Performance Recovery in the Presence of Uncertainty for Soft Landing on Asteroids

Maryam Taheri Shirazi, Peyman Setoodeh, and Alireza Khayatian

***Abstract*— A two-time-scale guidance law is proposed for soft landing on asteroids to recover the desired descent trajectory in the presence of uncertainties due to inaccurate model of the asteroid's gravitational field as well as solar gravity and radiation pressure. A high-gain observer estimates the uncertainties over the faster time scale and a nonlinear controller uses these estimates to cancel the effect of uncertainties over the slower time scale. This way, the spacecraft trajectory will converge to the desired one after a short transient phase. The two-time-scale guidance law is compared with the terminal-sliding mode control approach. The proposed method shows a better transient behavior and requires smoother control signals with lower amplitudes.**

***Index Terms*— Spacecraft, soft landing, performance Recovery, high-gain observer, sliding-mode control.**

I. INTRODUCTION

Enhancing scientific knowledge, mitigating the impact hazard, and utilizing extra-terrestrial resources are the driving force for launching space missions aimed at minor planets, asteroids, and comets in the Solar System [1], [2]. Data collection in such missions, which is performed both remotely and on the surface, calls for close-proximity maneuvers including *hovering* and *soft landing* (i.e., landing with an impact velocity of less than 3 m/s) [3]. As a successful mission, in 2005, the Huygens probe softly landed on Titan (Saturn's largest moon) after parachute descending for about 2.5 hours [4].

Compared to planets, asteroids and comets are very small objects with weak gravitational fields. Due to their irregular shapes, the gravitational force they exert on a spacecraft varies in both direction and intensity as a function of the spacecraft's location [5]–[8]. Hence, orbital trajectories can be complex and non-periodic. Spacecrafts are also affected by solar gravity as well as radiation pressure, and in effect therefore stability is an issue that deserves special attention [3]. Moreover, in order to guarantee a successful landing mission, the spacecraft must be able to investigate the safety of landing at the nominal site and in case of detecting a hazard, maneuver to a new safe site [9].

Sophisticated controllers are needed to handle the decent and landing phase of space missions. Such controllers must provide high levels of autonomy, flexibility, and navigation accuracy in guiding the spacecraft to the chosen landing site on the asteroid. An additional constraint that must be taken into account in the design and implementation of a desired trajectory is that the spacecraft must reach the desired point with (almost) zero velocity. Moreover, robustness against inaccurate models of the gravitational field, perturbations, and unmodeled dynamics is quite critical for autonomous landing [3].

Due to the robustness of *sliding-mode* (SM) control against model mismatch, it provides the main idea behind a class of nonlinear guidance algorithms for landing on asteroids [10], [11]. This class of guidance algorithms is built on the so called *terminal sliding mode* (TSM) control [12]. TSM control was developed at the Jet Propulsion Laboratory (JPL) in the early 1990s based on the notion of *terminal sliders*, which in turn, was inspired by the idea of *terminal attractors* [13]. Finite-time convergence of states is guaranteed in TSM control by designing the sliding surface as an attractor, which is done by introducing a nonlinear term in the sliding surface design. TSM control has been successfully applied to nonlinear systems, whose state-space model has the form of position-velocity equations. As an alternative to the TSM control-based guidance, it was proposed in [3] to use multiple sliding surfaces for generating desired trajectories, which remain stable under bounded perturbations. The multiple-sliding-surface guidance was inspired by the higher-order sliding mode control theory [14].

Decision making and control in the presence of uncertainty in a way to guarantee an acceptable level of performance for all realizations that belong to a bounded uncertainty set has been well addressed in the literature. However, an important issue that deserves special attention is how to control the performance degradation when the realization is outside the considered uncertainty set [15]. In other words, there is a need to move beyond robustness and aim at designing *antifragile* systems. While a robust or resilient system is supposed to resist shocks and stay the same, an antifragile system must improve. Antifargile systems should be immune against prediction errors. Moreover, in case of an adverse event, the antifragile system must be able to quickly restore its normal status and recover its normal performance [16].

As a step towards building antifragile systems, the problem of interest may be viewed as a multi-stage finite-horizon uncertain problem and extensions of robust methods may be employed to solve it [15]. Following this line of thinking, a *two-time scale* (TTS) control structure was proposed in [17] for *performance recovery*. In the proposed structure, a *high-gain observer* [18] is used to estimate the uncertainty. Regarding the faster dynamics of the observer with respect to the controller, the uncertainty estimates obtained by the observer are employed by the controller to cancel the effect

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of the perturbations in the system under control. This way, after a fast transient mode, state trajectories of the perturbed system will converge to the trajectories of the unperturbed system, which is controlled by a nominal controller. Thanks to a semi-global version of the *separation principle* [18], in the presence of uncertain nonlinearities, the TTS controller would be able to recover the trajectories, which are achieved by a nominal controller; hence, the term performance recovery [17].

Building on the idea of the TTS control structure, this paper presents a two-time-scale nonlinear guidance algorithm for soft landing on asteroids in the presence of uncertainties due to inaccurate models of the asteroid's gravitational field, solar gravity, and radiation pressure. The performance-recovery ability of the TTS guidance approach is evaluated and compared with that of the TSM method. By computer experiments, it is shown that the TTS method improves the transient response and requires a much smoother control input.

The rest of the paper is as follows. Formulation of the descent and landing guidance problem is presented in Section II. Section III is focused on the design of guidance laws for both TSM and TTS approaches. It also includes the structure of the high-gain observer and the nominal controller. Section IV presents the simulation results. Finally, the paper concludes in Section V.

II. SYSTEM MODEL

For soft landing on an asteroid, the guidance law computes a real-time acceleration command as the control input in a way to force the spacecraft to follow a desired trajectory and reach a target point on the asteroid's surface with zero velocity [3]. To be more precise, three coordinate frames are considered: the asteroid coordinate system, $o_a - x_a y_a z_a$, whose origin coincides with the asteroid's center of mass, the landing coordinate system, $o_\ell - x_\ell y_\ell z_\ell$, whose origin coincides with the landing target point, and the body coordinate system, $o_b - x_b y_b z_b$, whose origin coincides with the spacecraft's center of mass. Fig. 1 shows the geometric relationship between these three coordinate frames, where R, θ, and λ represent radius, latitude, and longitude, respectively. It is assumed that the asteroid rotation speed is uniform with the constant rate of revolution $[0\;0\;\omega_a]^T$.

A six-dimensional state vector is considered for the spacecraft, which includes positions and velocities along the three coordinate axes, $[x\;y\;z\;\dot{x}\;\dot{y}\;\dot{z}]^T$. Evolution of the spacecraft's state trajectories expressed in the landing coordinate frame is governed by the following equations:

$$\begin{aligned}
\dot{x}_1 &= x_4 \\
\dot{x}_2 &= x_5 \\
\dot{x}_3 &= x_6 \\
\dot{x}_4 &= b_1 x_5 + b_3 x_1 + b_5 x_3 + u_1 + g_1 + d_1 \\
\dot{x}_5 &= -b_1 x_4 - b_2 x_6 + \omega_a^2 x_2 + u_2 + g_2 + d_2 \\
\dot{x}_6 &= b_2 x_5 + b_5 x_1 + b_4 x_3 + u_3 + g_3 + d_3
\end{aligned} \tag{1}$$

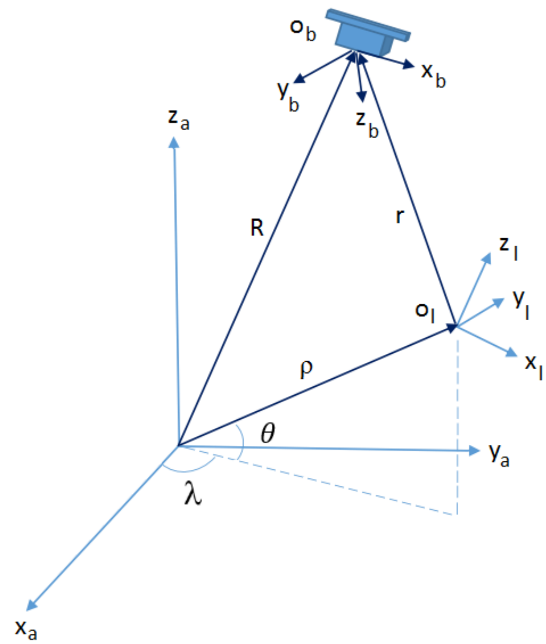

Fig. 1. Geometric relationship between the asteroid, landing, and body coordinate frames.

where $x_1 = x, x_2 = y, x_3 = z, x_4 = \dot{x}, x_5 = \dot{y}, x_6 = \dot{z}$. Also, $[u_1\;u_2\;u_3]^T$ denotes the input vector and $[d_1\;d_2\;d_3]^T$ represents the disturbance vector due to the solar gravity and radiation pressure. The b_i coefficients are defined as follows:

$$\begin{aligned}
b_1 &= 2\omega_a sin\theta \\
b_2 &= 2\omega_a cos\theta \\
b_3 &= \omega_a^2 sin^2\theta \\
b_4 &= \omega_a^2 cos^2\theta \\
b_5 &= \omega_a^2 sin\theta cos\theta
\end{aligned} \tag{2}$$

It is worth noting that $\theta = tan^{-1}\frac{x_3}{\sqrt{x_1^2+x_2^2}}$. It is worth noting that $\theta = tan^{-1}\frac{x_3}{\sqrt{x_1^2+x_2^2}}$. The vector $[g_1\;g_2\;g_3]^T$ represents the gradient of the gravitational potential, which in turn, can be expressed as a series expansion of spherical harmonics [5], [7]:

$$\begin{aligned}
V(R) &= \frac{GM}{R}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{R_0}{R}\right)^n P_{nm}(sin\theta)\times \\
&\quad [C_{nm}cos(m\lambda) + S_{nm}sin(m\lambda)]
\end{aligned} \tag{3}$$

where R_0, G, and M denote the largest equatorial radius, gravitational constant, and asteroid's mass, respectively; P_{nm}'s are the Legendre polynomials; C_{nm}'s, and S_{nm}'s are the associated coefficients, which are determined by the mass distribution within the asteroid. Also, $\lambda = tan^{-1}\frac{x_2}{x_1}$. Therefore, the state equation in (1) represents a nonlinear system and a nonlinear guidance law is required. In order to achieve soft landing, desired trajectories for descent altitude and velocity (i.e., z_d and $\dot{z}_d$) must be designed. A cubic polynomial is considered for the altitude desired trajectory

[9]:

$$z_d(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{4}$$

that should satisfy the following initial and final conditions:

$$\begin{aligned} z_d(0) &= z_0 \\ \dot{z}_d(0) &= \dot{z}_0 \\ z_d(\tau) &= 0 \\ \dot{z}_d(\tau) &= 0. \end{aligned} \tag{5}$$

where τ is the descent time, which is finite.

In summary, the guidance law should force the spacecraft error states

$$[x_1 \;\; x_2 \;\; x_3 - z_d \;\; x_4 \;\; x_5 \;\; x_6 - \dot{z}_d]^T \tag{6}$$

to go to zero. Next section is focused on how to design an appropriate guidance law.

III. CONTROLLER DESIGN

Two nonlinear guidance laws for the state-space model in (1) are presented in this section, which are built on two-time scale and terminal sliding mode controllers.

A. Two-Time Scale Guidance Law

The nonlinear state equation in (1) can be rewritten in the following compact form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}\left(\mathbf{u} + \boldsymbol{\delta}(\mathbf{x})\right) \tag{7}$$

where $\boldsymbol{\delta}(\mathbf{x}) = \mathbf{g} + \mathbf{d}$ and $\mathbf{G}_{6\times 3} = [\mathbf{0}_{3\times 3} \mid \mathbf{I}_{3\times 3}]^T$. In this form, the terms g_i's and d_i's are considered as uncertainty terms, which should be estimated by the observer. The two-time scale control structure has two main building blocks: a nominal controller and a high-gain observer [17].

1) Nominal Controller: The state-space model in (1) has the form of state-velocity equations. In order to simplify the design of the nominal controller, the sixth-order system of (1) is considered as three second-order systems and a one-dimensional control signal is designed for each one of them. Defining $z_1 = x_1$, $z_2 = x_4$, $z_3 = x_2$, $z_4 = x_5$, $z_5 = x_3$, and $z_6 = x_6$, the three second-order unperturbed systems will have the following state equations:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= b_1 z_4 + b_3 z_1 + b_5 z_5 + \bar{u}_1 \end{aligned} \tag{8}$$

$$\begin{aligned} \dot{z}_3 &= z_4 \\ \dot{z}_4 &= -b_1 z_2 - b_2 z_6 + \omega_a^2 z_3 + \bar{u}_2 \end{aligned} \tag{9}$$

$$\begin{aligned} \dot{z}_5 &= z_6 \\ \dot{z}_6 &= b_2 z_4 + b_5 z_1 + b_4 z_5 + \bar{u}_3 \end{aligned} \tag{10}$$

Using the following three feedback-linearization control laws for the above three systems,

$$\bar{u}_1 = -(b_1 z_4 + b_3 z_1 + b_5 z_5) - k_1 z_1 - k_2 z_2 \tag{11}$$

$$\bar{u}_2 = -(-b_1 z_2 - b_2 z_6 + \omega_a^2 z_3) - k_3 z_3 - k_4 z_4 \tag{12}$$

$$\bar{u}_3 = -(b_2 z_4 + b_5 z_1 + b_4 z_5) - k_5 z_5 - k_6 z_6 \tag{13}$$

the following closed-loop dynamic equations will be obtained:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -k_1 z_1 - k_2 z_2 \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{z}_3 &= z_4 \\ \dot{z}_4 &= -k_3 z_3 - k_4 z_4 \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{z}_5 &= z_6 \\ \dot{z}_6 &= -k_5 z_5 - k_6 z_6 \end{aligned} \tag{16}$$

which are linear. The nominal controller is formed by the control laws in (11), (12), and (13).

2) High-Gain Observer: The following observer is used to estimate the unknown $\boldsymbol{\delta}(\mathbf{x})$ [17]:

$$\dot{\hat{\mathbf{x}}} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{x} + \mathbf{G}\mathbf{u} - \frac{1}{\epsilon}\left(\hat{\mathbf{x}} - \mathbf{x}\right) \tag{17}$$

where the observer is initialized as $\hat{\mathbf{x}}(0) = \mathbf{x}(0)$. Then, based on the uncertainty estimates obtained from the observer, the nominal control signals of (11), (12), and (13) are corrected as:

$$\mathbf{u} = \bar{\mathbf{u}} + \frac{1}{\epsilon}\mathbf{G}^{\dagger}\left(\hat{\mathbf{x}} - \mathbf{x}\right) \tag{18}$$

where $\dagger$ denotes the pseudoinverse.

B. Terminal Sliding Mode Guidance Law

The formulation of the TSM guidance law for the system in (1) is recalled from [11]:

$$\begin{aligned} u_1 &= -b_1 e_5 - b_3 e_1 - b_5(e_3 + z_d) - g_1 \\ &\quad -\beta_1^{-1}\gamma_1^{-1}|\dot{e}_1|^{2-\gamma_1} sign(\dot{e}_1) - k_1 sign(s_1) \\ u_2 &= b_1 e_4 + b_2(e_6 + \dot{z}_d) - \omega_a^2 e_2 - g_2 \\ &\quad -\beta_2^{-1}\gamma_2^{-1}|\dot{e}_2|^{2-\gamma_2} sign(\dot{e}_2) - k_2 sign(s_2) \\ u_3 &= \ddot{z}_d - b_2 e_5 - b_5 e_1 - b_4(e_3 + z_d) - g_3 \\ &\quad -\beta_3^{-1}\gamma_3^{-1}|\dot{e}_3|^{2-\gamma_3} sign(\dot{e}_3) - k_3 sign(s_3) \end{aligned} \tag{19}$$

The sliding surfaces are designed as:

$$\begin{aligned} s_1 &= e_1 + \beta_1|\dot{e}_1|^{\gamma_1} sign(\dot{e}_1) \\ s_2 &= e_2 + \beta_2|\dot{e}_2|^{\gamma_2} sign(\dot{e}_2) \\ s_3 &= e_3 + \beta_3|\dot{e}_3|^{\gamma_3} sign(\dot{e}_3) \end{aligned} \tag{20}$$

where $\beta_i > 0$ and $\gamma_i \in (1, 2)$ for $i = 1, 2, 3$.

IV. COMPUTER EXPERIMENTS

In this section, the performance recovery ability of the proposed TTS guidance law in the face of uncertainties is evaluated by computer experiments. Here, we adopt the scenario presented in [10] and [11] for simulations and compare the results obtained by the TTS guidance law with those of the TSM guidance law in [11].

In the simulations, the parameter values are as follows. The predetermined landing time is $\tau = 4000s$, the disturbance terms are $d_1 = 1.5sin2t$, $d_2 = 1.6sin1.5t$, and $d_3 = 1.4sin3t$, the reference radius is $R_0 = 1.15km$, the rotation speed is $\omega_a = 2\pi/10.54\ rad/h$, $GM = 4.794 \times 10^{-4}$, and the initial state is

$$\mathbf{x}(0) = [350\ ,\ 300\ ,\ 2050\ ,\ -1.2\ ,\ 0.2\ ,\ -1]^T \qquad (21)$$

The units are meter and meter per second for position and velocity values, respectively. For the TSM, we choose $\beta_i = 2$ and $\gamma_i = 1.5$ for $i = 1, 2, 3$.

Unlike the scenario considered in [11], the gravitational force exerted on the spacecraft by the asteroid was considered as uncertainty. However, the gravitational force was calculated from the gradient of the potential function in (3) by considering the series expansion up to the fourth harmonic. If a homogeneous triaxial ellipsoid is considered as an approximation of the asteroid's shape, then the potential function will be quite simplified. With this approximation, $S_{nm} = 0$ for all n or m and $C_{nm} = 0$ for any odd n or m. The values of nonzero coefficients in the series expansions are: $C_{20} = -0.113$, $C_{22} = 0.0396$, $C_{40} = 0.068$, $C_{42} = -0.00323$, and $C_{44} = 0.000279$.

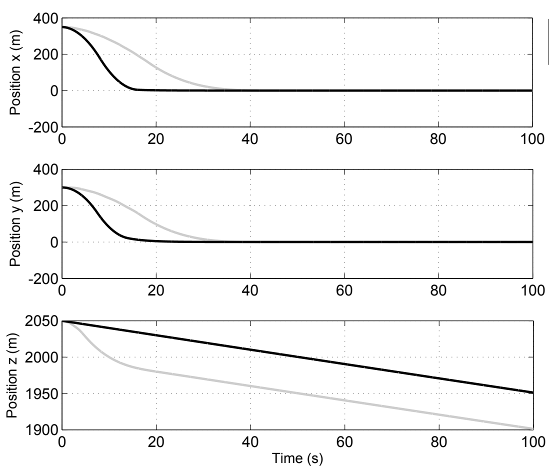

Fig. 2. Spacecraft state trajectories for positions for both terminal sliding mode (TSM) and two-time scale (TTS) controllers.

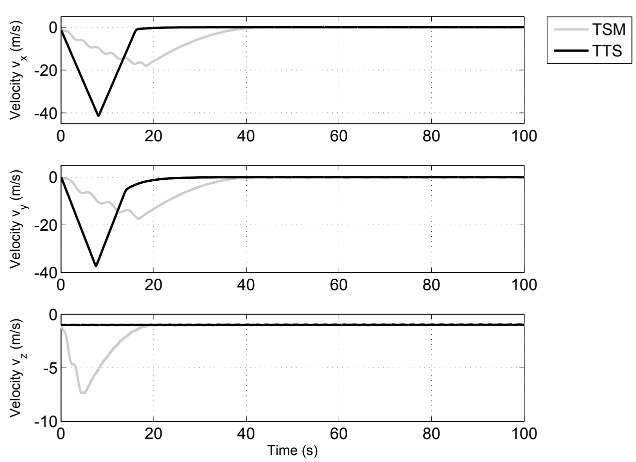

Fig. 3. Spacecraft state trajectories for velocities for both terminal sliding mode (TSM) and two-time scale (TTS) controllers.

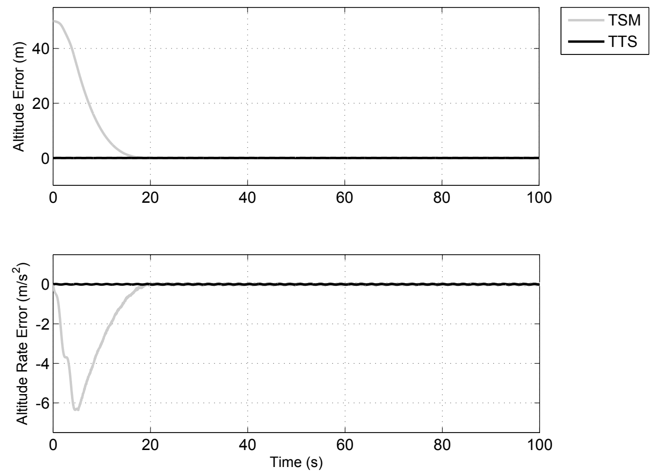

Fig. 4. Deviation of spacecraft's altitude and altitude rate from desired trajectories for both terminal sliding mode (TSM) and two-time scale (TTS) controllers.

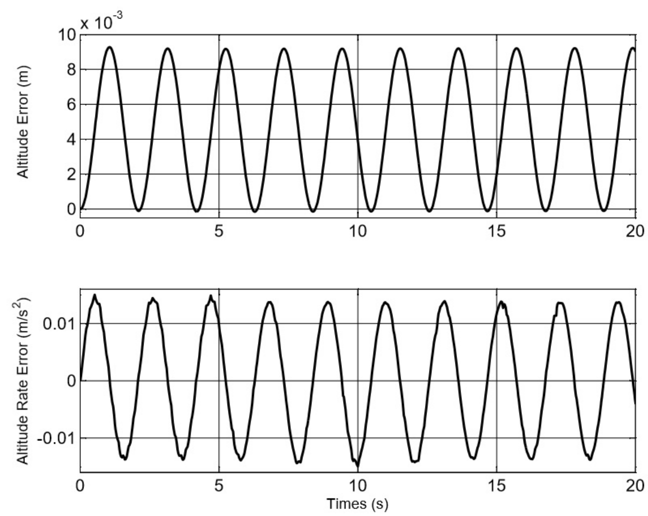

Fig. 5. Deviation of spacecraft's altitude and altitude rate from desired trajectories for the two-time scale (TTS) controller.

Position and velocity trajectories achieved by the two guidance laws are respectively shown in Fig. 2 and Fig. 3 for the first hundred seconds of the descent phase. Deviation of spacecraft's altitude and altitude rate from target designed trajectories are shown in Fig. 4 again for the first hundred seconds of the descent phase. A zoomed portion of the errors over time is shown in Fig. **??** for the TTS method. It is obvious from the figures that the TTS guidance law has improved the transient response compared to the TSM method.

The control effort required by the two guidance laws are compared in Fig. 6. Chattering is observed in the TSM control signals. It can be seen that the TTS guidance law needs much smoother control signals with lower amplitudes; hence, the superiority of the TTS method over the TSM controller. The sliding surfaces for the TSM controller are also shown in Fig. 7 for the first hundred seconds of the descent phase.

Fig. 8 depicts the three-dimensional trajectories of the spacecraft from the initial point to the landing site over the predetermined 4000-second descent time interval.

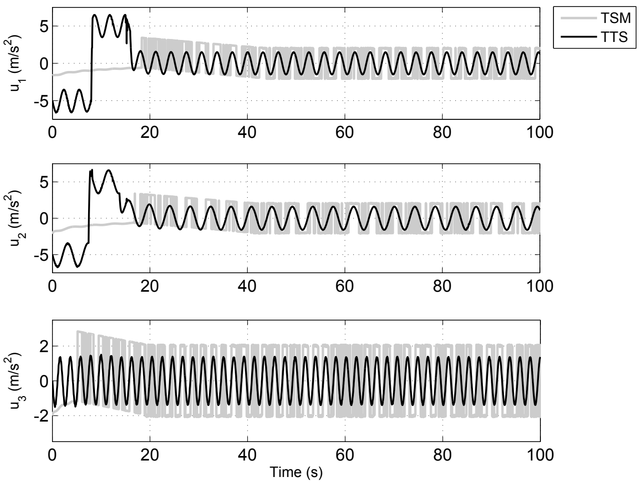

Fig. 6. Control signals for both terminal sliding mode (TSM) and two-time scale (TTS) controllers.

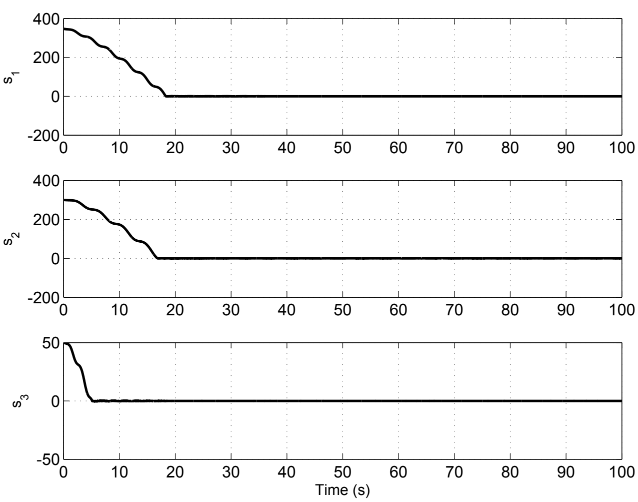

Fig. 7. Sliding surfaces for the terminal sliding mode (TSM) controller.

V. CONCLUDING REMARKS

Soft-landing on small space bodies calls for sophisticated guidance laws that are able to guarantee some acceptable level of performance in the face of adverse events and uncertainties. Such guidance laws must go beyond robustness and control or slow down the performance degradation, when the realization is outside of the uncertainty set that was considered in designing controllers that provide a level of robustness. This paper presented a two-time scale guidance law that solves the problem in two stages. In the first stage, the uncertainties are estimated over the faster time scale. Then, the controller uses the obtained estimates to cancel out the effect of uncertainties over the slower time scale. The proposed method is able to recover the nominal trajectories achieved by a nominal controller for the unperturbed system after a fast transient. Compared to the terminal sliding mode guidance law, the two-time scale method improves the transient response with less control effort. Also, it requires much smoother control signals.

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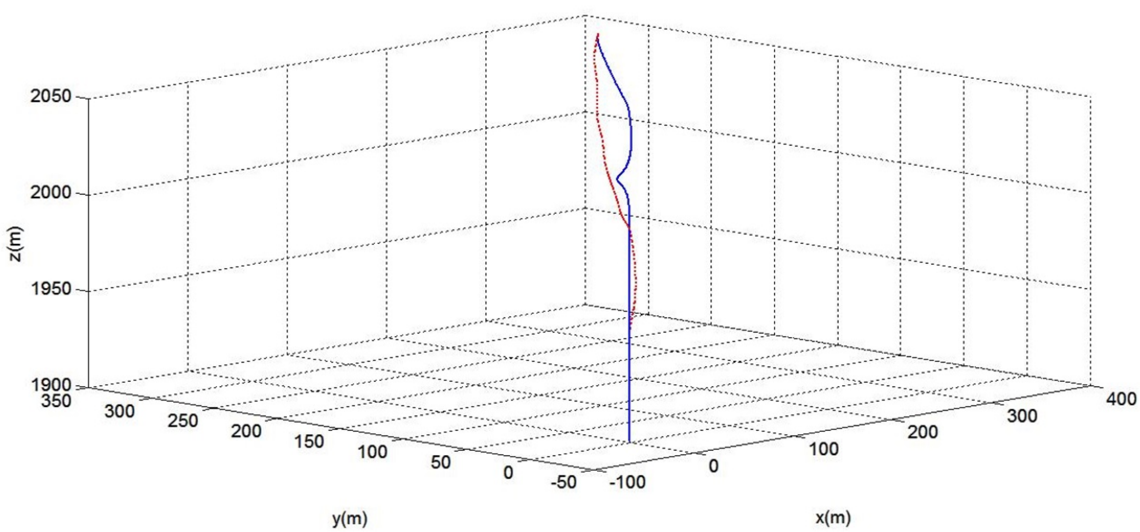

Fig. 8. The three-dimensional trajectory of the spacecraft for the terminal sliding mode control (dotted line) and the two-time scale control (solid line).