Adaptive Neural Decentralized Control for Nonlinear Large-Scale Systems

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Abstract—**This paper presents an adaptive decentralized control method for a class of nonlinear large-scale systems with unknown nonlinear functions and bounded time varying state delays. The adaptive compensation controller is constructed by utilizing Neural Networks (NN) and a backstepping design method. With the help of NNs to approximate the unknown nonlinear functions, the novel adaptive control approach is developed by using the backstepping design method. The appropriate Lyapunov-Krasovskii type functionals are introduced to design new adaptive laws to compensate the unknown nonlinearities as well as uncertainties from unknown state delays. The proposed design method does not require a priori knowledge of the bounds of the unknown time delays. The boundedness of all the closed-loop signals is guaranteed and the tracking error is proved to converge to a small neighborhood of the origin. As an application, the proposed approach is employed for a two inverted pendulums. The simulation results show effectiveness of the proposed adaptive decentralized control approach.**

Keywords—Large-scale systems; Nonlinear time delay systems; Backstepping; Neural Networks.

I. INTRODUCTION

In the past decade, Neural Networks (NNs) and Fuzzy Logic Systems (FLSs) have been extensively used for modeling and controller design for uncertain nonlinear systems [1-3]. In [2], a neural network based adaptive control scheme was proposed for a class of uncertain nonlinear systems. The fuzzy adaptive control scheme was presented in [3] for a class of uncertain nonlinear systems. Due to NNs and FLSs ability to approximate the uncertain nonlinear smooth functions, various NN and fuzzy based control approaches were proposed for nonlinear large scale systems [4]-[13]. Large-scale system is considered as a dynamical system that is composed of some lower-order subsystems with interconnections and often exists in many practical applications such as electric power systems, computer network systems and aerospace systems. In addition, because of the presence of the uncertainties in interconnections terms of the large-scale systems, adaptive control has been an effective tool to design controllers for these systems [6]-[13].

Many engineering systems, such as chemical reactors, networked control systems, electrical networks and so on have the characteristics of time delays. Due to the effect of time delays, these systems may own instability and poor performance [14]. Consequently, the stability analysis and control design for time delay systems attracted considerable attention over the past years [14-18]. So far, considerable attention has been devoted to the stability analysis and control design for linear time delay systems [15]-[18]. For nonlinear time delay systems, the adaptive control problem was considered in [19]-[23] for strict-feedback nonlinear time delay systems. The proposed methods in [19]-[22] proved that all the closed loop signals were uniformly ultimately bounded; however, the tracking problem was not considered. The considered nonlinear systems in [19]-[23] contained known control gains, unknown constant delays or unknown time varying delays with known bounds. However, the considered nonlinear functions in [19]-[23] were known or should be linearly parameterized. To remove this restrictive condition, the latest work in [24]-[27] considered the control problem of uncertain nonlinear systems, in which NNs or FLSs were employed to approximate the unknown nonlinearities. For large-scale nonlinear time delay systems, the adaptive control approaches were presented in [28]-[30]. The proposed controllers in [28]-[30] were delay dependent and the upper bounds of constant or time varying delays should be known.

In this manuscript, an adaptive decentralized state feedback controller is proposed for a class of nonlinear large-scale time delay systems. The considered nonlinear system contains the unknown nonlinear functions, unknown control gains functions and bounded time varying delays. The state delays values are considered to be unknown and time varying with unknown bounds. In engineering practice, it may be difficult to obtain exact information of time varying delays. Therefore, this paper proposed the decentralized time delay independent control scheme for nonlinear large-scale time delay systems. the considered large-scale nonlinear time delay system is more general from the considered systems in the previous literature. In this paper, the unknown nonlinear functions are approximated with the help of NNs and the adaptive control approach is constructed by using the backstepping design method. In addition, appropriate Lyapunov-Krasovskii type functionals are introduced to design new adaptive laws. The proposed method proves that, not only all the signals in the closed loop system are bounded, but also the tracking errors converge to a small neighborhood of the origin.

The paper is organized as follows. In section 2, the system description is given along with the necessary assumptions. In section 3, function approximation by using Neural Networks is explained. In section 4, design and stability analysis of an adaptive NN based decentralized control scheme is investigated. In section 5, simulation results are presented to illustrate the effectiveness of the proposed control scheme. Finally, the paper is concluded in section 6.

II. PROBLEM STATEMENT

Consider a class of interconnected large-scale nonlinear time delay systems, which is composed of N subsystems, the ith subsystem for $i = 1, \dots, N$, is given as

$$\dot{x}_{i,1}(t) = x_{i,2} + f_{i,1}(\bar{x}_{i,1}) + h_{i,1}\left(\bar{x}_{i,1}(t - \tau_{i1}(t))\right) + \Delta_{i,1}(\bar{y}) \quad (1)$$

$$\dot{x}_{i,2}(t) = x_{i,3} + f_{i,2}(\bar{x}_{i,2}) + h_{i,2}\left(\bar{x}_{i,2}(t - \tau_{i2}(t))\right) + \Delta_{i,2}(\bar{y})$$

$$\vdots$$

$$\dot{x}_{i,n_i}(t) = u_i(t) + f_{i,n_i}(\bar{x}_{i,n_i}) + h_{i,n_i}\left(\bar{x}_{i,n_i}\left(t - \tau_{i,n_i}(t)\right)\right) + \Delta_{i,n_i}(\bar{y})$$

$$y_i(t) = x_{i,1}(t),$$

where $\bar{x}_{i,j} = [x_{i,1}, x_{i,2}, \dots, x_{i,j}]^T$, $1 \le i \le N, 1 \le j \le n_i$ and $\bar{y} = [y_1, y_2, \dots, y_N]^T \in R^N$ are state vectors and output vector of the system, respectively. $f_{i,j}(\bar{x}_{i,j})$ and $h_{i,j}(\bar{x}_{i,j})$ are unknown smooth nonlinear functions, $\Delta_{i,j}(\bar{y})$ are unknown smooth nonlinear functions that represent the interconnection among subsystems and $\tau_{i,j}(t)$ are unknown state delays.

The control objective is to design a neural-network-based adaptive decentralized controller for plant (1) in order to assure that all the closed loop signals are bounded and the plant output $y_i(t), i = 1, \dots, N$ tracks a reference signal $y_{d_i}(t), i = 1, \dots, N$, despite the presence of unknown nonlinear functions and time varying delays. For this purpose, the following assumptions are considered.

Assumption 1. Nonlinear functions $h_{i,j}(.), 1 \le i \le N, 1 \le j \le n_i$, satisfy the following inequality

$$\left|h_{i,j}(\bar{x}_{i,j})\right|^2 \le (\bar{x}_{i,1} - y_{d_i})\bar{H}_{i,j}((\bar{x}_{i,1} - y_{d_i})) + \bar{h}_{i,j}\left(y_{d_i}(t)\right) + d_{i,j}$$

where $\bar{H}_{i,j}(.)$ is an unknown nonlinear function, $\bar{h}_{i,j}(.)$ is an unknown bounded function with $\bar{h}_{i,j}(0) = 0$ and $d_{i,j}$ is an unknown positive scalar.

Assumption 2. The interconnected nonlinear function $\Delta_{i,j}(\bar{y})$ satisfy

$$\left|\Delta_{i,j}(\bar{y}) - \Delta_{i,j}(\bar{y}_d)\right| \le \sum_{l=1}^{N} \left|\Phi_{i,j,l}(\bar{y}_d)\right| \left|y_l - y_{d_l}\right|$$

where $\Phi_{i,j,l}(.)$ are unknown smooth functions and $\bar{y}_d = [y_{d_1}, \dots, y_{d_N}]^T$.

Assumption 3. The reference signal $y_{d_i}(t), i = 1, \dots, N$ and its first n_{max}-th order derivatives $y_{d_i}^{(k)}(k = 1, \dots, n_{max}), n_{max} = max\{n_1, \dots, n_N\}$ are bounded and piecewise continuous.

Assumption 4. The unknown time varying delays $\tau_{i,j}(t), i = 1, \dots, N, j = 1, \dots, n_i$, are differentiable functions satisfying

$$0 \le \tau_{i,j}(t) \le \bar{\tau}_{i,j}, \quad \dot{\tau}_{i,j}(t) \le \vartheta_{i,j} < 1$$

where $\bar{\tau}_{i,j}$ and $\vartheta_{i,j}$ are unknown positive constants.

III. RADIAL BASIS FUNCTION NEURAL NETWORKS

In this paper, Radial Basis Function Neural Networks (RBF NNs) are employed to approximate unknown continuous function $P(Z): R^q \to R$ [1]. Then, we have

$$P(Z) = W^{*T} S(Z) + \epsilon(Z) \quad (2)$$

where $S(Z) = [S_1(Z), S_2(Z), \dots, S_l(Z)]^T \in R^l$ is the basis function vector, $S_i(Z) = e^{-\frac{(Z-\mu_i)^T(Z-\mu_i)}{\zeta_i^2}}, i = 1,2,\dots,l$, $\mu_i = [\mu_{i1}, \mu_{i2}, \dots, \mu_{iq}]^T$ is the center of the receptive field and ξ_i is the width of the Gaussian function and W is an unknown ideal constant weight vector. It is defined as $W^* = arg\,min_{W \in R^l} sup_{Z \in \Omega_Z}\{|W^T S(Z) - P(Z)|\}$ where $\epsilon(Z)$ is the approximation error, satisfying $|\epsilon(Z)| \le \epsilon^*$ with ϵ^* being an unknown positive constant.

IV. ADAPTIVE CONTROLLER DESIGN

In this section, the procedure of designing an adaptive controller based on the NN backstepping design method for the large-scale time delay system (1) is explained. At first, the following state transformation

$$z_{i,1} = x_{i,1} - y_{d_i}, i = 1, \dots, N \quad (3)$$

$$z_{i,j} = x_{i,j} - \alpha_{i,j-1}, j = 2, \dots, n_i$$

is considered for system (1) in which $\alpha_{i,j}, i = 1, \dots, N, j = 1, \dots, n_i - 1$, are the intermediate control functions. The transformed system in the new coordination is obtained as

$$\dot{z}_{i,1}(t) = z_{i,2} + \alpha_{i,1} + f_{i,1}(\bar{x}_{i,1}) - \dot{y}_{d_i} + h_{i,1}\left(\bar{x}_{i,1}(t - \tau_{i1}(t))\right) + \Delta_{i,1}(\bar{y}) \quad (4)$$

$$\dot{z}_{i,2}(t) = z_{i,3} + \alpha_{i,2} + f_{i,2}(\bar{x}_{i,2}) - \dot{\alpha}_{i,1} + h_{i,2}\left(\bar{x}_{i,2}(t - \tau_{i2}(t))\right) + \Delta_{i,2}(\bar{y})$$

$$\vdots$$

$$\dot{z}_{i,n_i}(t) = u_i(t) + f_{i,n_i}(\bar{x}_{i,n_i}) + \Delta_{i,n_i}(\bar{y}) - \dot{\alpha}_{i,n_i-1} + h_{i,n_i}\left(\bar{x}_{i,n_i}\left(t - \tau_{i,n_i}(t)\right)\right)$$

The detailed design procedure is given as follows.

Step1: In the first step, the $z_{i,1}, i = 1, \dots, N$, subsystems are considered. At first, the Lyapunov functions are selected as

$$V_{z_{i,1}}(t) = \frac{z_{i,1}^2}{2} \quad (5)$$

$$V_{h_{i,1}} = \frac{1}{2(1 - \vartheta_{i,1})} \int_{t-\tau_{i,1}(t)}^{t} e^{\gamma_{i,1}(\xi - t)} z_{i,1}(\xi) \bar{H}_{i,1}\left(z_{i,1}(\xi)\right) d\xi \quad (6)$$

$$V_{W_{i,1}^*} = \frac{1}{2} \tilde{W}_{i,1}^T \Gamma_{i,1}^{-1} \tilde{W}_{i,1} \quad (7)$$

$$V_{i,1} = V_{z_{i,1}} + V_{h_{i,1}} + V_{W_{i,1}^*} \quad (8)$$

$$V_1 = \sum_{i=1}^{N} V_{i,1} \quad (9)$$

where $\gamma_{i,1}, i = 1, \dots, N$, are positive constants, $\Gamma_{i,1} = \Gamma_{i,1}^T > 0$ and $\tilde{W}_{i,1} = \hat{W}_{i,1} - W_{i,1}^*$ in which $\hat{W}_{i,1}$ being the estimate of $W_{i,1}^*$. Along system (4), the time derivative of $V_{z_{i,1}}$ satisfies

$$\dot{V}_{z_{i,1}} = z_{i,1}(t)\left[z_{i,2} + \alpha_{i,1} + f_{i,1}(\bar{x}_{i,1}) - \dot{y}_{d_i} + h_{i,1}\left(\bar{x}_{i,1}(t - \tau_{i1}(t))\right) + \Delta_{i,1}(\bar{y})\right] \quad (10)$$

By applying assumption 2 and using the Young's inequality, the time derivative of $V_{z_{i,1}}$ becomes

$$\dot{V}_{z_{i,1}} \le z_{i,1}(t)[z_{i,2} + \alpha_{i,1} + f_{i,1}(\bar{x}_{i,1}) + \Delta_{i,1}(\bar{y}_d) - \dot{y}_{d_i}] + \frac{z_{i,1}^2(t)}{2}\sum_{l=1}^{N}\Phi_{i,1,l}^2(\bar{y}_d) + \sum_{l=1}^{N}\frac{z_{l,1}^2}{2} + \frac{e^{\gamma_{i,1}\bar{\tau}_{i,1}}}{2} z_{i,1}^2(t) + \frac{e^{-\gamma_{i,1}\bar{\tau}_{i,1}}}{2} h_{i,1}^2(\bar{x}_{i,1}(t - \tau_{i1}(t)))$$

Based on assumption 4, the time derivative of $V_{h_{i,1}}$ becomes

$$\dot{V}_{h_{i,1}} \le \frac{1}{2(1-\vartheta_{i,1})} z_{i,1}(t)\bar{H}_{i,1}\left(z_{i,1}(t)\right) - \frac{1}{2}e^{-\gamma_{i,1}\bar{\tau}_{i,1}} z_{i,1}(t - \tau_{i,1}(t))\bar{H}_{i,1}\left(z_{i,1}(t-\tau_{i,1}(t))\right) - \gamma_{i,1}V_{h_{i,1}}$$

By applying assumptions 1-2, $\dot{V}_{z_{i,1}} + \dot{V}_{h_{i,1}}$ becomes

$$\dot{V}_{z_{i,1}} + \dot{V}_{h_{i,1}} \le z_{i,1}(t)z_{i,2}(t) + z_{i,1}(t)\alpha_{i,1} + z_{i,1}(t)Q_{i,1}(z_{i,1}) - \gamma_{i,1}V_{h_{i,1}} + \sum_{l=1}^{N}\frac{z_{l,1}^2}{2} + d_{i,1}^* \quad (11)$$

where

$$Q_{i,1}(Z_{i,1}) = f_{i,1}(\bar{x}_{i,1}) + \Delta_{i,1}(\bar{y}_d) - \dot{y}_{d_i} + \frac{z_{i,1}(t)}{2}\sum_{l=1}^{N}\Phi_{1,1,l}^2(\bar{y}_d) + \frac{e^{\gamma_{i,1}\bar{\tau}_{i,1}}}{2}z_{i,1}(t) + \sum_{k=1}^{n_i}\sum_{j=1}^{k}\frac{1}{2(1-\vartheta_{i,j})}\bar{H}_{i,j}\left(z_{i,1}(t)\right) \quad (12)$$

and $d_{i,1}^*$ is as follows.

$$d_{i,1}^* = \frac{e^{-\gamma_{i,1}\bar{\tau}_{i,1}}}{2}\bar{h}_{i,1}\left(y_{d_i}(t)\right) + \frac{e^{-\gamma_{i,1}\bar{\tau}_{i,1}}}{2}d_{i,1}$$

and $Z_{i,1} = [x_{i,1}, \bar{y}_d^T, \dot{y}_{d_i}]^T$.

According to (11)-(12), the intermediate control function is selected as

$$\alpha_{i,1}\left(\bar{x}_{i,1}(t)\right) = -Q_{i,1}(Z_{i,1}) - k_{i,1}z_{i,1} \quad (13)$$

The proposed controller in (13) is not feasible due to the existence of the function $Q_{i,1}(Z_{i,1})$. As defined in (12), $Q_{i,1}(Z_{i,1})$ contains the nonlinear functions which are completely unknown. Besides, $Q_{i,1}(Z_{i,1})$ is continuous and well defined for all values of $Z_{i,1}$, thus, it can be approximated by RBF neural networks such that

$$Q_{i,1}(Z_{i,1}) = {W_{i,1}^*}^T S_{i,1}(Z_{i,1}) + \epsilon_{i,1}(Z_{i,1})$$

where $|\epsilon_{i,1}(Z_{i,1})| \le \epsilon_{z_{i,1}}^*$ is the approximation error, $W_{i,1}^*$ is an unknown constant weights and $S_{i,1}(Z_{i,1})$ is a basis function. Therefore, the time derivative of $V_{i,1}(t)$ becomes

$$\dot{V}_{i,1} = \dot{V}_{z_{i,1}} + \dot{V}_{h_{i,1}} + \dot{V}_{W_{i,1}^*} \le z_{i,1}(t)z_{i,2} + z_{i,1}(t)\alpha_{i,1} + z_{i,1}(t){W_{i,1}^*}^T S_{i,1}(Z_{i,1}) + z_{i,1}(t)\epsilon_{i,1}(Z_{i,1}) - \gamma_{i,1}V_{h_{i,1}} + \sum_{l=1}^{N}\frac{z_{l,1}^2}{2} + d_{i,1}^* + \tilde{W}_{i,1}^T\Gamma_{i,1}^{-1}\dot{\hat{W}}_{i,1} \quad (14)$$

Accordingly, the intermediate control function becomes

$$\alpha_{i,1}\left(\bar{x}_{i,1}(t)\right) = -\hat{W}_{i,1}^T S_{i,1}(Z_{i,1}) - k_{i,1}z_{i,1} \quad (15)$$

where $\hat{W}_{i,1}$ is the estimate of $W_{i,1}$. The updating law for $\hat{W}_{i,1}$ is chosen as

$$\dot{\hat{W}}_{i,1} = \Gamma_{i,1}\left(z_{i,1}S_{i,1}(Z_{i,1}) - \sigma_{i,1}\hat{W}_{i,1}\right) \quad (16)$$

where $\sigma_{i,1}$ is a small positive constant and $\Gamma_{i,1} = \Gamma_{i,1}^T > 0$.

By using the inequality $-\sigma_{i,1}\tilde{W}_{i,1}^T\hat{W}_{i,1} \le \frac{-1}{2}\sigma_{i,1}\|\tilde{W}_{i,1}\|^2 + \frac{1}{2}\sigma_{i,1}\|W_{i,1}^*\|^2$, the time derivative of $V_{i,1}$ becomes

$$\dot{V}_{i,1} \le z_{i,1}z_{i,2} - k_{i,1}z_{i,1}^2 + \epsilon_{i,1}(Z_{i,1})z_{i,1} - \frac{1}{2}\sigma_{i,1}\|\tilde{W}_{i,1}\|^2 + \frac{1}{2}\sigma_{i,1}\|W_{i,1}^*\|^2 + \sum_{l=1}^{N}\frac{z_{l,1}^2}{2} - \gamma_{i,1}V_{h_{i,1}} + d_{i,1}^* \quad (17)$$

where $k_{i,1} = k_{i,1}^0 + \frac{1}{4} + k_{i,1}' + k_{i,1}''$, in which $k_{i,1}'' = \sum_{i=1}^{N}\sum_{s=1}^{n_i}\frac{s}{2}$ and $k_{i,1}^0$ and $k_{i,1}'$ are positive constants. By using the Young's inequality, $\dot{V}_{i,1}$ becomes

$$\dot{V}_{i,1} \le z_{i,2}^2 - k_{i,1}'z_{i,1}^2 - \gamma_{i,1}V_{h_{i,1}} + \mu_{i,1} - k_{i,1}''z_{i,1}^2 + \sum_{l=1}^{N}\frac{z_{l,1}^2}{2} - \frac{1}{2}\sigma_{i,1}\|\tilde{W}_{i,1}\|^2 \quad (18)$$

$$\mu_{i,1} = \frac{1}{4k_{i,1}^0}{\epsilon_{i,1}^*}^2 + \frac{1}{2}\sigma_{i,1}\|W_{i,1}^*\|^2 + d_{i,1}^*$$

By considering (9), the time derivative of $V_1(t)$ becomes

$$\dot{V}_1 \le -\sum_{i=1}^{N}[\gamma_{i,1}V_{h_{i,1}} + k_{i,1}'z_{i,1}^2 + k_{i,1}''z_{i,1}^2 + \frac{1}{2}\sigma_{i,1}\|\tilde{W}_{i,1}\|^2] + \sum_{i=1}^{N}(z_{i,2}^2 + \mu_{i,1} + \sum_{l=1}^{N}\frac{z_{l,1}^2}{2}) \quad (19)$$

Step 2: Now, consider the $z_{i,j}$ subsystem for $i = 1, \dots, N, j = 2, \dots, n_i - 1$:

$$\dot{z}_{i,j}(t) = z_{i,j+1} + \alpha_{i,j} + f_{i,j}(\bar{x}_{i,j}) - \dot{\alpha}_{i,j-1}(t) + h_{i,j}\left(\bar{x}_{i,j}\left(t - \tau_{i,j}(t)\right)\right) + \Delta_{i,j}(\bar{y}) \quad (20)$$

For the $z_{i,j}$ subsystems, the following Lyapunov functions are considered.

$$V_{z_{i,j}}(t) = \frac{z_{i,j}^2}{2} \quad (21)$$

$$V_{h_{i,j}} = \sum_{k=1}^{j}\frac{1}{2(1-\vartheta_{i,k})}\int_{t-\tau_{i,k}(t)}^{t} e^{\gamma_{i,j}(\xi - t)}z_{i,1}(\xi)\bar{H}_{i,k}\left(z_{i,1}(\xi)\right)d\xi \quad (22)$$

$$V_{W_{i,j}^*} = \frac{1}{2}\tilde{W}_{i,j}^T\Gamma_{i,j}^{-1}\tilde{W}_{i,j} \quad (23)$$

$$V_{i,j} = V_{z_{i,j}} + V_{h_{i,j}} + V_{W_{i,j}^*} \quad (24)$$

$$V_j = V_{j-1} + \sum_{i=1}^{N}V_{i,j} \quad (25)$$

where $\gamma_{i,j}$ is a positive constant, $\Gamma_{i,j} = \Gamma_{i,j}^T > 0$ and $\tilde{W}_{i,j} = \hat{W}_{i,j} - W_{i,j}^*$ in which $\hat{W}_{i,j}$ being the estimate of $W_{i,j}^*$. Along subsystem (20), the time derivative of $V_{z_{i,j}}$ becomes

$$\dot{V}_{z_{i,j}} = z_{i,j}(t)[z_{i,j+1} + \alpha_{i,j} + f_{i,j}(\bar{x}_{i,j}) + h_{i,j}\left(\bar{x}_{i,j}\left(t - \tau_{ij}(t)\right)\right) + \Delta_{i,j}(\bar{y}_d) + \Delta_{i,j}(\bar{y}) - \Delta_{i,j}(\bar{y}_d) - \dot{\alpha}_{i,j-1}(t)] \quad (26)$$

Since $\alpha_{i,j-1}$ is a function of $\bar{x}_{i,j-1}, y_{d_1}, \dots, y_{d_1}^{(j-1)}, \dots, y_{d_N}, \dots, y_{d_N}^{(j-1)}, \hat{W}_{i,1}, \dots, \hat{W}_{i,j-1}$, the time derivative of $\alpha_{i,j-1}$ can be expressed as

$$\dot{\alpha}_{i,j-1} = \sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial x_{i,k}}\dot{x}_{i,k} + \sum_{l=1}^{N}\sum_{k=1}^{j}\frac{\partial\alpha_{i,j-1}}{\partial y_{d_l}^{(k-1)}}y_{d_l}^{(k)} + \sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{W}_{i,k}}\dot{\hat{W}}_{i,k} \quad (27)$$

Based on assumption 4, the time derivative of $V_{h_{i,j}}$ becomes

$$\dot{V}_{h_{i,j}} = \sum_{k=1}^{j}[\frac{1}{2(1-\vartheta_{i,k})} z_{i,1}(t)\bar{H}_{i,k}\left(z_{i,1}(t)\right) - \frac{e^{-\gamma_{i,j}\bar{\tau}_{i,k}}}{2} z_{i,1}(t - \tau_{i,k}(t))\bar{H}_{i,k}\left(z_{i,1}\left(t - \tau_{i,k}(t)\right)\right)] - \gamma_{i,j}V_{h_{i,j}} \tag{28}$$

By applying assumptions 1-2 and using the Young's inequality, the time derivative of $V_{z_{i,j}} + V_{h_{i,j}}$ becomes

$$\dot{V}_{z_{i,j}} + \dot{V}_{h_{i,j}} \leq z_{i,j}z_{i,j+1} + z_{i,j}\alpha_{i,j} + z_{i,j}Q_{i,j}(Z_{i,j}) - \gamma_{i,j}V_{h_{i,j}} + \sum_{k=1}^{j}\sum_{l=1}^{N}\frac{z_{l,1}^2}{2} + \sum_{k=1}^{j} d_{i,k}^* \tag{29}$$

where

$$\begin{aligned} Q_{i,j}(Z_{i,j}) = {} & f_{i,j}(\bar{x}_{i,j}) + \Delta_{i,j}(\bar{y}_d) + \frac{e^{\gamma_{i,j}\bar{\tau}_{i,j}}}{2} z_{i,j} + \frac{z_{i,j}(t)}{2}\sum_{l=1}^{N}\Phi_{i,j,l}^2(\bar{y}_d) \\ & - \sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial x_{i,k}}\left(x_{i,k+1} + f_{i,k}(\bar{x}_{i,k}) + \Delta_{i,k}(\bar{y}_d)\right) \\ & - \sum_{l=1}^{N}\sum_{k=1}^{j}\frac{\partial\alpha_{i,j-1}}{\partial y_{d_l}^{(k-1)}} y_{d_l}^{(k)} - \sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{W}_{i,k}}\dot{\hat{W}}_{i,k} \\ & + \frac{1}{2}\sum_{k=1}^{j-1} z_{i,j}\left(\frac{\partial\alpha_{i,j-1}}{\partial x_{i,k}}\right)^2 \sum_{l=1}^{N}\Phi_{i,k,l}^2(\bar{y}_d) + \sum_{k=1}^{j-1}\frac{e^{\gamma_{i,j}\bar{\tau}_{i,k}}}{2} z_{i,j}\left(\frac{\partial\alpha_{i,j-1}}{\partial x_{i,k}}\right)^2 \end{aligned} \tag{30}$$

and $d_{i,k}^*$ is defined as follows.

$$d_{i,k}^* = \frac{e^{-\gamma_{i,j}\bar{\tau}_{i,k}}}{2}\bar{h}_{i,k}(y_{d_k}) + \frac{e^{-\gamma_{i,j}\bar{\tau}_{i,k}}}{2} d_{i,k} \tag{31}$$

and

$$Z_{i,j} = [\bar{x}_{i,j}^T, \alpha_{i,j-1}, \frac{\partial\alpha_{i,j-1}}{\partial x_{i,1}}, \frac{\partial\alpha_{i,j-1}}{\partial x_{i,2}}, \ldots, \frac{\partial\alpha_{i,j-1}}{\partial x_{i,j-1}}, \sum_{l=1}^{N}\sum_{k=1}^{j}\frac{\partial\alpha_{i,j-1}}{\partial y_{d_l}^{(k-1)}} y_{d_l}^{(k)} + \sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{W}_{i,k}}\dot{\hat{W}}_{i,k}]^T \tag{32}$$

By considering (29), the intermediate control function is selected as

$$\alpha_{i,j} = -k_{i,j}z_{i,j} - Q_{i,j}(Z_{i,j}) \tag{33}$$

where $k_{i,j} = k_{i,j}^0 + k_{i,j}^{'} + \frac{5}{4}$ in which $k_{i,j}^0$ and $k_{i,j}^{'}$ are positive constants. Since $f_{i,j}(.)$and $\Delta_{i,j}(.)$ are completely unknown functions, the $Q_{i,j}(Z_{i,j})$ function defined in (30) is also unknown; therefore, the proposed intermediate controller in (33) is not feasible. Besides, $Q_{i,j}(Z_{i,j})$ is continuous and well defined for all $Z_{i,j}$, thus, it can be approximated by RBF neural networks such that

$$Q_{i,j}(Z_{i,j}) = W_{i,j}^{*T} S_{i,j}(Z_{i,j}) + \epsilon_{i,j}(Z_{i,j}) \tag{34}$$

where $|\epsilon_{i,j}(Z_{i,j})| \leq \epsilon_{z_{i,j}}^*$ is the approximation error, $W_{i,j}^*$ is an unknown constant weights and $S_{i,j}(Z_{i,j})$ is a basis function. Accordingly, the time derivative of $V_{i,j}$ becomes

$$\begin{aligned} \dot{V}_{i,j} = {} & \dot{V}_{z_{i,j}} + \dot{V}_{h_{i,j}} + \dot{V}_{W_{i,j}^*} \\ \leq {} & z_{i,j}[z_{i,j+1} + \alpha_{i,j} + W_{i,j}^{*T} S_{i,j}(Z_{i,j}) + \epsilon_{i,j}(Z_{i,j})] - \gamma_{i,j}V_{h_{i,j}} + \tilde{W}_{i,j}^T\Gamma_{i,j}^{-1}\dot{\hat{W}}_{i,j} \\ & + \sum_{k=1}^{j}\sum_{l=1}^{N}\frac{z_{l,1}^2}{2} + \sum_{k=1}^{j} d_{i,k}^* \end{aligned} \tag{35}$$

Therefore, the intermediate controller becomes

$$\alpha_{i,j} = -k_{i,j}z_{i,j} - \hat{W}_{i,j}^T S_{i,j}(Z_{i,j}) \tag{36}$$

The updating law for $\hat{W}_{i,j}$ is chosen as

$$\dot{\hat{W}}_{i,j} = \Gamma_{i,j}(S_{i,j}(Z_{i,j})z_{i,j} - \sigma_{i,j}\hat{W}_{i,j}) \tag{37}$$

where $\sigma_{i,j}$ is a small positive constant and the matrix $\Gamma_{i,j} = \Gamma_{i,j}^T$. By using the inequality $-\sigma_{i,j}\tilde{W}_{i,j}^T\hat{W}_{i,j} \leq \frac{-1}{2}\sigma_{i,j}\|\tilde{W}_{i,j}\|^2 + \frac{1}{2}\sigma_{i,j}\|W_{i,j}^*\|^2$, $\dot{V}_{i,j}$ becomes

$$\begin{aligned} \dot{V}_{i,j} \leq {} & z_{i,j}z_{i,j+1} - k_{i,j}z_{i,j}^2 - \frac{1}{2}\sigma_{i,j}\|\tilde{W}_{i,j}\|^2 + \frac{1}{2}\sigma_{i,j}\|W_{i,j}^*\|^2 + \epsilon_{i,j}(Z_{i,j})z_{i,j} \\ & - \gamma_{i,j}V_{U_{i,j}} + \sum_{k=1}^{j}\sum_{l=1}^{N}\frac{z_{l,1}^2}{2} + \sum_{k=1}^{j} d_{i,k}^* \end{aligned} \tag{38}$$

By applying the Young's inequality, the time derivative of $V_{i,j}$ becomes

$$\dot{V}_{i,j} \leq z_{i,j+1}^2 - k_{i,j}^{\prime}z_{i,j}^2 - z_{i,j}^2 - \frac{1}{2}\sigma_{i,j}\|\tilde{W}_{i,j}\|^2 - \gamma_{i,j}V_{U_{i,j}} + \mu_{i,j} + \sum_{k=1}^{j}\sum_{l=1}^{N}\frac{z_{l,1}^2}{2} \tag{39}$$

$$\mu_{i,j} = \frac{1}{4k_{i,j}^0}\epsilon_{i,j}^{*\,2} + \frac{1}{2}\sigma_{i,j}\|W_{i,j}^*\|^2 + \sum_{k=1}^{j} d_{i,k}^*$$

By considering (25), the time derivative of V_j becomes

$$\begin{aligned} \dot{V}_j = {} & \dot{V}_{j-1} + \sum_{i=1}^{N}\dot{V}_{i,j} \\ \leq {} & \sum_{i=1}^{N} z_{i,j+1}^2 + \sum_{i=1}^{N}\sum_{k=1}^{j}\{\mu_{i,k} - k_{i,k}^{\prime}z_{i,k}^2 + \sum_{s=1}^{k}\sum_{l=1}^{N}\frac{z_{l,1}^2}{2} - \frac{1}{2}\sigma_{i,k}\|\tilde{W}_{i,k}\|^2 \\ & - \gamma_{i,k}V_{h_{i,k}}\} - \sum_{i=1}^{N} k_{i,1}^{"}z_{i,1}^2 \end{aligned} \tag{40}$$

Step 3: In the final step, the $z_{i,n_i}, i = 1, \ldots, N$, subsystems are considered. For these subsystems, the Lyapunov functions are considered as (21)-(25) for $j = n_i$. Based on the design procedure similar to the previous step, the controller becomes

$$u_i = -k_{i,j}z_{i,n_i} - \hat{W}_{i,n_i}^T S_{i,n_i}(Z_{i,n_i}) \tag{41}$$

and the updating laws for $\hat{W}_{i,n_i}$ becomes similar to (37).

By using the inequalities $-\sigma_{i,n_i}\tilde{W}_{i,n_i}^T\hat{W}_{i,n_i} \leq -\frac{1}{2}\sigma_{i,n_i}\|\tilde{W}_{i,n_i}\|^2 + \frac{1}{2}\sigma_{i,n_i}\|W_{i,n_i}^*\|^2$, the time derivative of V_{i,n_i} becomes

$$\dot{V}_{i,n_i} \leq -k_{i,n_i}^{\prime}z_{i,n_i}^2 - \gamma_{i,n_i}V_{h_{i,n_i}} + \sum_{k=1}^{n_i}\sum_{l=1}^{N}\frac{z_{l,1}^2}{2} - \frac{1}{2}\sigma_{i,n_i}\|\tilde{W}_{i,n_i}\|^2 + \mu_{i,n_i} \tag{42}$$

Therefore, the time derivative of $V(t)=\sum_{i=1}^{N}\sum_{j=1}^{n_i}V_{i,j}$ becomes

$$\dot{V}=\dot{V}_{n_i-1}+\sum_{i=1}^{N}\dot{V}_{i,n_i} \quad (43)$$
$$\leq \sum_{i=1}^{N}\sum_{k=1}^{n_i}\{\mu_{i,k}-k'_{i,k}z_{i,k}^2 + \sum_{s=1}^{k}\sum_{l=1}^{N}\frac{z_{l,1}^2}{2}-\frac{1}{2}\sigma_{i,k}\|\tilde{W}_{i,k}\|^2 - \gamma_{i,k}V_{h_{i,k}}\} - \sum_{i=1}^{N}k''_{i,1}z_{i,1}^2$$

Since $k''_{i,1}=\sum_{i=1}^{N}\sum_{s=1}^{n_i}(\frac{s}{2})$, the time derivative of $V(t)$ becomes

$$\dot{V}\leq -cV+\mu$$

$$c=min\{c_{1,1},c_{1,2},\dots,c_{1,n_1},\dots,c_{i,n_i},\dots,c_{N,1},\dots,c_{N,n_N}\} \quad (44)$$
$$c_{i,k}=min\{2k'_{i,k},\gamma_{i,k},\frac{\sigma_{i,k}}{\lambda_{max}(\Gamma_{i,n_i}^{-1})}\},\mu=\sum_{i=1}^{N}\sum_{k=1}^{n_i}\mu_{i,k}$$

Therefore, V(t) is bounded and accordingly all the closed loop signals are bounded.

$$V(t)\leq\left[V(0)-\frac{\mu}{c}\right]e^{-ct}+\frac{\mu}{c}$$

Consequently,

$$\sum_{i=1}^{N}\sum_{j=1}^{n_i}z_{i,j}^2\leq 2[(V(0)-\frac{\mu}{c})e^{-ct}+\frac{\mu}{c}]$$

It can be concluded that all the closed loop signals are bounded and $z(t)=[z_{1,1},z_{1,2},\dots,z_{1,n_1},z_{2,1},\dots,z_{2,n_2},\dots,z_{N,n_N}]^T$ will eventually converges to the compact set Λ_z .

$$\Lambda_z=\left\{z\,\middle|\,\|z\|\leq\sqrt{\frac{2\mu}{c}}\right\}$$

The compact set Λ_z can be minimized to the desired size by an appropriate selection of the design parameters. The above design procedures and analysis can be summarized as the following theorem.

Theorem 1. For the nonlinear system (1), if assumptions 1-4 are satisfied, the proposed decentralized neural-network-based control approach can guarantee that all the signals in the closed loop system remain bounded and the tracking errors converge to a small neighborhood of the origin by choosing the design parameters appropriately.

V. SIMULATION RESULTS

In this section, the obtained results are simulated to verify the effectiveness of the proposed method. For this purpose, as an application, the adaptive control scheme is proposed for two inverted pendulums.

The two inverted pendulums system is shown in Figure 1. Each pendulum may be positioned by a torque input u_i applied by a servomotor as its base. It is only assumed that θ_i are available to the ith controller for $i=1,2$. Let $\theta_1=x_{1,1},\theta_2=x_{2,1},\dot{\theta}_1=x_{1,2}$ and $\dot{\theta}_2=x_{2,2}$. Thus, the inverted pendulum equation can be described as

$$\dot{x}_{1,1}(t)=f_{1,1}(x_{1,1})+x_{1,2}+h_{1,1}(x_{1,1}(t-\tau_{1,1}(t))) +\Delta_{1,1}(y_1,y_2)$$
$$\dot{x}_{1,2}(t)=f_{1,2}(\bar{x}_{1,2})+u_1+h_{1,2}(\bar{x}_{1,2}(t-\tau_{1,2}(t))) +\Delta_{1,2}(y_1,y_2)$$
$$y_1=x_{1,1}$$
$$\dot{x}_{2,1}(t)=f_{2,1}(x_{2,1})+x_{2,2}+h_{2,1}(x_{2,1}(t-\tau_{2,1}(t))) +\Delta_{2,1}(y_1,y_2)$$
$$\dot{x}_{2,2}(t)=f_{2,2}(\bar{x}_{2,2})+u_2+h_{2,2}(x_{2,2}(t-\tau_{2,2}(t))) +\Delta_{2,2}(y_1,y_2)$$
$$y_2=x_{2,1}$$

where $f_{1,1}(x_{1,1})=0,h_{1,1}=0,\Delta_{1,1}(y_1,y_2)=0,f_{1,2}(\bar{x}_{1,2})=\left(\frac{m_1gr}{J_1}-\frac{kr^2}{4J_1}\right)\sin(x_{1,1}),h_{1,2}=\frac{x_{11}(t-\tau_{1,2}(t))}{1+x_{1,1}^2(t-\tau_{1,2}(t)},\Delta_{1,2}(y_1,y_2)=\frac{kr^2}{4J_1}\sin(x_{2,1}),f_{2,1}(x_{1,1})=0,h_{2,1}=0,\Delta_{2,1}(y_1,y_2)=0,f_{2,2}(\bar{x}_{2,2})=\left(\frac{m_2gr}{J_2}-\frac{kr^2}{4J_2}\right)\sin(x_{2,1}),h_{2,2}=\frac{x_{21}(t-\tau_{2,2}(t))}{1+x_{2,1}^2(t-\tau_{2,2}(t)},\Delta_{2,2}(y_1,y_2)=\frac{kr^2}{4J_2}\sin(x_{1,1}),\tau_{1,2}=\tau_{2,2}=0.4(1+sin^2(t))$.

Hence, θ_1 and θ_2 are the angular displacements of the pendulums from vertical. The parameters $m_1=2\,kg$ and $m_2=2.5\,kg$ are the pendulum end masses, $J_1=5\,kg$ and $J_2=6.25\,kg$ are the moments of inertia, $k=100\,N/m$ is the spring constant of the connecting spring, $r=0.5\,m$ is the pendulum height, $l=0.5\,m$ is the natural length of the spring and $g=9.81\,\frac{m}{s^2}$ is the gravitational acceleration. The distance between the pendulum hinges is defined as $b=0.5\,m$. The control objective is to track the desired signals $y_{d_1}(t)=y_{d_2}(t)=0.5(\sin(0.5t)+\sin(t))$.

The following design parameters are selected in the simulation:

$$x_1(0)=x_2(0)=[0.5,0.5]^T,\Gamma_{12}=10I,\Gamma_{22}=5I,W_{12}(0)=W_{22}(0)=0,\sigma_{1,2}=\sigma_{2,2}=0.5.$$

The simulation results of this example are shown in Figures 2-3.

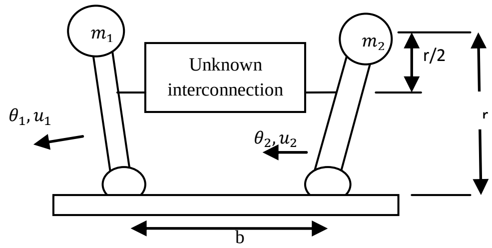

Fig. 1 Two inverted pendulums connected by a spring

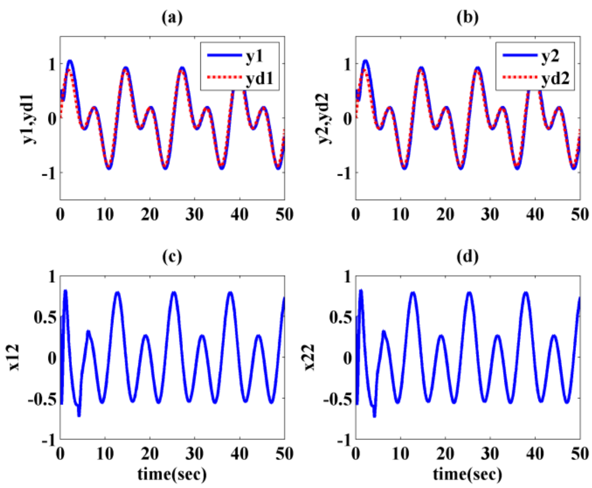

Fig.2 (a) The plant and reference signal outputs y_1 (t) and $y_{d_1}(t)$. (b) The plant and reference signal outputs y_2 (t) and $y_{d_2}(t)$. (c) State $x_{12}(t)$. (d) State $x_{22}(t)$.

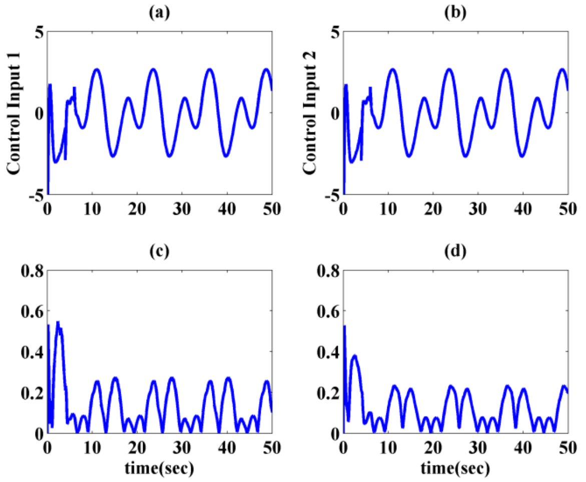

Fig.3 (a) Control input $u_1(t)$. (b) Control input $u_2(t)$. (c) $\widehat{W}_{12}(t)$. (d) $\widehat{W}_{22}(t)$

It can be seen from Figure 2 that the output tracking is ensured. Figures 2 (b)-(c) and (3) show the boundedness of the state variables, the control inputs and the estimates of the parameters of the control loop system.

It can be seen from Figures 2-3, that the proposed adaptive control scheme can guarantee that all the variables are bounded and the outputs $y_i(t), i = 1, \dots, N$ can track the desired signals $y_{d_i}(t), i = 1, \dots, N$, appropriately.

VI. CONCLUSION

In this paper, a neural-network-based adaptive decentralized control scheme is proposed for the nonlinear large-scale systems with time varying state delays. The state delays values are considered to be unknown and time varying with unknown bounds. The offered method is based on the Neural Networks and backstepping design method. The uncertainties from unknown nonlinearities and time varying delays have been compensated by using appropriate Lyapunov-Krasovskii functionals. The proposed systematic design method can guarantee global boundedness of all the closed loop signals in addition to the convergence of the system outputs to a small neighborhood of the desired signals. Simulation results have been conducted to verify the effectiveness of the proposed control method.

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