Robust Discrete-time Linear Control of Heart Rate During Treadmill Exercise

Clément Girard[1], Asier Ibeas[2], Ramon Vilanova[2], Ali Esmaeili[2]

***Abstract*—This paper investigates the ability to control the human heart rate during treadmill exercise which is an important issue in the development of rehabilitation protocols after surgery or weight loss programs and in the prevention of heart failure. In this way, a nonlinear model describing the heart rate response to treadmill exercise is used in this paper to design a PID-type controller adjusting the treadmill speed in a way that the heart rate tracks a reference input profile. The model parameters are estimated individually to best describe the person exercising. Afterwards, two extreme linear systems are defined from the nonlinear model. These extreme linear systems are then used to design a linear robust controller capable of providing an adequate closed-loop response for all the linear systems contained between the two extreme models. The so-designed controller is finally discretized by using a ZOH in order to obtain a discrete-time controller suitable to be implemented in practice. The designed control system is tested on the original nonlinear model by computer simulation to demonstrate the effectiveness of the proposed method to achieve the required objective.**

***Index Terms*—Heart-rate control, Nonlinear systems, PID, robust controllers.**

I. INTRODUCTION

Regular physical activity is beneficial for the overall health of people. It improves heart capacity to send blood to muscles due to their higher demand in oxygen and by this way reduces the risk of heart diseases or cardiac failure. Exercise practice was also proven to be beneficial for people recovering after cardiac disease or surgery as well as for people involved in weight loss programs, [1]. For these people, physical exercise has to be carefully supervised by health professionals to check that it does not present hazards and to adapt activity's intensity level according to patients health conditions. The strong correlation between the heart rate and the intensity of the exercise allows medical experts to supervise the good practice of the activity thanks to the ease to monitor the heart rate response. The knowledge of the cardiovascular system response during exercise can thus improve training protocols for recovery and weight loss programs by an individual adaptation of the process. In this way, a typical training exercise consist in defining a time-varying profile for the heart rate response profitable to patient cardiovascular recovery and in adapting exercise intensity to track this profile. A large range of activities can be performed during heart recovery programs, one of the most frequently employed being the treadmill exercise because of its efficiency and ease of use.

Several studies have been carried out in order to model the heart rate response during treadmill exercise, [2]- [6]. It concluded in a nonlinear system depending both on the running speed and on individual characteristics of the exerciser. Two nonlinearities can be observed: the first one comes from a nonproportional increase of the heart rate with the walking speed increase while the second one appears during prolonged exercise where the heart rate steady increases with a slope depending on the walking speed maintained by the patient (i.e. the heart rate increases despite treadmill speed is constant). This last effect is referred to as *drift phenomenon* and increases with the intensity of the exercise, [7]. Therefore, unlike linear systems whose step response proportionally follows the input, the response of this system is dependent on the intensity of the input applied being, thus, nonlinear. Some articles describe simple models of heart rate response during exercise but based on short duration exercise which do not take into account the drift phenomenon, [2]. A more workable model, however, would require the description of the aforementioned nonlinear effects. In this way, [3] and [4] model the heart rate response during treadmill exercise as a Hammerstein system composed of a static nonlinearity followed by a linear dynamical system. Moreover, a fully nonlinear dynamical model was proposed in [5] and takes into consideration the nonlinear particularities of the system during prolonged exercise.

The model proposed in [5] is described as a state space model with two state variables. It includes five parameters depending on miscellaneous environmental and personal items such as temperature and differ from one individual to another in respect to his physical and health conditions. Therefore, an individual evaluation of these parameters should be made in order to best describe the evolution of the heart rate with the walking speed of each person. The individualized model of each patient is then used to design a controller able to regulate the speed of the treadmill in order to make patient's heart rate accurately follow the reference profile.

When the system is modeled as a Hammerstein system, the control strategy consists in cancelling the input nonlinearity by applying its inverse and then control the remaning dynamical system by using linear control approaches such as H_∞, *LQ* or model predictive control, [3], [4]. On the other hand, [5], [6] and [8] propose the design and implementation of nonlinear controllers based on the Lyapunov theory, whose starting point is the original nonlinear system. The first approach faces the modeling of the static nonlinearity and its inverse by using suport vector regression (SVM) while the second one has to deal with the generation of complex control laws due to the system nonlinearities. The main purpose of this paper is to

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ease off the design of controllers by using a novel approach that leads to simple but powerful control laws.

Initially, two extreme linear models are obtained from the original nonlinear system. These linear models are then used to generate a nominal linear system for which a robust controller is afterwards designed. Therefore, the robust approach is used to cope with the nonlinear behaviour of the actual system when a linear controller is employed. Thus, we maintain the simplicity of working with linear models to design the controller but in a way that the prolonged excercise effects can be taken into consideration. The controller is finally discretized by using a zero-order hold (ZOH) in order to obtain a controller suitable to be implemented in practice since most of industrial controllers are currently implemented in discrete-time. Some simulation examples will show the usefulness of the proposed approach when the controller is applied to the original nonlinear system.

II. Problem Formulation

The first element we need to design a controller is an accurate and rigorous model of the exerciser's heart rate response during treadmill exercise. The proposed starting model is the nonlinear state space controlled system considered in [5]:

$$\begin{cases} \dot{x}_1(t) = -a_1 x_1(t) + a_2 x_2(t) + a_2 u^2(t) \\ \dot{x}_2(t) = -a_3 x_2(t) + a_4 x_1(t)\psi(x_1(t)) \\ \psi(x_1(t)) = \frac{1}{1+\exp(-(x_1(t)-a_5))} \\ HR(t) = \frac{x_1(t)+HR_0}{4} \end{cases} \tag{1}$$

The input variable $u(t)$ corresponds to the speed of the treadmill. The state variable $x_1(t)$ represents the heart rate variation from the at-rest value and $x_2(t)$ represents the influence of local peripheral effects (like temperature, hydratation or sweat) on the heart rate. In this way, this model allows considering heart rate fluctuations not only with the running speed but also with environmental and physiological conditions. The variable $HR(t)$ is the actual heart rate of the exerciser given by the model while HR_0 is a constant representing the at-rest heart rate value of this same person. By this formulation the state space variable $x_1(t)$ is not directly the value of heart rate deviation but the quarter of this. This fact does not change substantially the model description but it has to be taken into account in the system design.

Considering positive values for the model parameters a_i, $i = 1,2...5$, it is easy to see that an increase in the input $u(t)$ will increase $x_1(t)$ which will in turn raise $x_2(t)$, fact that will further increase $x_1(t)$. This aspect of the model reflects the drift phenomenon observed in body's response to exercise. It has to be noticed that the input variable $u(t)$ is not directly the speed of the treadmill in this model but it is normalized to 8 km/h so that $u(t) = \frac{s(t)}{8}$ were $s(t)$ is the actual speed of the treadmill in km/h. Therefore, $u(t)$ will vary in the interval [0,1] to define a speed value between 0 km/h and 8 km/h. The speed of 8 km/h is defined as the maximal speed beyond which the previous model is not suitable because of the non linearity predominance.

The control problem is the design of a speed controller that makes user's heart rate follow a specific given profile. The controller's role is to regulate the treadmill speed in order to change exercise intensity and, as a consequence, exerciser heart rate. For this purpose, the controller takes as input, the error signal between the reference profile and the current heart rate of the user to generate as output a control value that is the speed of the treadmill. Since the output of the model is taken as $y(t) = x_1(t)$, this one represents the quarter of the patient heart rate deviation. Therefore, the error value will be actually taken as the difference between an input signal adapted to be the quarter of the desired heart rate deviation and the output signal $y(t)$. The overall control loop can be seen in Fig. (1)[1].

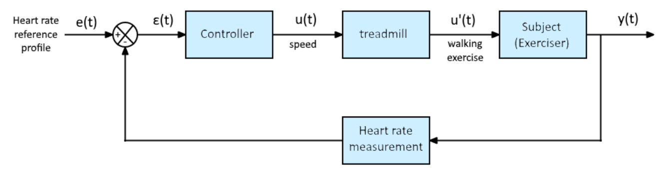

Figure 1. Block diagram of the controlled treadmill system.

The control method is based on a PID controller tuned by classical tools. The main contribution of the paper relies on the approach of manipulating the system model in a way that allows using well-known and well-establised techniques to cope with the design of a robust controller for this nonlinear system instead of using advanced control techniques for the original system (e.g. SVM or Lyapunov theory) that lead to complex control laws. This viewpoint simplifies a lot the design procedure along with the controller implementation while obtaining adequate results. Thus, the simulation examples in Section IV will show the effectiveness of the proposed approach.

An important issue remaining at this point is the determination of the model parameters. In previous analysis [5], [6], model parameters $(a_1, a_2, a_3, a_4, a_5)$ were determined by studies based on a panel of people. Each person had to walk at a determined speed during a fixed period of time to study his heart rate response from at-rest to a certain intensity level and then during the recovery period after activity. By compiling data of each person for several walking speeds, an estimation of averaged model parameters for the complete panel can be made (for instance, by using the methods of [9]). It thus allows obtaining a model independent from the subject considered. However, by studying heart rate response of various people to a same fixed exercise it turns out that it can differ significantly from person to person because of the age, sex and physical condition of each one. This is why the procedure proposed in this paper consists in the following steps:

1) Firstly, an individual estimation of the model parameters is performed.
2) Secondly, two extreme linear models are generated for the original individual nonlinear system.
3) A robust controller is designed with the aim of controlling the whole family of linear systems between the two extreme ones.

[1]For the sake of simplicity the transfer function of the treadmill motor, relating the output of the controller u' with the actual speed u in Figure 1, will be assumed to be unity.

4) The controller is discretized by using a ZOH in order to obtain a discrete-time controller.
5) The so-obtained controller is applied to the original nonlinear system.

The individualized controller design procedure is introduced in the following section.

III. Controller Design

This Section contains how the model is manipulated in order to obtain a nominal one suitable for controller design purposes and the description of the controller structure.

A. Model Definition

Since the variation of the five a_i parameters from one individual to another can be consistent, a personal approach has to be made. Thus, the starting point is the nonlinear model given by equations (1) where the parameters a_i are now assumed to be known and individually adapted. The model contains two nonlinearities. The first one comes from the term $u^2(t)$ which can be overcome by considering a new function $w(t) = u^2(t)$ so that the new input of the process will be $w(t)$. In this way the term $a_2u^2(t)$ in (1) will be replaced by $a_2w(t)$ and $w(t)$ will be the control action to be designed. Therefore, the actual input to the system is obtained from:

$$u = \sqrt{max(0, w(t))} \tag{2}$$

The other nonlinearity is the $\psi(x_1(t))$ function which depends on the state variable $x_1(t)$ in a nonlinear way. In order to avoid this nonlinear dependence, the ψ function can be inset between its two extreme values which are the limits of this function when $x_1(t)$ tends respectively to zero (ψ_1) and to infinity (ψ_2):

$$\lim_{x_1 \to 0} \psi(x_1) = \psi_1 = \frac{1}{1+\exp(a_5)} \tag{3}$$

$$\lim_{x_1 \to \infty} \psi(x_1) = \psi_2 = 1 \tag{4}$$

so that

$$\psi_1 \le \psi(x_1(t)) < \psi_2 \tag{5}$$

In this way, the above values define the two extreme linear systems given by Σ_1 and Σ_2:

$$\Sigma_1 : \begin{cases} \dot{x}_1(t) = -a_1x_1(t) + a_2x_2(t) + a_2w(t) \\ \dot{x}_2(t) = -a_3x_2(t) + a_4x_1(t)\psi_1 \end{cases} \tag{6}$$

and

$$\Sigma_2 : \begin{cases} \dot{x}_1(t) = -a_1x_1(t) + a_2x_2(t) + a_2w(t) \\ \dot{x}_2(t) = -a_3x_2(t) + a_4x_1(t)\psi_2 \end{cases} \tag{7}$$

while a general system in-between both is given by:

$$\Sigma_\lambda : \begin{cases} \dot{x}_1(t) = -a_1x_1(t) + a_2x_2(t) + a_2w(t) \\ \dot{x}_2(t) = -a_3x_2(t) + a_4x_1(t)\psi_\lambda \end{cases} \tag{8}$$

with:

$$\psi_\lambda = \frac{1}{1+\exp(a_5)} + \frac{\lambda}{1+\exp(-a_5)} \tag{9}$$

for $\lambda \in [0,1]$. The design procedure will choose the model in the middle (i.e. for $\lambda = 0.5$) as nominal model while the controller should be robust enough to cope with the whole family. In this way, the controller design problem for the nonlinear system is converted into designing a linear controller able to achieve appropriate tracking and stability properties for the whole family of linear systems defined by (8)-(9). Thus, the controller will be designed for $\psi_{1/2} = \frac{\psi_1+\psi_2}{2} = \frac{2+exp(a_5)}{2+2exp(a_5)}$ while the state space model taken for controller design purposes is:

$$\begin{cases} \dot{x}_1(t) = -a_1x_1(t) + a_2x_2(t) + a_2w(t) \\ \dot{x}_2(t) = -a_3x_2(t) + a_4x_1(t)\frac{2+\exp(a_5)}{2+2\exp(a_5)} \end{cases} \tag{10}$$

which can be cast into the traditional state space representation as:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bw(t) \\ y(t) = Cx(t) + Dw(t) \end{cases} \tag{11}$$

where $x(t)$ is the state vector, $w(t)$ is the input variable and $y(t)$ the output variable. The A, B, C and D matrices are:

$$A = \begin{pmatrix} -a_1 & a_2 \\ a_4\frac{2+\exp(a_5)}{2+2\exp(a_5)} & -a_3 \end{pmatrix} B = \begin{pmatrix} a_2 \\ 0 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 0 \end{pmatrix} \tag{12}$$

while the transfer function corresponding to this linear model is given by $H(s) = D + C(sI - A)^{-1}B$:

$$H(s) = \frac{a_2s + a_2a_3}{s^2 + (a_1+a_3)s + a_1a_3 - \frac{a_2a_4(2+\exp(a_5))}{2+2\exp(a_5)}} \tag{13}$$

In the next subsection a suitable controller for this model is proposed in order to robustly stabilize the whole family (8) what, in turn, will be able to adequately control the nonlinear system as the simulation examples in Section IV will show.

B. Controller Structure

The aim of the controller is to make the heart rate follow the predefined profile set up as reference. In the case of recovery and training programs, the most frequent shape of heart rate reference profile is a trapezium. In this way, during the first part of the exercise, the heart rate will slowly increase from its at-rest value to a maximal one determined according to individual needs. This value is often established by a certain percentage of the maximal heart rate of the individual. For instance, people in rehabilitation after heart surgery will target between 50% and 60% while people involved in weight loss program will focus on 60% or 70% and athletes will be able to achieve between 70% and 90% of their maximal heart rate. The value thus defined will be maintained for a period of time and during the last part of the exercise its intensity will slowly decrease to make the heart rate progressively recover its at-rest value. Therefore, the reference input can be regarded as a ramp followed by a constant value and another ramp at the end. In order to cancel the tracking error during the complete exercise, the corrector has been chosen with a double integral action.Then, the corrector will be a PII controller of the form:

$$K(s) = \frac{W(s)}{E(s)} = K_0\left(\frac{1+\tau s}{\tau s}\right)^2 \tag{14}$$

where $W(s)$ denotes the Laplace transform of the controller's output, $w(t)$, while $E(s)$ denotes the Laplace transform of the tracking error[2]. Moreover, the controller will then be discretized by using the zero-order hold in order to obtain a numerical controller corresponding to the control technique applied in the actual system. The equation of the discrete-time PII controller is of the form:

$$K_d(z^{-1}) = \frac{d_0 + d_1 z^{-1} + d_2 z^{-2}}{b_0 + b_1 z^{-1} + b_2 z^{-2}} \tag{15}$$

that yields the recurrence equation:

$$w(k) = \frac{d_0\varepsilon(k) + d_1\varepsilon(k-1) + d_2\varepsilon(k-2) - b_1 w(k-1) - b_2 w(k-2)}{b_0} \tag{16}$$

were $w(k)$ is the calculated control variable which represent the square of the treadmill speed to apply.

Since the control system will act on the human body and, especially, on his heart rate, the aspects of accuracy and stability are of most importance in the closed-loop system. The speed aspect is of less importance. Thus, a too high speed of the system would increase the variations of the control signal which control the speed of the treadmill. A compromise has to be met in order to limit the control signal deviation while keeping an accurate and robust system with a satisfactory celerity. In order to tune the parameters τ and K_0 of the continuous controller $K(s)$, the phase margin (PM) and the frequency at unity gain (ω_u) of the open loop controlled system will be progressively adjusted. The phase margin will be chosen sufficiently high to reduce the oscillations on the control variable but with an acceptable value for not slowing down the system and augmenting the overshoot. The value of ω_u, on the other hand, will be chosen in way to give an acceptable speed to the system, not too high to not involve oscillations in the output and large deviations of the control signal. The next Section IV will show the specific tuning of the controller and the results when it is discretized and used to control the original nonlinear system.

IV. Simulation Examples

This Section contains some numerical simulation examples showing the tuning method and the results achieved by the proposed controller. Particular values for the model, extracted from [6], will be used to tune the controller and show some examples of the attained results[3]:

$$a_1 = \frac{2.2}{60}, a_2 = \frac{19.96}{60} \quad , a_3 = \frac{0.0831}{60}, \quad a_4 = \frac{0.002526}{60} \quad , \quad a_5 = 8.32 \tag{17}$$

Since these values are provided in [6] in minutes they have been divided by 60 in order to obtain the corresponding values in the units of the International System (seconds). The software used is the powerful development environment Matlab.

The Fig. 2 shows the Bode diagram of both extreme system $\psi(x_1) = \psi_1$ in green and $\psi(x_1) = \psi_2$ in blue. It can be observed a small deviation of gain and phase values at low frequency between the systems. This phenomenon comes from the model nonlinearity and its intensity depends, as can be seen in the figure, on the value of the function $\psi(x_1(t))$. The Bode diagram also reveals a transition frequency (this one to be taken at $-3dB$ lower the gain at low frequency of the system with $\psi(t) = \psi_{1/2}$ corresponding to the red curve in Fig. 2). This cutting frequency of the process is measured as $\omega_c = 0.037 rad/s$. From this value an estimation of the unity gain frequency ω_u of the future open loop controlled system can be made about $5\omega_c < \omega_u < 10\omega_c$ ($0.18 rad/s < \omega_u < 0.37 rad/s$), [10]. In order to obtain a sufficiently stable closed-loop system for the whole family of models (8)-(9) without oscillations in the output and a small overshoot, the phase margin will be chosen high enough, ranging in $60° \leq PM \leq 80°$. The input of the system takes the form of a typical reference profile that could be applied during a training program. It will consist in a 20 minutes exercise started by 4 minutes during which the heart rate will linearly increase to reach 40 bpm above HR_0. This value will be maintained during 11 minutes and will then be progressively decreased to recover the at-rest value.

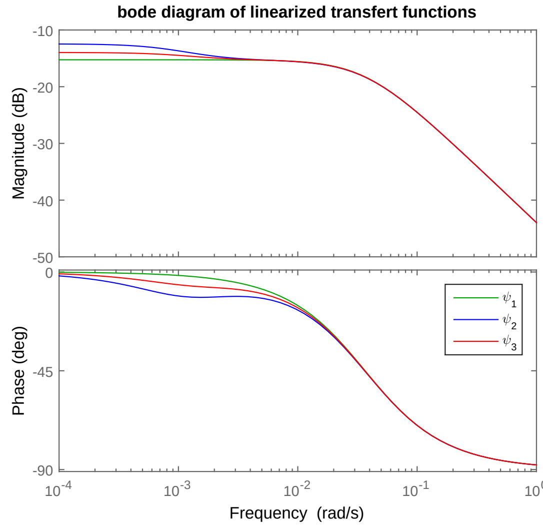

Figure 2. Bode diagram of the linearized models with $\psi = \psi_1$ (green), $\psi = \psi_2$ (blue) and $\psi = \psi_{1/2}$ (red).

A simulation sensitivity analysis is carried out to find the best parameters of the controller. Thus, several values of ω_u and PM within the previously defined intervals will be taken to design a panel of continuous-time controllers which will be discretized by using a ZOH to generate the discrete-time controller (15) and used to test the response of the controlled system to the exercise outlined above. The sample time used to discretize the controller is 0.15 s. This value has been chosen for computer resources reasons, but also because of the updating time of the output value. Indeed, if the human body is a continuous system, the heart rate monitor only gives a new value at each heart pulsation. Considering that the maximum heart rate will be about 180 bpm, the output value is only updated at most each 0.33 s. If we take a little less of the half

[2]We must bear in mind that the actual input is calculated from Eq. (2).

[3]These parameters could be obtained for a specific individual by using an estimation procedure.

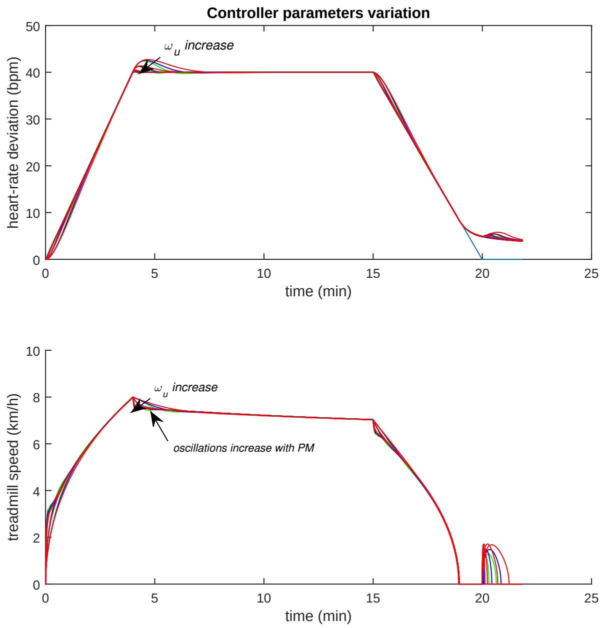

Figure 3. Controller parameters influence on the heart rate response to treadmill exercise. $PM = 60°$ (green), $PM = 70°$ (blue), $PM = 80°$ (red).(Top) heart rate deviation of the exerciser during treadmill exercise. (Bottom) treadmill speed calculated by the numerical PII controller.

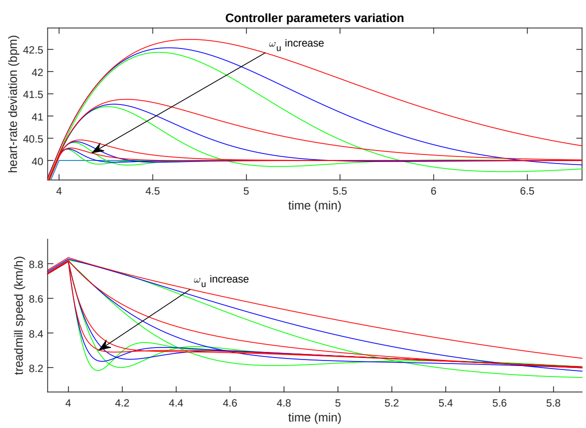

Figure 4. Zoom on the controller parameters influence on the heart rate response to treadmill exercise (Fig. 3). $PM = 60°$ (green), $PM = 70°$ (blue), $PM = 80°$ (red).

of this value in order to not lose information we have that the sample time has been fixed for the controller discretization at 0.15 s. The influence of both parameters on the controller efficiency is shown in Figs. 3 and 4.

The values used in this figure are, for the phase margin, $PM = 60°$, $PM = 70°$ and $PM = 80°$ and for the unity gain frequency, $\omega_u = 0.05 rad/s$, $\omega_u = 0.1 rad/s$, $\omega_u = 0.3 rad/s$ and $\omega_u = 0.5 rad/s$. It can be seen in Figure 4 that the increase of ω_u makes the system faster and reduces the overshoot but also generates quick variations of the treadmill speed that could jeopardize the exerciser. The increase of the phase margin will, on the contrary, slow down the system and augment the overshoot but allows to reduce the oscillations in the heart rate and in the treadmill speed by smoothing the response, what makes the exercise safer. By selecting the best compromise to obtain a controller with a satisfying velocity and low output oscillations, the values have been chosen as $\omega_u = 0.2 rad/s$ and $PM = 70°$. The values of the continuos-time controller (14) satisfying these design criteria are:

$$K_0 = 0.1074, \ \tau = 0.0543 \tag{18}$$

while the discrete-time controller (15) is defined by:

$$K_d(z) = \frac{36.4375 - 70.8825z^{-1} + 34.4719z^{-2}}{1 - 2z^{-1} + z^{-2}} \tag{19}$$

It can be observed in Fig. 3 that the heart rate does not totally recover its at-rest value after the 20 minutes. This situation reflects the fact that the heart is not controllable when it comes to reducing its rate. Therefore, it will recover the at-rest value according to the dynamics of the unforced system (1). This figure also shows an undesired rise on the treadmill speed at 20 minutes. This defect comes from the sharp discontinuity of the input value when the reference signal reaches the at-rest value. It can be overcome by given a smoother input using a low pass filter $F(s) = \frac{1}{(1+10s)^2}$ applied to the input (which is discretized at 0.15 s to cope with the running period of the discrete-time controller).

To show the efficiency and usefulness of the controller, it has been tested with several inputs of different intensities. These reference heart rate signals have been filtered by the low pass filter $F(s)$ to prove that it improves the activity safety and system performances. The results of these simulations are shown in Fig. 5 where the reference input and system output are practically superimposed. This figure shows that the use of a double integration in the controller allows canceling the dynamic error during the tracking of the ramp signal. The effect of the filter $F(s)$ is also clearly noticed by giving an output signal smoother and very close to the reference. It should also be noted that the higher reference profile that corresponds to an increase of 60 bpm of the exerciser heart rate is only shown to prove the efficiency of the controller for high value inputs. Indeed, the treadmill speed exceeds the 8 km/h threshold to reach this heart rate value which normally is the speed limit of the model availability. However, 8 km/h is not a precise value and it is possible that the model remain valid for slightly higher speed.

Since the nonlinear model, even with an individual identification of the parameters, cannot exactly reproduce the variations of the heart rate during an exercise, the designed controller has to be able to ensure good performances even for a heart rate response relatively distant from the model taken into account. To verify this further robustness, the controller can be tested on models with different values of parameters. With a variation of 100% increasing or decreasing of the parameter values from the initial model, the corrected system using always the same numerical controller designed from the nominal model remains stable and the response continues to track the input appropriately what shows the validity of the control approach. In order to illustrate these claims, Figure 6 shows the output of the controlled system for different exercise

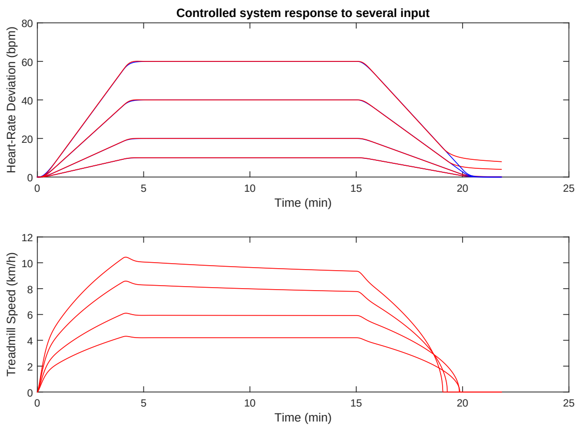

Figure 5. Controlled system response to inputs with different intensities. (top) Heart rate deviation measured in the output of the nonlinear model (blue) following the input reference signal (red). (bottom) Treadmill speed evolution during the exercise.

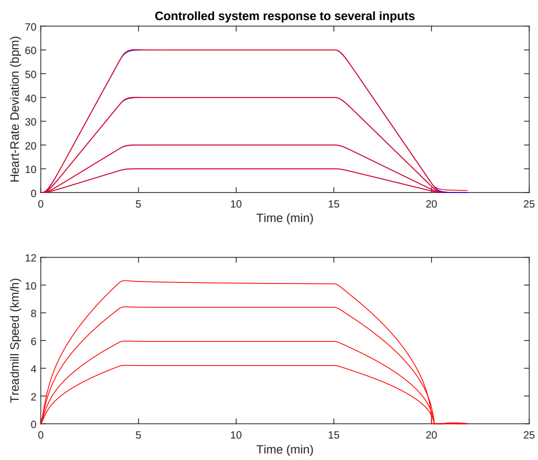

Figure 6. Controlled system response to inputs with different intensities and a 100% of variation of model parameters. (top) Heart rate deviation measured in the output of the non linear model (blue) following the input reference signal (red). (bottom) Treadmill speed evolution during the exercise.

intensities with the previously designed controller (19) and an increase of a 100% of the actual parameters of the model (17).

V. Conclusion

In this paper, a PID-type controller is proposed to regulate a nonlinear model describing the heart rate response during treadmill exercise. The nonlinearity produced by the drift phenomenon depending on the walking speed is firstly inset within its extremal values in order to define a family of linear models covering the original nonlinear system. A linear model located in the middle of these two extreme values is then considered in order to design a robust PII controller. Moreover, a simulation program describing the overall control loop is implemented in order to heuristically tune the parameters of the controller by performing a sensitivity analysis of its critical parameters. Afterwards, a trade-off tuning balancing adequate robustness and speed for the closed-loop is selected for the controller. An input reproducing a typical heart rate reference profile used in a recovery program is defined to test the controller performances in an as realistic way as possible. The controller thus obtained is discretized to define a numerical controller whose recurrence equation can be implemented in a computer-based controller.

The results obtained by simulation show that the controller is efficient to control the nonlinear model. The reference profile is properly followed and the treadmill speed does not undergo sudden changes to ensure a gentle activity. The use of a double integral action on the controller is efficient to cancel the tracking error under ramp and step inputs. This outcome confirms that this control method based on a PID-type controller shows good performances and satisfying robustness. In this way, an effective control is perfomed without needing advanced control strategies nor complex control laws.

Since the model is only valid for treadmill speeds lower than 8 km/h, it may be interesting to obtain a more general one in order to design controllers for healthy people, who can employ speeds in the range of 8-12 km/h to improve their physical condition or for athletic training programs.

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