 = (\mathbf{R}_{f_1}^* \otimes \mathbf{R}_{f_1}) - \text{vec}(\bar{\mathbf{f}}_1\bar{\mathbf{f}}_1^H)\text{vec}(\bar{\mathbf{f}}_1\bar{\mathbf{f}}_1^H)^H$$
$$\mathbf{E}_2 = (\mathbf{R}_{f_2}^* \otimes \mathbf{R}_{f_2}) - \text{vec}(\bar{\mathbf{f}}_2\bar{\mathbf{f}}_2^H)\text{vec}(\bar{\mathbf{f}}_2\bar{\mathbf{f}}_2^H)^H$$

The total noise powers at the transceivers 1 and 2 are expressed, respectively, as

$$P_{n_1} = E\{|\text{tr}(\mathbf{W}\mathbf{v}\mathbf{f}_1^H)|^2\} + \sigma_{n_1}^2 = E\{|\mathbf{w}^H\text{vec}(\mathbf{v}\mathbf{f}_1^H)|^2\} + \sigma_{n_1}^2$$
$$= \mathbf{w}^H E\{\text{vec}(\mathbf{v}\mathbf{f}_1^H)\text{vec}(\mathbf{v}\mathbf{f}_1^H)^H\} + \sigma_{n_1}^2 \tag{12}$$

$$P_{n_2} = E\{|\text{tr}(\mathbf{W}\mathbf{v}\mathbf{f}_2^H)|^2\} + \sigma_{n_2}^2 = E\{|\mathbf{w}^H\text{vec}(\mathbf{v}\mathbf{f}_2^H)|^2\} + \sigma_{n_2}^2$$
$$= \mathbf{w}^H E\{\text{vec}(\mathbf{v}\mathbf{f}_2^H)\text{vec}(\mathbf{v}\mathbf{f}_2^H)^H\} + \sigma_{n_2}^2 \tag{13}$$

Using $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = \mathbf{AC} \otimes \mathbf{BD}$, (12) and (13) can be rewritten as

$$P_{n_1} = \mathbf{w}^H E\{(\mathbf{f}_1^* \otimes \mathbf{v})(\mathbf{f}_1^* \otimes \mathbf{v})^H\}\mathbf{w} + \sigma_{n_1}^2$$
$$= \mathbf{w}^H \mathbf{Q}_1 \mathbf{w} + \sigma_{n_1}^2 \tag{14}$$

$$P_{n_2} = \mathbf{w}^H E\{(\mathbf{f}_2^* \otimes \mathbf{v})(\mathbf{f}_2^* \otimes \mathbf{v})^H\}\mathbf{w} + \sigma_{n_2}^2$$
$$= \mathbf{w}^H \mathbf{Q}_2 \mathbf{w} + \sigma_{n_2}^2 \tag{15}$$

Where $\mathbf{Q}_1 = \mathbf{R}_{f_1}^* \otimes \mathbf{R}_v$ and $\mathbf{Q}_2 = \mathbf{R}_{f_2}^* \otimes \mathbf{R}_v$. As a result, we can express the SINRs in the transceivers 1 and 2 that defined as the desired signal power divided by the sum of undesired signal power and noise power, respectively, by

$$\text{SINR}_1 = \frac{P_2\mathbf{w}^H\mathbf{R}_h\mathbf{w}}{P_1\mathbf{w}^H\mathbf{E}_1\mathbf{w} + \mathbf{w}^H\mathbf{Q}_1\mathbf{w} + \sigma_{n_1}^2} \tag{16}$$

$$\text{SINR}_2 = \frac{P_1\mathbf{w}^H\mathbf{R}_h\mathbf{w}}{P_2\mathbf{w}^H\mathbf{E}_2\mathbf{w} + \mathbf{w}^H\mathbf{Q}_2\mathbf{w} + \sigma_{n_2}^2} \tag{17}$$

III. Achievable Rate Region

In this part, we investigate the achievable rate region. Let us denote the rate from s_1 to s_2 and from s_2 to s_1 by R_{12} and R_{21}, respectively. Considering two time slots used for exchanging the data, the rates R_{12} and R_{21} can be presented as

$$R_{12} = \frac{1}{2}\log_2(1 + \text{SINR}_2) \qquad R_{21} = \frac{1}{2}\log_2(1 + \text{SINR}_1)$$

Similar to [7] for determining the achievable rate region, the following optimization problem is considered

$$\begin{aligned} \min_{\mathbf{w}} \quad & P_r = \mathbf{w}^H\mathbf{T}\mathbf{w} \\ \text{s.t} \quad & R_{12} \geq \alpha r \\ & R_{21} \geq (1-\alpha) r \end{aligned} \tag{18}$$

Where $\mathbf{T} = \mathbf{I} \otimes (P_1\mathbf{R}_{f_1} + P_2\mathbf{R}_{f_2} + \mathbf{R}_v)$, $\alpha \in [0, 1]$ and r is the sum-rate. In above problem, if the optimal value of P_r satisfies the inequality $P_r \leq P_r^{max}$, the rates αr and $(1-\alpha)r$ will be located in the achievable rate region, otherwise they will fall outside of the achievable rate region. For solving the optimization problem, the bisection search method is utilized. In bisection search method, it is first needed to determine an initial interval. We set the initial interval for the parameter r as

$$r_l = 0 \qquad r_u = \log_2(1 + \min(\text{SINR}_1^{max}, \text{SINR}_2^{max}))$$

In recent consideration, SINR_1^{max} and SINR_2^{max} are determined according to Rayleigh-Ritz ratio inequality [2]. The process of finding optimum value is described in algorithm (1). In this algorithm, ϵ specifies the accuracy of finding the optimal values, and we consider it equal to 10^{-3}.

Algorithm 1 Achievable Rate Region

```
Iinitializing:
r_l = 0
r_u = log_2(1 + min(SINR_1^max(P_r^max), SINR_2^max(P_r^max)))
while r_u − r_l ≥ ϵ do
    r = (r_l + r_u)/2
    Set r in (18) for obtaining P_r
    if P_r ≤ P_r^max then
        r_l ← r
    else
        if P_r ≥ P_r^max then
            r_u ← r
        end if
    end if
end while
```

In the next section, assuming the availability of second-order statistics of CSI, we determine the general-rank beamforming matrix for sum-rate maximization approach under the total transmit power constraint at the multi-antenna relay node.

IV. Sum-Rate Maximization

In this part, we design the general-rank complex beamforming matrix for the sum-rate maximization under the transmitted relay power constraint in the following problem

$$\begin{aligned} \max_{\mathbf{w}} \quad & R_{sum} = R_{12} + R_{21} \\ \text{s.t} \quad & P_r \leq P_r^{max} \end{aligned} \tag{19}$$

Or, equivalently, as

$$\begin{aligned} \max_{\mathbf{w}, R_{sum}} \quad & R_{sum} \\ \text{s.t} \quad & R_{12} \geq \alpha^* R_{sum} \\ & R_{21} \geq (1-\alpha^*) R_{sum} \\ & P_r \leq P_r^{max} \end{aligned} \tag{20}$$

In recent equation, α^* denotes the optimal value of α determined from (18). Using definition $\mathbf{X} = \mathbf{w}^H\mathbf{w}$, the optimal problem can be equivalently represented as

$$\begin{aligned} \max_{\mathbf{X},R_{sum}} \quad & R_{sum} \\ \text{s.t} \quad & tr(\mathbf{X}\mathbf{L}_1) \geq (2^{\beta_1 R_{sum}} - 1)\sigma_{n_1}^2 \\ & tr(\mathbf{X}\mathbf{L}_2) \geq (2^{\beta_2 R_{sum}} - 1)\sigma_{n_2}^2 \\ & tr(\mathbf{X}\mathbf{T}) \leq P_r^{max} \\ & \text{and rank}\mathbf{X} = 1 \quad , \quad \mathbf{X} \succeq 0 \end{aligned} \tag{21}$$

Where $\beta_1 = 2\alpha^*$ and $\beta_2 = 2(1-\alpha^*)$. Also $\mathbf{L}_1$ and $\mathbf{L}_2$ are defined, respectively, as

$$\mathbf{L}_1 = P_2\mathbf{R}_h - (2^{\beta_1 R_{sum}} - 1)P_1\mathbf{E}_1 - (2^{\beta_1 R_{sum}} - 1)\mathbf{Q}_1$$

$$\mathbf{L}_2 = P_1\mathbf{R}_h - (2^{\beta_2 R_{sum}} - 1)P_2\mathbf{E}_2 - (2^{\beta_2 R_{sum}} - 1)\mathbf{Q}_2$$

In (21), the rank constraint causes the optimization problem is a non-convex problem. Regardless of the rank constraint, the problem will be a quasi-convex problem. We find the optimal value of R_{sum} by use of bisection search method. So the following convex feasibility problem can be expressed

$$\begin{aligned} & \text{find} \\ \text{s.t} \quad & tr(\mathbf{X}\mathbf{L}_1) \geq (2^{\beta_1 R_{sum}} - 1)\sigma_{n_1}^2 \\ & tr(\mathbf{X}\mathbf{L}_2) \geq (2^{\beta_2 R_{sum}} - 1)\sigma_{n_2}^2 \\ & tr(\mathbf{X}\mathbf{T}) \leq P_r^{max} \\ & \mathbf{X} \succeq 0 \end{aligned} \tag{22}$$

In resent convex feasibility problem, using SDP, we research the optimal R_{sum} in an initial interval $[R_{sum,l}, R_{sum,u}]$. The process of finding optimum value is described in algorithm (2). Similar to algorithm (1), we consider $\epsilon = 10^{-3}$.

Algorithm 2 Sum-Rate Maximization

Iinitializing:
$R_{sum,l} = 0$
$R_{sum,u} = \log_2(1 + \min(\text{SINR}_1^{max}(P_r^{max}), \ \text{SINR}_2^{max}(P_r^{max})))$
while $R_{sum,u} - R_{sum,l} \geq \epsilon$ **do**
  $R_{sum} = \frac{R_{sum,l}+R_{sum,u}}{2}$
  Set R_{sum} in (22) for checking the feasibility of the problem
  if the problem is feasible **then**
    $R_{sum,l} \leftarrow r$
  else
    if the problem is not feasible **then**
      $R_{sum,u} \leftarrow R_{sum}$
    end if
  end if
end while

V. Simulation Results

In this part we provide some simulations to evaluate the performance of our proposed methods. In all simulations we consider the transmission powers of transceivers as $P_1 = P_2 = 0dB$, and the noises' variances as $\sigma_v^2 = \sigma_{n_1}^2 = \sigma_{n_2}^2 = 0dB$. As previously mentioned, assumption used in this paper is that the second-order statistics of CSI are only available. Like [2], the channel vectors $\mathbf{f}_1$ and $\mathbf{f}_2$ are generated, respectively, as

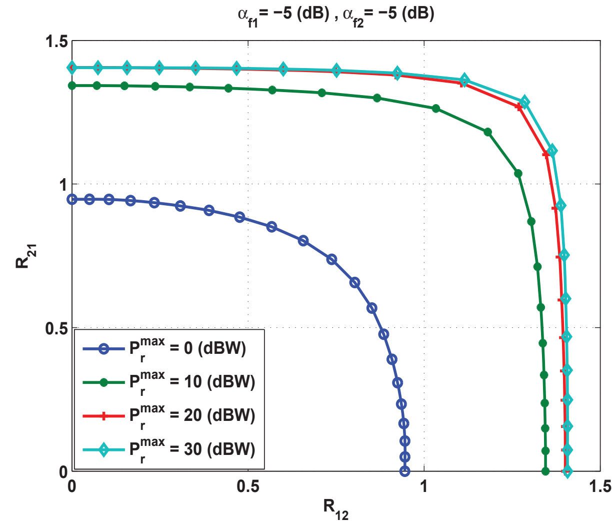

Fig. 2. The achievable rate region

$$\mathbf{f}_1 = \bar{\mathbf{f}}_1 + \tilde{\mathbf{f}}_1 \qquad \mathbf{f}_2 = \bar{\mathbf{f}}_2 + \tilde{\mathbf{f}}_2 \tag{23}$$

Where $\bar{\mathbf{f}}_1$ and $\bar{\mathbf{f}}_2$ are the known mean vectors, and $\tilde{\mathbf{f}}_1$, $\tilde{\mathbf{f}}_2$ are variations of the channel vectors around their nominal values. $\bar{\mathbf{f}}_1$ and $\bar{\mathbf{f}}_2$ are generated, respectively, as

$$\bar{\mathbf{f}}_1 = \frac{e^{j\boldsymbol{\theta}_{f_1}}}{\sqrt{1+\alpha_{f_1}}} \qquad \bar{\mathbf{f}}_2 = \frac{e^{j\boldsymbol{\theta}_{f_2}}}{\sqrt{1+\alpha_{f_2}}} \tag{24}$$

Where α_{f_1} and α_{f_2} are parameters which determine the level of uncertainty in the channel coefficients and $\boldsymbol{\theta}_{f_1}$ and $\boldsymbol{\theta}_{f_2}$ are random variable uniformly vectors distributed in $[0, 2\pi]$. $\tilde{\mathbf{f}}_1$ and $\tilde{\mathbf{f}}_2$ are zero-mean random variables whose variances are determined, respectively, as

$$var(\tilde{f}_1) = \frac{\alpha_{f_1}}{1+\alpha_{f_1}} \qquad var(\tilde{f}_2) = \frac{\alpha_{f_2}}{1+\alpha_{f_2}} \tag{25}$$

The correlation matrices of the channel vectors $\mathbf{f}_1$ and $\mathbf{f}_2$ are expressed, respectively, as

$$\mathbf{R}_{f_1} = \bar{\mathbf{f}}_1 \, \bar{\mathbf{f}}_1^H + \frac{\alpha_{f_1}}{1+\alpha_{f_1}}\mathbf{I} \qquad \mathbf{R}_{f_2} = \bar{\mathbf{f}}_2 \, \bar{\mathbf{f}}_2^H + \frac{\alpha_{f_2}}{1+\alpha_{f_2}}\mathbf{I}$$

Fig. 2 illustrates the achievable rate region for different values of the maximum allowable total transmit relay power (P_r^{max}). In this simulation, the number of antennas in the multi-antenna relay node are considered 10. Since the conditions of two transceivers are considered similar, the figure is formed symmetrically. As can be seen in Fig. 2, by increasing P_r^{max}, the available rates increase. These increments continue to roughly $P_r^{max} = 30dB$, and after that increasing P_r^{max} doesn't substantially impact on the rates.

Fig. 3 shows the maximum sum-rate versus the maximum allowable total transmit relay power for $\alpha_{f_1} = -5dB$ and different values of α_{f_2}. Note that different values of α_{f_2} are

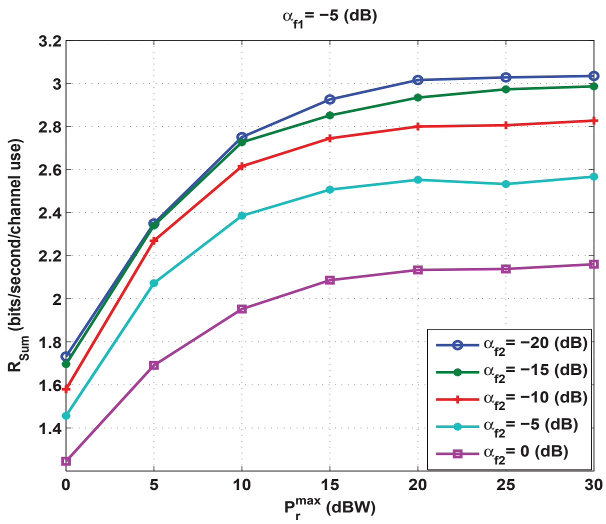

Fig. 3. The maximum sum-rate (R_{sum}) versus the maximum allowable total transmit relay power (P_r^{max}) for $\alpha_{f_1} = -5$dB and different values of α_{f_2}.

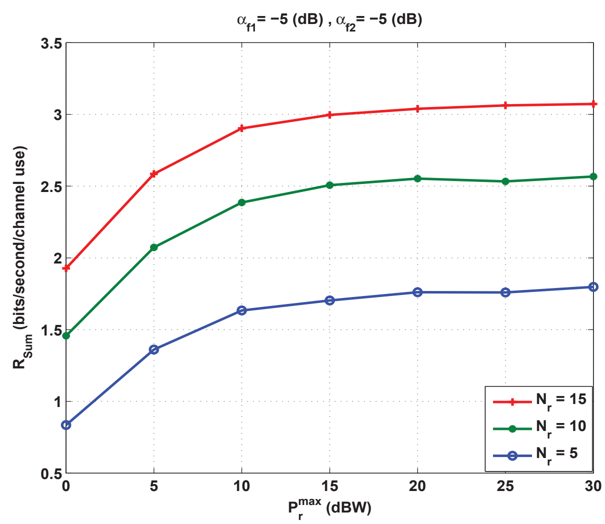

Fig. 4. The maximum sum-rate (R_{sum}) versus the maximum allowable total transmit relay power (P_r^{max}) for different number of antenna (N_r).

considered to test the impact of uncertainty in the channel coefficients. As can be seen in this figure, rising certainty about the channel coefficients, the maximum sum-rate increases.

Next, we investigate impact of increasing the number of antennas in multi-antenna relay node, i.e. N_r. Fig. 4 represents the maximum sum-rate versus the maximum allowable total transmit relay power for three different values of N_r. As can be seen in this figure, by increasing the number of antennas in the multi-antenna relay node, the maximum sum-rate increases, and the performance of network improves.

VI. Conclusion

In this paper we investigated a two-way MIMO relay network where two transceivers are serviced for exchanging their data by a multi-antenna relay node. First, the achievable rate region is determined and then the complex beamforming weights are designed with the aim of sum-rate maximization under the total transmit power relay node constraint with the SOS of CSI assumption. Using the MIMO relay causes we benefit receive and transmit beamforming jointly, and so the specified complex beamforming matrix is a general-rank matrix. For solving the consequence non-convex optimization problems, we used SDR technique and bisection method. Simulation results revealed to us that increment in the number of antennas in the relay node causes improvement in performance, i.e. sum-rate maximization.

References

[1] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," *IEEE Trans. Inf. Theory*, vol. 55, no. 6, pp. 2499 – 2517, 2009.

[2] V. Havary-Nassab, S. Shahbazpanahi, A. Grami, and Z.-Q. Luo, "Distributed beamforming for relay networks based on second-order statistics of the channel state information," *IEEE Trans. Signal Process.*, vol. 56, no. 9, pp. 4306 – 4316, Sep. 2008.

[3] J. Joung and A. H. Sayed, "Multiuser two-way amplify-and-forward relay processing and power control methods for beamforming systems," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1833 – 1846, 2010.

[4] S. M. Rezvani, S. H. Safavi, R. Sadeghzadeh, A. Haghbin, and R. Davarpanah, "Relay power minimization approach based on general-rank beamforming for multi-antenna relaying schemes," in *New Technologies, Mobility and Security (NTMS), 2012 5th International Conference on*, Istanbul, Turkey, May 2012, pp. 1–5.

[5] S. H. Safavi, M. Ardebilipour, and S. Salari, "Relay beamforming in cognitive two-way networks with imperfect channel state information," *IEEE Wireless Commun. Letter*, vol. 1, pp. 344–347, Aug. 2012.

[6] Y. Wu, P. A. Chou, and S. Y. Kung, "Information exchange in wireless networks with network coding and physical-layer broadcast," in *inProc. 39th Ann. Conf. Inf. Sci. Syst. (CISS05)*, Mar. 2005.

[7] R. Zhang, Y.-C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE J. Select. Areas Commun.*, pp. 699 – 712, 2009.

[8] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, "Optimal distributed beamforming for two-way relay networks," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1238 – 1250, 2010.

[9] S. M. Rezvani and H. Keshavarz, "Distributed beamforming design for sinr balancing approach in cooperative two-way networks based on second-order statistics," in *Electrical Engineering (ICEE), 2015 23rd Iranian Conference on*, Tehran, Iran, May 2015, pp. 358–362.