On Interference Alignment for Crosstalk Mitigation in VDSL2

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***Abstract*— One of the main impairments for advanced digital subscriber line systems is crosstalk. Crosstalk can be effectively cancelled using vectoring. However, some challenges such as implementation and computational complexities associated with full users' coordination when the number of DSL lines is large or when they are not co-located at any end make full vectoring impractical in many scenarios. In this paper, we consider interference alignment (IA) as an alternative technique when full vectoring is not practical. We apply this technique to both non-coordinated and partially coordinated very fast digital subscriber line 2 (VDSL2) systems. The computational complexity of IA can be prohibitive, however, we reduce it by applying IA to subsets of tones resulting in a computational complexity much smaller than that of vectoring. We also use iterative IA algorithms for performance improvement. Simulation results show that IA increases the achievable rates of VDSL2 loops considerably without a need for signal coordination among users. Moreover, when IA and partial vectoring are applied together, the users can achieve bit rates moderately close to that of a crosstalk-free network.**

***Index Terms*— digital subscriber line, interference alignment, maximum SINR, minimum interference leakage, vectoring.**

I. Introduction

Digital subscriber line (DSL) systems provide broadband services to customers using ordinary telephone loops. One of the most significant sources of performance degradation in advanced DSL systems is crosstalk, i.e., the undesired signal of an unintended DSL transmitter being received by the intended DSL receiver due to the electromagnetic induction between the twisted wire pairs inside a telephone cable. Crosstalk can be a few orders of magnitude larger than the background noise. As a result, crosstalk mitigation is essential in DSL to achieve data rates comparable to those of alternative access technologies such as broadband over coaxial cable.

Vectoring is an effective approach for eliminating crosstalk [1]. When all DSL lines are co-located at one side (usually at the central office or a street cabinet) vectored DSL systems can be implemented. Vectored DSL precoding and decoding techniques are applied at the transmitter and receiver sides for crosstalk mitigation in downstream and upstream directions, respectively. Vectoring increases achievable data rates significantly, but it has some challenges in practical implementations [2], [3]. Vectored DSL requires perfect signal coordination between users. When the number of users is large or the DSL lines are not connected to the backbone network at the same location (e.g. as in the central office/ remote terminal (CO/ RT) deployments) vectoring can not be applied to all users.

To resolve this problem, users can be divided into a few groups. Crosstalk between users in each group (intra-group crosstalk) can be eliminated using vectoring, however, crosstalk between users in different groups (inter-group crosstalk) still remains. Interference alignment (IA) has been proposed as a method for inter-group crosstalk mitigation [4].

IA employs precoding at the transmitter and interference suppression at the receiver for crosstalk mitigation between users. IA aligns the interference seen at each receiver in a subspace of the signal space. Hence, each receiver can recover its original signal by projecting the received signal on the nullspace of the interference subspace [5]. IA is a major approach for scenarios with multiple interfering users [6]. IA is used in wireless networks for dealing with interferences in low SNR and high interference scenarios [7], [8]. The DSL crosstalk interference channel is similar to the wireless interference channel which motivate us to use IA in DSL.

The computational complexity of IA grows linearly with the number of users, $\mathcal{N}$, and quadratically with the number of tones, K, i.e., $\mathcal{O}(\mathcal{N}K^2)$. Since K is in the order of 4000 in advanced DSL systems such as VDSL2 and G.fast, the computational complexity is simply prohibitive. However, in [4] it was shown that the computational complexity can be reduced by applying IA on different subsets of tones resulting in a system with computational complexity smaller than that of vectored DSL. Considering this advantage, we are motivated to use this approach to reduce all crosstalk (both inter-group and intra-group) in DSL system. We apply IA independently and also jointly with vectoring to evaluate the quality of this technique.

In general IA is achieved by using precoding and decoding matrices over all signal dimensions such as time, frequency and space [9]. In wireless systems, IA is applied on space dimension, however, in DSL systems, it is applied on frequency dimension, since the DSL system use many frequency tones. There are several algorithms for updating precoding and suppression matrices. These algorithms are implemented in an alternative fashion. In [10], two algorithms have been introduced for IA in wireless channels, namely, the minimum interference leakage (minIL) algorithm and the maximum SINR (maxSINR). These algorithms are investigated in this paper for observing the potential of the IA approach for crosstalk mitigation in DSL systems.

We use the two IA algorithm for crosstalk mitigation of VDSL2 users with different SNR's. Simulation results show that the maxSINR algorithm outperforms minIL algorithm for low SNR users significantly. Moreover it achieves the capacity of crosstalk free network at much lower SNR's compared to the minIL algorithm. Using IA, the achievable bit rates are about 80% of a crosstalk free network which are considerably higher than those achieved by partial vectoring.

The paper is organized as follows: the DSL system model is described in Section II. In Section III the concept of IA is introduced in more details. The IA algorithms used in this paper are introduced in Section IV. Simulation results are presented in Section V. Finally, the paper is concluded in Section VI.

II. System Model

We consider a DSL system with K tones and $\mathcal{N}$ users. We assume that the users are divided into G groups, where $\mathcal{N}_i, i = 1, ..., G$ denotes the set of users in group i. In this paper, we assume that all users use synchronous discrete multi-tone transmission (DMT). Under this assumption the transmitted signal on tone k is written by [1]

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k, \quad (1)$$

where $\mathbf{y}_k = [(\mathbf{y}_k^{(1)})^T, ..., (\mathbf{y}_k^{(G)})^T]^T = [y_k^{(1)}, ..., y_k^{(\mathcal{N})}]^T$, $\mathbf{x}_k = [(\mathbf{x}_k^{(1)})^T, ..., (\mathbf{x}_k^{(G)})^T]^T = [x_k^{(1)}, ..., x_k^{(\mathcal{N})}]^T$ and $\mathbf{z}_k = [(\mathbf{z}_k^{(1)})^T, ..., (\mathbf{z}_k^{(G)})^T]^T = [z_k^{(1)}, ..., z_k^{(\mathcal{N})}]^T$ are the vectors of the received, transmitted and Gaussian noise, respectively, on tone k, and $(\cdot)^T$ denotes the vector and matrix transpose operation. Parameters $\mathbf{y}_k^{(i)}, \mathbf{x}_k^{(i)}$ and $\mathbf{z}_k^{(i)}$ denote the vectors of the received, transmitted and Gaussian noise signal for group i, respectively, where $y_k^{(n)}$, $x_k^{(n)}$ and $z_k^{(n)}$ denote the received, transmitted and noise signals of user n on tone k. The transmit power spectral density (PSD) for user n on frequency tone k is denoted by $p_k^{(n)} = \frac{\mathcal{E}[|x_k^{(n)}|^2]}{\Delta f}$, where $\mathcal{E}$ denotes the expected value, and Δf denotes the frequency tone spacing and $\sigma_k^{(n)} = \frac{\mathcal{E}[|z_k^{(n)}|^2]}{\Delta f}$ denotes the noise PSD.

The channel matrix on tone k, $\mathbf{H}_k$, is $\mathcal{N} \times \mathcal{N}$ and can be written as

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{H}_k^{11} & \mathbf{H}_k^{12} & \cdots & \mathbf{H}_k^{1G} \\ \mathbf{H}_k^{21} & \mathbf{H}_k^{22} & \cdots & \mathbf{H}_k^{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_k^{G1} & \mathbf{H}_k^{G2} & \cdots & \mathbf{H}_k^{GG} \end{bmatrix}, \quad (2)$$

where $[\mathbf{H}_k]_{n,m} = h_k^{(n,m)}$ is the channel response from the m-th user to the n-th user on frequency tone k. The sub-matrix $\mathbf{H}_k^{ij}$ for $i \neq j$ denotes the crosstalk channel from group j to group i and $\mathbf{H}_k^{ii}$ denotes the direct and crosstalk channel matrix in group i. In this paper, we consider downstream (DS) transmission. However, the technique can be applied to upstream (US) transmission as well. We assume that all sub-matrices $\mathbf{H}_k^{ii}$ are perfectly known and can be used for inter-group crosstalk calculation. We consider the diagonalizing precoder [11] as the crosstalk cancellation scheme. The diagonalizing precoder multiplies the transmitted signal vector by a precoding matrix $\mathbf{B}_k^{(i)}$ as follows:

$$\mathbf{y}_k^{(i)} = \mathbf{H}_k^{ii}\mathbf{B}_k^{(i)}\mathbf{x}_k^{(i)} + \sum_{j \neq i} \mathbf{H}_k^{ij}\mathbf{B}_k^{(j)}\mathbf{x}_k^{(j)} + \mathbf{z}_k^{(i)}, \quad (3)$$

where $\mathbf{B}_k^{(i)} = (\beta_k^{(i)})^{-1}(\mathbf{H}_k^{ii})^{-1}\text{diag}\{\mathbf{H}_k^{ii}\}$ is called a precoding matrix, diag(C)denotes a diagonal matrix with diagonal elements the same as those of C and $\beta_k^{(i)} = \max_{n \in \mathcal{N}_i} \|[(\mathbf{H}_k^{ii})^{-1}\text{diag}\{\mathbf{H}_k^{ii}\}]_{\text{row}n}\|$ [11]. With this notation we can rewrite the combination of the per-group precoders and the channel matrix as follows:

$$\tilde{\mathbf{H}}_k = \begin{bmatrix} \tilde{\mathbf{H}}_k^{11} & \tilde{\mathbf{H}}_k^{12} & \cdots & \tilde{\mathbf{H}}_k^{1G} \\ \tilde{\mathbf{H}}_k^{21} & \tilde{\mathbf{H}}_k^{22} & \cdots & \tilde{\mathbf{H}}_k^{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{H}}_k^{G1} & \tilde{\mathbf{H}}_k^{G2} & \cdots & \tilde{\mathbf{H}}_k^{GG} \end{bmatrix}, \quad (4)$$

where $\tilde{\mathbf{H}}_k^{ii} = (\beta_k^i)^{-1}\text{diag}\{\mathbf{H}_k^{ii}\}$ and $\tilde{\mathbf{H}}_k^{ij} = \mathbf{H}_k^{ij}\mathbf{B}_k^j, i \neq j$ are the modified intra-group crosstalk channels after partial vectoring.

In DSL, the number of bits that can be transmitted on tone k for user n is obtained as follows:

$$b_k^{(n)} = \log_2(1 + \frac{1}{\Gamma}\text{SINR}_k^{(n)}), \quad (5)$$

where $\text{SINR}_k^{(n)}$ denotes the signal to interference plus noise ratio (SINR) of user n on frequency tone k, and Γ denotes the SNR gap which is a function of the bit error rate. The SINR of user n on frequency tone k is given by

$$\text{SINR}_k^{(n)} = \frac{p_k^{(n)}|\tilde{h}_k^{(n,n)}|^2}{\sum_{m \notin \mathcal{N}_i} p_k^{(m)}|\tilde{h}_k^{(n,m)}|^2 + \sigma_k^{(n)}}, \quad (6)$$

where $\tilde{h}_k^{(n,m)} = [\tilde{\mathbf{H}}_k]_{(n,m)}$ denotes the crosstalk channel gain from user m to user n on tone k. In practical DSL systems $b_k^{(n)}$ is truncated to an integer number and is bounded by b_{max} where $2^{b_{max}}$ is the maximum QAM constellation size.

Finally, the sum-rate of user n is given by

$$\mathcal{R}^{(n)} = f_s \sum_{k=1}^{K} b_k^{(n)}, \quad (7)$$

where f_s denotes the DMT symbol rate.

III. Interference Alignment

In this section we introduce the IA approach for inter-group crosstalk mitigation. The interference is aligned by using precoder and decoder matrices applied on $\tilde{\mathbf{x}}^{(n)} = [\tilde{x}_1^{(n)}, ..., \tilde{x}_{d_n}^{(n)}]^T$ where d_n is the number of degrees of freedom for user n. That is the precoder and decoder matrices are applied to the signals transmitted by each user in the frequency dimension only. Therefore, we do not need signal level coordination between users in different groups to apply IA. For user $n \in \mathcal{N}_i$, the $K \times d_n$ precoder and decoder matrices, $\mathbf{V}^{(n)}$ and the $\mathbf{U}^{(n)}$ are used to transform the modified channel matrix $\mathbf{H}^{(n,n)}$ where $\tilde{\mathbf{H}}^{(n,n)} = \text{diag}\{\tilde{h}_1^{(n,n)}, ..., \tilde{h}_K^{(n,n)}\}$. Then, the received signal over all K tones for user n is

$$\begin{aligned} \tilde{\mathbf{y}}^{(n)} &= (\mathbf{U}^{(n)})^H\tilde{\mathbf{H}}^{(n,n)}\mathbf{V}^{(n)}\tilde{\mathbf{x}}^{(n)} \\ &+ \textstyle\sum_{m \notin \mathcal{N}_i} (\mathbf{U}^{(n)})^H\tilde{\mathbf{H}}^{(n,m)}\mathbf{V}^{(m)}\tilde{\mathbf{x}}^{(m)} \\ &+ \sum_{\substack{m \in \mathcal{N}_i; \\ m \neq n}} (\mathbf{U}^{(n)})^H \underbrace{\tilde{\mathbf{H}}^{(n,m)}}_{=0} \mathbf{V}^{(m)}\tilde{\mathbf{x}}^{(m)} + \tilde{\mathbf{z}}^{(n)}, \end{aligned} \quad (8)$$

where $(\cdot)^H$ denotes the transpose conjugate operator, $\tilde{\mathbf{z}}^{(n)} = (\mathbf{U}^{(n)})^H\mathbf{z}^{(n)}$, $\tilde{\mathbf{y}}^{(n)} = (\mathbf{U}^{(n)})^H\mathbf{y}^{(n)}$ and $\tilde{\mathbf{H}}^{(n,m)}$ is zeros for $m \in \mathcal{N}_i, m \neq n$ thanks to the diagonalizing precompensator. In order to cancel inter-group crosstalk we need the second term in (8) to be zero. The main IA problem for $n \in \mathcal{N}_i$ can be written as follows:

$$\begin{aligned}(\mathbf{U}^{(n)})^H\tilde{\mathbf{H}}^{(n,m)}\mathbf{V}^{(m)} = \mathbf{0}_{d_n\times d_n},\ \forall m \notin \mathcal{N}_i \\ \text{rank}((\mathbf{U}^{(n)}))^H\tilde{\mathbf{H}}^{(n,n)}\mathbf{V}^{(n)}) = d_n.\end{aligned} \quad (9)$$

This problem may or may not have a solution depending on the channel gains and d_n. However, it has been shown that when $d_n = \frac{K}{2}$ perfect IA can be achieved [8]. However, in practice, it may not be possible to achieve the perfect alignment. Therefore, we choose a greater value for d_n such as $d_n = K$ for which $\mathbf{U}^{(n)}$ and $\mathbf{V}^{(n)}$ can be calculated more efficiently [4].

After optimization of precoder and decoder matrices using algorithms which is presented in Section IV, the IA sum rate is written as follows:

$$\text{R}_{\text{IA}} = \sum_{n=1}^{\mathcal{N}} f_s \log_2 |\mathbf{I} + \frac{1}{\Gamma}\text{SINR}^{(n)}|, \quad (10)$$

where the SINR for user n is

$$\text{SINR}^{(n)} = \frac{(\mathbf{U}^{(n)})^H\tilde{\mathbf{H}}^{(n,n)}\mathbf{V}^{(n)}\mathbf{P}^{(n)}(\mathbf{V}^{(n)})^H(\tilde{\mathbf{H}}^{(n,n)})^H\mathbf{U}^{(n)}}{(\mathbf{U}^{(n)})^H\mathbf{A}^{(n)}\mathbf{U}^{(n)}}, \quad (11)$$

where

$$\mathbf{A}^{(n)} = \sigma_n^2\mathbf{I} + \sum_{m\notin\mathcal{N}_i}\tilde{\mathbf{H}}^{(n,m)}\mathbf{V}^{(m)}\mathbf{P}^{(m)}(\mathbf{V}^{(m)})^H(\tilde{\mathbf{H}}^{(n,m)})^H \quad (12)$$

and $\mathbf{I}$ is the identity matrix of size d_n.

IV. Algorithms for Interference Alignment

In this section we introduce two IA algorithms which are originally proposed for the wireless interference channel. In order to solve the linear IA problems in (9), most studies have focused on iterative algorithms. By iteratively minimizing leakage interference and maximizing SINR, two distributed IA algorithms have been proposed in [10] that require only local channel state information (CSI) at each node. The minIL algorithm minimizes the leakage interference by updating the precoder and decoder matrices. This algorithm minimizes the interference leakage provided that the signal power is high. The maxSINR algorithm can be used for low or moderate SNR signals by maximizing the signal to interference plus noise ratio (SINR). Previous studies show that the maxSINR algorithm achieves much higher rates than the minIL algorithm in wireless channels. Inspired by these works, with different optimization goals, several iterative algorithms to numerically find the alignment solutions have been proposed. These solutions try to achieve the optimal tradeoff balance between performance and complexity for IA in MIMO interference channels [12], [13]. In this paper we use two iterative IA algorithm for crosstalk mitigation in DSL systems. These algorithms are introduced in the following:

A. The MinIL Algorithm

This algorithm aims to achieve IA by reducing the interference leakage. The quality of the algorithm is measured by the power of the leakage interference at each receiver, i.e. the interference power remaining in the received signal after decoding. The algorithm starts with arbitrary precoder and decoder matrices $\mathbf{V}^{(n)}, \mathbf{U}^{(n)}$ which are updated iteratively.

From (8) the total interference leakage at user n due to all interfering users is given by [10]

$$\text{IL}^{(n)} = \text{Tr}[(\mathbf{U}^{(n)})^H\mathbf{Q}^{(n)}\mathbf{U}^{(n)}], \quad (13)$$

where Tr[C] denotes the trace of matrix C,

$$\mathbf{Q}^{(n)} = \sum_{m\notin\mathcal{N}_i}\tilde{\mathbf{H}}^{(n,m)}\mathbf{V}^{(m)}\mathbf{P}^{(m)}(\mathbf{V}^{(m)})^H(\tilde{\mathbf{H}}^{(n,m)})^H, \quad (14)$$

is the interference covariance matrix for user n at tone k, and $\mathbf{P}^{(m)} = \text{diag}\{p_1^{(m)}, p_2^{(m)}, ..., p_K^{(m)}\}$ is the transmit PSD of user m over K tones arranged on the diagonal of a diagonal matrix of size K. The optimization problem for user n with fixed $\mathbf{V}^{(m)}$ and $\mathbf{P}^{(m)}$ is

$$\begin{aligned}\min_{\mathbf{U}^{(n)}}\ \text{IL}^{(n)} \\ \text{subject to } \mathbf{U}^{(n)}(\mathbf{U}^{(n)})^H = \mathbf{I}_{d_n},\end{aligned} \quad (15)$$

In other words, user n chooses its interference decoder matrix, $\mathbf{U}^{(n)}$, to minimize the leakage interference due to all interfering users. The space which is spanned by the eigenvectors corresponding to the d_n smallest eigenvalues of the interference covariance matrix $\mathbf{Q}^{(n)}$ is the subspace of the received signal that contains the least interference. Hence, the d_n columns of $\mathbf{U}^{(n)}$ are obtained as follows:

$$\mathbf{U}^{(n)} = \nu_d[\mathbf{Q}^{(n)}],\ d = 1, ..., d_n, \quad (16)$$

where $\nu_d[\mathbf{C}]$ denotes the eigenvectors corresponding to the d smallest eigenvalues of $\mathbf{C}$. Conversely, when $\mathbf{U}^{(n)}$ is fixed, the covariance matrix is written as follows:

$$\mathbf{Q}^{(n)} = \sum_{m\notin\mathcal{N}_i}\tilde{\mathbf{H}}^{(n,m)}\mathbf{U}^{(m)}\mathbf{P}^{(m)}(\mathbf{U}^{(m)})^H(\tilde{\mathbf{H}}^{(n,m)})^H. \quad (17)$$

Similar to (16), the d_n columns of $\mathbf{V}^{(n)}$ are obtained as follows:

$$\mathbf{V}^{(n)} = \nu_d[\mathbf{Q}^{(n)}],\ d = 1, ..., d_n. \quad (18)$$

The above process continues until the algorithm converges. The convergence of this algorithm has been proven in [10], by showing that the total leakage interference reduces in each iteration.

B. The MaxSINR Algorithm

In the last section we introduced minIL algorithm for perfect IA. This algorithm seeks to produce an interference free subspace of the required number of dimensions, that is designed as the desired signal subspace. This algorithm can not be used for low power signals, i.e. it makes no attempt to maximize the desired signal power within desired signal subspace [10]. In the minIL algorithm, we just eliminate the interference. In fact, the algorithm is optimal when all signal powers approach infinity and it is suboptimal for low or moderate SNRs. It has been shown that for low to moderate SINR the maxSINR algorithm yields higher bit rates [10].

In this section we briefly explain the maxSINR algorithm. This algorithm optimizes the precoders and decoders (i.e. $\mathbf{V}^{(n)}, \mathbf{U}^{(n)}$) in order to maximize the SINR instead of only minimizing the leakage interference. This algorithm updates $\mathbf{V}^{(n)}$ and $\mathbf{U}^{(n)}$ iteratively for maximizing the SINR. In this algorithm, it is not necessary for the precoder to be orthonormal. On the contrary orthogonal precoding is not optimal for SINR maximization. The SINR for user n is presented in (11).

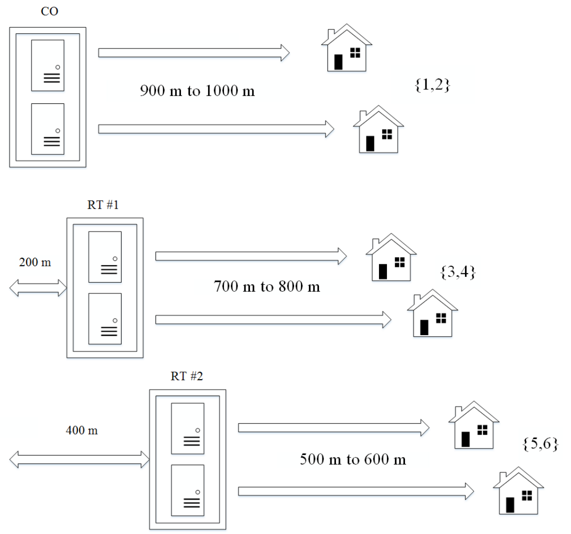

Figure 1: Scenario 1.

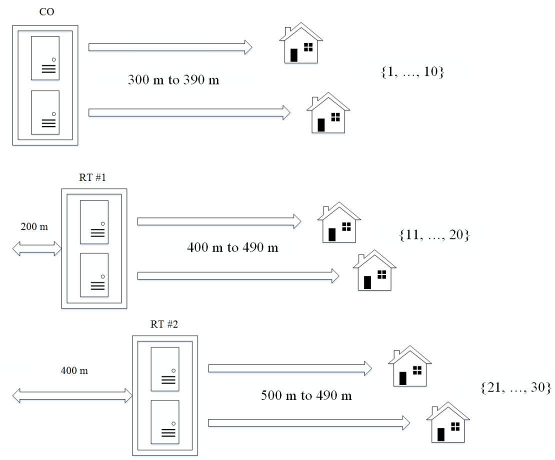

Figure 2: Scenario 2.

The decoder matrix $\mathbf{U}^{(n)}$ that maximizes $\text{SINR}^{(n)}$ is given by [10]

$$\mathbf{U}^{(n)} = \frac{(\mathbf{A}^{(n)})^{-1}\tilde{\mathbf{H}}^{(n,n)}\mathbf{V}^{(n)}}{\|(\mathbf{A}^{(n)})^{-1}\tilde{\mathbf{H}}^{(n,n)}\mathbf{V}^{(n)}\|}. \quad (19)$$

This algorithm starts with an arbitrary precoder matrix $\mathbf{V}^{(n)}$ with linearly independent unit vector columns. Then the algorithm begins the iteration and computes $\mathbf{U}^{(n)}$ using (19). In the next step, it uses the obtained $\mathbf{U}^{(n)}$ as a precoder and updates the decoder matrix. The iteration continues until convergence.

V. Simulation Results

We consider two scenarios in our simulations. The first one is designed for comparing the performance of IA algorithms and the second one is designed for evaluating the performance of IA used jointly with vectoring. The loop lengths are shorter and the number of users is larger in the second scenario. We use the 26 AWG [14] cable model and the standard ANSI crosstalk model [14]. The frequency tone spacing is set to $\Delta_f = 4.3125$ kHz, and the DMT symbol error rate is set to $f_s = 4$ kHz. The maximum transmit power for each modem is set to 11.5 dBm. We use VDSL2 E17 bandplan [15], which consists of three downstream bands 0.276 - 3.75 MHz, 5.2 - 8.5 MHz, and 12 - 17.664 MHz. The SINR gap is set to 9.75 dB. The number of tones in each subset is set to 10. We use flat -60 dBm/Hz PSD in our simulations. The PSD of noise is set to -140 dBm/Hz.

Scenario 1: This scenario consist of 6 users in three groups of size 2 (i.e. $\{1,2\}$,$\{3,4\}$ and $\{5,6\}$) as shown in Fig. 1. The loop lengths of the users in group one are 900 m and 1000 m. Group one contains low SNR users which is because of relatively long loop lengths and strong crosstalk power due to proximity to the other groups' downstream modems. Group two is offset by 200 m. The loop lengths of the users in group two are 700 m and 800 m. Group two contains moderate SNR users (i.e., medium loop lengths and crosstalk). Group three is offset by 400 m. The loop lengths of the users in group three are 500 m and 600 m. This group contains high SNR users which is because of short loop lengths and weak crosstalk due to remote crosstalking users in groups 1 and 2.

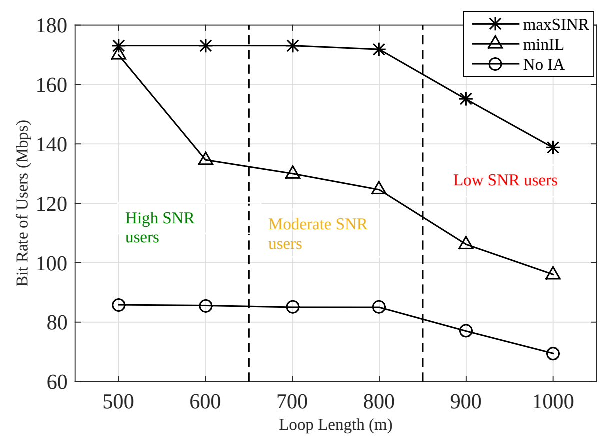

Figure 3: The sum achievable rates vs. loop length for users in Scenario 1 using the minIL and maxSINR algorithms compared to the "No IA" case.

Simulation results for Scenario 1 is illustrated in Fig. 3 where the users achievable rates are plotted vs. loop length for the two IA algorithms and the case without using IA (labeled "No IA"). Note that we are not using any vectoring technique in Scenario 1. As can be seen, using IA we can achieve much higher bit rates than those achieved without crosstalk cancellation in the simulated scenario. The results show that for low, moderate and high SNR users, the maxSINR algorithm achieves much higher rates than the minIL algorithm. The achievable bit rates of users decrease by increasing the loop lengths. However, by increasing the loop lengths the minIL algorithm performance degrades much faster than the maxSINR algorithm which is not affected by loop length except for the loops longer than 900 m. Therefore, the maxSINR algorithm can be the IA approach of choice particularly when low or moderate SNR users exist in the network.

Scenario 2: This scenario consists of 30 users in three groups of size 10 (i.e., $\{1, 2, ..., 10\}$, $\{11, ..., 20\}$ and

$\{21, ..., 30\}$) as follows: The second and third groups are offset by 200 m and 400 m from the central office, respectively. The loop lengths of the first group (i.e., users 1 to 10) are 300, 310, ..., 390 m, respectively. The loop lengths of the users in the second group are 200, 210, ..., 290 m, respectively and the loop lengths of the users in the third group are 100, 110, ..., 190 m, respectively. In this scenario, we assume that the intra-group crosstalk is canceled in each group using the diagonalizing precompensator. A schematic of this scenario is illustrated in Fig. 2.

Simulation results for Scenario 2 is summarized in Table I. The sum achievable rate of users is listed for the following cases 1) full vectoring 2) IA (without vectoring) 3) partial vectoring and IA, 4) partial vectoring (without IA), and 5) no crosstalk cancellation scheme and is compared to the sum achievable rate of a crosstalk free network . Full vectoring can almost achieve the capacity of crosstalk free network. IA can independently achieve 69.10% of network capacity which is 26.91% higher than the achievable rates when no interference cancellation scheme is used. When partial vectoring and IA are applied jointly the achievable sum rate increases from 69.10% to 82.46% of the crosstalk free rates. Finally, when partial vectoring is applied without IA, the sum achievable rates are only 51.24% of the crosstalk free network. Obviously, IA is capable of achieving higher bit rates than partial vectoring with smaller computational complexity. Moreover, the computational complexity of IA and joint IA and partial vectoring is considerably smaller than full vectoring which makes IA a practical solution for large scenarios.

Table I: The summary of the achievable sum rate of the simulated crosstalk mitigation techniques for Scenario 2.

Technique	Sum rate (Mbps)	% of crosstalk free channel
Full vectoring	5560.4	99.8
IA and partial vectoring	4659.7	82.46
IA (without partial vectoring)	3905.5	69.10
Partial vectoring (without IA)	2850.6	51.24
No crosstalk cancellation scheme	2347.2	42.19

VI. CONCLUSION

Full vectoring can achieve crosstalk free rates in VDSL2 scenarios. Unfortunately, full vectoring is impractical when the number of users is large or in RT/CO deployments. In this paper, IA is proposed as an alternative crosstalk cancellation scheme for VDSL2 when full vectoring is impractical. We considered two IA algorithms, namely the minIL and the maxSINR algorithms, which were introduced for wireless systems. These algorithms work iteratively to update precoder and decoder matrices. We compared the two algorithms for low, moderate and high SNR users. Our simulation results show that IA outperforms partial vectoring considerably. In general, IA can increase the achievable rates of the users significantly particularly when it is applied jointly with partial vectoring. In contrast to vectoring, IA does not require signal level coordination among users and has much smaller computational complexity. Moreover, IA is implemented by the users separately making it much more practical from the computational and implementational complexity points of view.

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