Adaptive Control of Dual User Teleoperation with Time Delay and Dynamic Uncertainty

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***Abstract*—This technical note aims at proposing an adaptive control scheme for dual-user teleoperation systems in the presence of time-delay and dynamic uncertainty in the parameters. The majority of existing control schemes for trilateral teleoperation systems have been developed for linear systems or nonlinear systems without dynamic uncertainty or time delay. However, in the practical teleoperation applications, the dynamics equations are nonlinear and contain uncertain parameters. In addition, the time delay in the communication channel mostly exists in the real applications and can affect the stability of closed loop system. As a result, an adaptive control methodology is proposed in this paper that can ensure the stability and performance of the system despite nonlinearity, dynamic uncertainties and time delay. Simulation results are presented to show the effectiveness of the proposed adaptive controller methodology.**

I. Introduction

By using teleoperation systems, a human operator has the ability to safely handle dangerous or unapproachable environments. Owing to this ability, teleoperation systems are useful in applications such as outer space, underwater explorations, and handling poisonous materials. The reader is referred to [1] for a historical survey. A more recent application of teleoperation systems is robotic surgery in which it has several features such as filtering tremor, scaling of position. and force, and enhanced sensitivity. In a bilateral teleoperation systems a single operator controls a master robot to send his desired commands to a slave robot.

An important area of teleoperation systems is dual user teleoperation systems where two human operators can carry out a task cooperatively. Robotic tele-rehabilitation and surgical training are considered as the main applications of this systems. So far, several control architectures have been proposed for dual user or multiuser teleoperation systems. In [2], a robust H_∞ controller is proposed for surgury training in a dual user teleoperation system. The authority of the users over the task is adjusted through a dominance factor according to the their relative level of expertise. However, the proposed methodology is a unilateral position-based architecture, which means that no kinesthetic feedback is reflected back from the environment to the users which is critical in aplications such as telesurgery. Inspired by [2], a six-channel control methodology is presented in [3] for a dual-user teleoperation system in the absence of time delay which provides kinesthetic feedback between both masters and the slave. In [4]), a criterion for absolute stability of a dual user teleoperation system is proposed based on the Llewellyns criterion [5]. In [6] a method for stability analysis in dual-user linear teleoperation systems is developed based on extended ZehebWalach (ZW) criteria for absolute stability [7]. An important note is that, all of the mentioned control methodologies for dual user teleopreation does not consider dynamic nonlinearities of the robots which is one of the most important complications in teleoperation systems.

In addition to the above schemes for linear dual user teleoperation, some control methods have been proposed for nonlinear dual-user teleoperation. For instance, in [8], an impedance control is proposed for a multiuser teleoperation system. The closed-loop stability analysis is ensured utilizing the small-gain theorem. Notwithstanding the fact that the proposed methodology is developed for nonlinear teleoperation systems, the stability of the whole system is not analysed in the presence of nonlinear dynamics. As an illustration, the presented methodology assumes that all the nonlinear terms are known; therefore, a simple inverse dynamics is able to eliminate all the nonlinearities. In [9] an adaptive nonlinear controller scheme is proposed for dual-user teleoperation system. However, the time delay in communication channel is not considered in the stability analysis. In [10], a control architecture is proposed for nonlinear dual user teleoperation based on $PD + d$ algorithm which is a generalization of the control algorithm proposed in [11]. However, the tracking performance of PD-based algorithm is not satisfactory when the nonlinear dynamics become complicated.

This paper addresses the adaptive control design problem for trilateral teleoperation in the presence of dynamic uncertainties. For a single robot manipulator, several adaptive controllers have been proposed to deal with dynamic uncertainties such as [12], [13]. Up to now, several adaptive control schemes have been proposed for bilateral teleoperation systems in the presence of dynamic uncertainties. In [14], a general nonlinear adaptive control scheme is presented to synchronize the master and slave positions and velocities. Afterwards, it was shown in [15] that the adaptive scheme proposed in [14] is generally appropriate in the absence of gravity forces, and an extended adaptive control algorithm is proposed to resolve this problem. In [16], a control methodology is proposed which is composed of an inner loop adaptive control and an outer loop robust control to deal with both parametric and non-parametric uncertainties.

Inspired by [12] and [13] as well as the adaptive controllers presented in [14] and [15] for bilateral teleoperation, an adaptive control scheme is proposed in this paper for dual-user teleoperation in the presence of time delay. Our presented methodology is mostly an extension of [15]. However, such an extension is not straightforward owing to some reasons. First, the proposed Lyapunov function candidate needs several modifications to be useful for dual-user teleoperation system. Besides, the analysis related to negative definiteness of the derivative of Lyapunov function is more complicated in the dual user case than in the single user case. In addition, the convergence of tracking error cannot be easily verified. To the best of our knowledge, no nadaptive control has been proposed for dual-user nonlinear teleoperation in the presence of time delay.

The rest of the paper is organized as follows. The dual user teleoperation system and its control objectives are described in Section II. Section III presents designing of adaptive controller for the system in the presence of dynamic uncertainties. Sections IV depicts simulation results. Finally, conclusions are stated in Section V.

II. System Description

Dual-user teleoperation systems include two master manipulators which are controllerd by human operators. The commands exerted by the operators are sent to the slave robot. In these systems, the authority of the the master manipulators over the slave manipulator is defined by dominance factor which will be discussed later. The connection between the robotic manipulators are established through communication channels. Note that, when the time delay in the communication channel generally influence the stability of closed loop system. Therefore, time delay should be considered in the stability analysis. The schematics of a general dual user teleoperation is shown in Fig. 1.

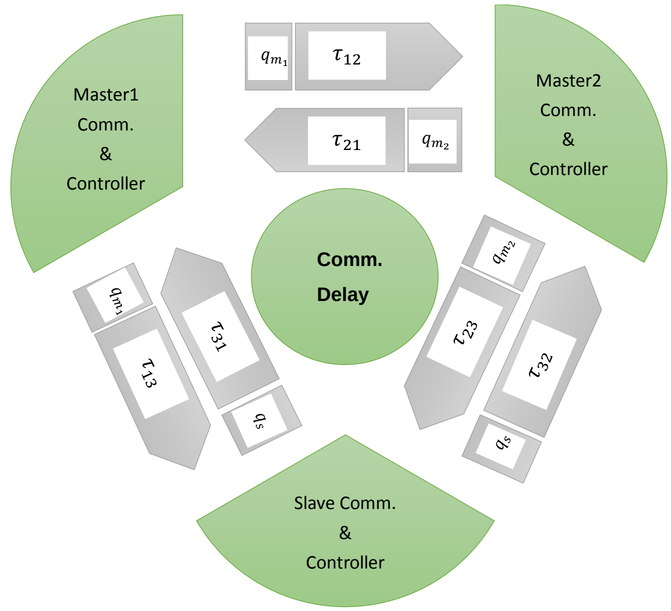


Fig. 1: Dual user trilateral teleoperation

The following n-DOF dynamics are considered for the master and slave robots [17]

$$M_{m1}(q_{m1})\ddot{q}_{m1} + C_{m1}(q_{m1},\dot{q}_{m1})\dot{q}_{m1} + G_{m1}(q_{m1}) = \tau_{m1} - \tau_{m1x} \tag{1}$$

$$M_{m2}(q_{m2})\ddot{q}_{m2} + C_{m2}(q_{m2},\dot{q}_{m2})\dot{q}_{m2} + G_{m2}(q_{m2}) = \tau_{m2} - tau_{m2x} \tag{2}$$

$$M_s(q_s)\ddot{q}_s + C_s(q_s,\dot{q}_s)\dot{q}_s + G_s(q_s) = \tau_s - \tau_{sx} \tag{3}$$

where $q_i \in \mathbb{R}^{n\times 1}$ are position vectors in joint space, $M_i(q_i) \in \mathbb{R}^{n\times n}$ are the inertia matrices, $C_i(\dot{q}_i, q_i) \in \mathbb{R}^{n\times n}$ are the centrifugal and Coriolis matrices, $G_i(q_i) \in \mathbb{R}^{n\times 1}$ are the gravity vectors, and $\tau_i \in \mathbb{R}^{n\times 1}$ are the control torque vectors all for $i = m, s1, s2$ where the subscript i denotes the masters for $i = m1, m2$ and slave robot for $i = s$. In addition, τ_{ix} represent the hand force of operators $\#1$, $\#2$ and the environment force for $i = m1$, $i = m2$, and $i = s$ respectively.

Some useful properties of the dynamic equations 1, 2, 3 are as follows ([17],[18])

Property 1. The inertia matrix $M_i(q_i)$ is symmetric and positive definite for all $q_i \in \mathbb{R}$

Property 2. The matrix $\dot{M}_{ii}(q_i) - 2C(q_i,\dot{q}_i)$ is skew symmetric which means that

$$x^T\big(\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i)\big)x = 0 \quad \forall x \in \mathbb{R}^n \tag{4}$$

Property 3. The left hand side of the dynamic models is linear in a set of physical parameters

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) = Y_i(q_i,\dot{q}_i,\dot{q}_i,\ddot{q}_i)\theta_i \tag{5}$$

the function, Y is called the Regressor and the Θ is the vector of physical parameters.

Finally, the desired positions of the robots in task space are defined as

$$\begin{aligned} q_{m1d}(t) &= \alpha_{12}q_{m2}(t-T_{12}) + \alpha_{13}q_s(t-T_{13}) \\ q_{m2d}(t) &= \alpha_{21}q_{m1}(t-T_{21}) + \alpha_{23}q_s(t-T_{23}) \\ q_{sd}(t) &= \alpha_{31}q_{m1}(t-T_{31}) + \alpha_{32}q_{m2}(t-T_{32}) \end{aligned} \tag{6}$$

where α_{ij} are dominance factors which determine authority sharing of masters and slave. α_{ij} should be selected such that

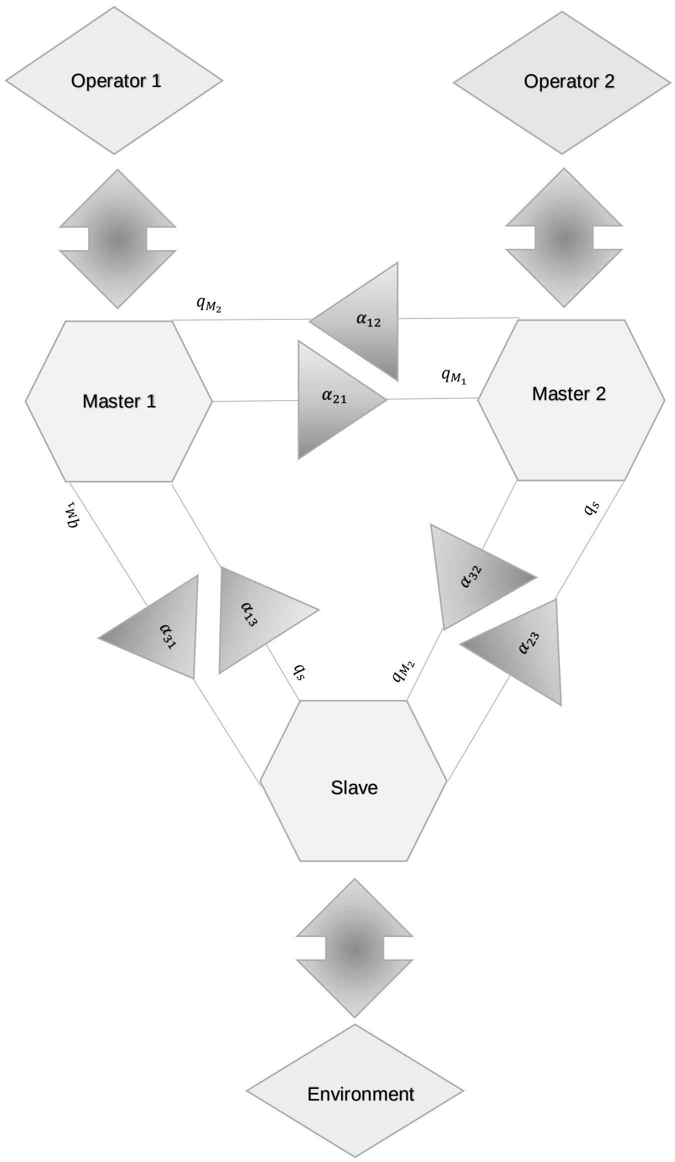


Fig. 2: Desired positions of the manipulators according to dominance factors

$0 \leq \alpha_{ij} \leq 1$ and

$$\begin{aligned} \alpha_{12} &= 1 - \alpha_{13} \\ \alpha_{21} &= 1 - \alpha_{23} \\ \alpha_{31} &= 1 - \alpha_{32} \end{aligned} \tag{7}$$

In order to choose reasonable values for α_{ij}, some performance metrics have been proposed in [19]. Fig. 2 shows the architecture of the system with dominance factors. Note that, our proposed architecture is a generalization of the one considered in [3], [10] with some useful advantages. As an illustration, just one and two parameter can be independently determined in the previously proposed schemes, but three independent parameters can be defined by the operators which gives more freedom. As a case in point, supposing that one of the operators want to only feel the feedback reflected from the environment. In this case, the related parameters can be independently defined without affecting the other parameters.

III. Adaptive Controller Design

In this section, the proposed adaptive control in the presence of dynamic uncertainty is illustrated. The analysis are illustrated for one robot with subscript i described by the following dynamic equation.

$$\begin{aligned} M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_s(q_i) \\ = \tau_i - \tau_{ix} \end{aligned} \tag{8}$$

For the rest of this paper, $i = 1$, 2, and 3 denotes master #1, master #2, and slave robot, respectively. Next, the error signal in joint space is defined as

$$e_i = q_i - q_{id} \tag{9}$$

where q_{id} is defined in (6). Afterwards, the variable r_i is defined as the new output.

$$r_i = \dot{q}_i + \Lambda_i e_i \tag{10}$$

where Λ_i is diagonal matrix of constant and positive values. Then, the control law is given by

$$\begin{aligned} \tau_i = & - \hat{M}_i(q_i)\Lambda\dot{e}_i - \hat{C}_i(q_i, \dot{q}_i)\Lambda e_i + \hat{G}_i(q_i) \\ & - K_p r_i - K_d \dot{e}_i \end{aligned} \tag{11}$$

where K_p and Kid are diagonal positive definit matrices and $\hat{M}_i(q_i)$, $\hat{C}_i(q_i, \dot{q}_i)$, and $\hat{G}_i$ are the estimates of their corresponding matrices. Using property 3, we have

$$\begin{aligned} & - \hat{M}_i(q_i)\Lambda\dot{e}_i - \hat{C}_i(q_i, \dot{q}_i)\Lambda e_i + \hat{G}_i(q_i) = \\ & Y_i(q_i, \dot{q}_i, -\Lambda e, -\Lambda\dot{e})\hat{\theta}_i \end{aligned} \tag{12}$$

where $\hat{\theta}_i$ is the estimate of parameter vector θ_i. Therefore, the control law (11) can be written as

$$\tau_i = Y_i(q_i, \dot{q}_i, -\Lambda e, -\Lambda\dot{e})\hat{\theta}_i - K_p r_i - K_d \dot{e}_i \tag{13}$$

Substituting the controller (11) on the dynamic equation (8) and using (10) yields

$$\begin{aligned} & M_i(q_i)\dot{r}_i + C_i(q_i, \dot{q}_i)r_i + K_p r_i + K_d \dot{e}_i = \\ & Y_i(q_i, \dot{q}_i, -\Lambda e, -\Lambda\dot{e})\tilde{\theta}_i - \tau_{ix} \end{aligned} \tag{14}$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$. The update law for uncertain parameter vector is given by

$$\hat{\theta}_i = -\Gamma_i Y_i^T(q_i, \dot{q}_i, -\Lambda e, -\Lambda\dot{e})r_i \tag{15}$$

Theorem: For the teleoperation system (1), (2), and (3) in free motion($\tau_{ix} = 0$) with the desired positions defined by (6), the adaptive control laws (11) and the parameter update law (15) all the signals of the closed loop system are bounded. Furthermore, position errors (e_i) and velocities $\dot{q}_i$ converge to zero.

Proof: we consider the following Lyapunov-Krasovskii candidate functional for each robot

$$V_i = (\frac{1}{2})\big(r_i^T(t)M_i(q_i)r_i(t) + \tilde{\theta}_i^T(t)\Gamma^{-1}\tilde{\theta}_i(t) + e_i(t)^T\Lambda K_d e_i(t) + \sum_{j\neq i}\alpha_{ij}\int_{t-T_{ij}}^{t}\dot{q}_j^T(s)K_d\dot{q}_j(s)ds\big) \tag{16}$$

Obviously, the above functional is positive definite and radially unbounded in r_i, $\tilde{\theta}_i$, and e_i. Using (4), the time derivative of V_i along the trajectories of (8) is given by

$$\dot{V}_i = -r_i^T(t)K_p r_i(t) + \dot{q}_i(t)^T K_d \dot{e}_i(t) + \sum_{j\neq i}\alpha_{ij}\big(\dot{q}_j^T(t)K_d\dot{q}_j(t) - \dot{q}_j^T(t-T_{ij})K_d\dot{q}_j(t-T_{ij})\big) \tag{17}$$

Note that

$$\dot{e}_i(t) = \dot{q}_i(t) - \sum_{j\neq i}\alpha_{ij}\dot{q}_j(t-T_{ij}) \tag{18}$$

Therefore, (17) is equivalent with

$$\begin{aligned}\dot{V}_i &= -r_i^T(t)K_p r_i(t) \\ &- \sum_{j\neq i}\alpha_{ij}\big(\dot{q}_i - \dot{q}_j(t-T_{ij})\big)^T K_d\big(\dot{q}_i - \dot{q}_j(t-T_{ij})\big) \\ &+ \dot{q}_i(t)^T K_d\dot{q}_i(t) - \sum_{j\neq i}\alpha_{ij}\big(\dot{q}_j^T(t)K_d\dot{q}_j(t)\big)\end{aligned} \tag{19}$$

Next, the following matrix is defined

$$\Upsilon = \begin{bmatrix}-1 & \alpha_{12} & \alpha_{13}\\ \alpha_{21} & -1 & \alpha_{23}\\ \alpha_{31} & \alpha_{32} & -1\end{bmatrix} \tag{20}$$

From (7), it is easy to see that $\det(\Upsilon) = 0$. Therefore, there exists a vector $\beta = \begin{bmatrix}\beta_1 & \beta_2 & \beta_3\end{bmatrix}^T$ such that $\beta^T\Upsilon = 0$. To be more specific, we can simply set $\beta = \begin{bmatrix}1 & 1 & 1\end{bmatrix}^T$ and see that $\beta\Upsilon = 0$. Afterwards, inspired by [20] and [21], consider

$$V = \sum_{i=1}^{3}\beta_i V_i(t)$$

Therefore, using (19) we have

$$\begin{aligned}\dot{V} &= -\sum_{i=1}^{3}\beta_i\big(r_i^T(t)K_p r_i(t)\big) - \sum_{i=1}^{3}\beta_i \times ... \\ &\Big(\sum_{j\neq i}\alpha_{ij}\big(\dot{q}_i - \dot{q}_j(t-T_{ij})\big)^T K_d\big(\dot{q}_i - \dot{q}_j(t-T_{ij})\big)\Big) \\ &+ \sum_{i=1}^{3}\beta_i\big(\dot{q}_i(t)^T K_d\dot{q}_i(t)\big) \\ &- \sum_{i=1}^{3}\beta_i\Big(\sum_{j\neq i}\alpha_{ij}\big(\dot{q}_j^T(t)K_d\dot{q}_j(t)\big)\Big)\end{aligned} \tag{21}$$

Then, the following equation can be stated

$$\begin{aligned}\dot{V} &= -\sum_{i=1}^{3}\beta_i\big(r_i^T(t)K_p r_i(t)\big) - \sum_{i=1}^{3}\beta_i \times ... \\ &\Big(\sum_{j\neq i}\alpha_{ij}\big(\dot{q}_i - \dot{q}_j(t-T_{ij})\big)^T K_d\big(\dot{q}_i - \dot{q}_j(t-T_{ij})\big)\Big) \\ &+ \beta^T\Upsilon Q\end{aligned} \tag{22}$$

where

$$Q = \begin{bmatrix}K_d(1)\dot{q}_1^2 & K_d(2)\dot{q}_1^2 & K_d(3)\dot{q}_1^2\end{bmatrix}^T \tag{23}$$

On the grounds that $\beta^T\Upsilon = 0$ the last term vanishes. As a result, $\dot{V} \leq 0$ which means that $r_i, e_i, \tilde{\theta}_i \in \mathcal{L}_\infty$. By integrating $\dot{V}$ from 0 to t, we can conclude that $r_i \in \mathcal{L}_2$. Thus, from (10) and (9), we can say that $\dot{e}_i, \dot{q}_i \in \mathcal{L}_\infty$. Afterwards, from (14) it is easy to conclude that $\dot{r} \in \mathcal{L}_\infty$. Then, since $r_i \in \mathcal{L}_2$ and $\dot{r} \in \mathcal{L}_\infty$ by using Barbalat's lemma, we conclude that $\lim_{t\to\infty} r_i = 0$.

On the other hand, with a similar reasoning which stated for r_i, we can say that $\dot{e}_i \in \mathcal{L}_2$. Also, $\dot{e}_i, \dot{r}_i \in \mathcal{L}_\infty$ means that $\ddot{q}_i, \ddot{e}_i \in \mathcal{L}_\infty$. Therefore, $\lim_{t\to\infty}\dot{e}_i = 0$ which implies that e_i converges to a constant value. In addition,

$$\lim_{t\to\infty}|r_i| = \lim_{t\to\infty}|\dot{q}_i + \Lambda e_i| = 0$$

which means that $\lim_{t\to\infty}\dot{q}_i = c$ where c is a constant value. Then, we can say that

$$\lim_{t\to\infty}\sum_{i=1}^{3}(\sum_{j=1, j\neq i}^{3}\alpha_{ij})r_i = 3c$$

Since r_i converges to zero so does c. Therefore, we have $\lim_{t\to\infty}|e_i| = \lim_{t\to\infty}|\dot{q}_i| = 0$. This means that the proof is complete. □

IV. Simulation Results

The proposed adaptive controller is utilized for position synchronization of a dual user teleoperation system. Three identical 2-DOF serial links with revolute joints are considered as the master #1, master #2, and slave manipulators. Their dynamic model is expressed in (8) It is possible to find the components of dynamic model by using the Lagrange method [17]. Thus, the inertia matrix is

$$M = \begin{bmatrix}m_{11} & m_{12}\\ m_{12} & m_{22}\end{bmatrix}$$

where

$$\begin{aligned}M_{11} &= m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}cos(q_2)) + I_1 + I_2 \\ M_{12} &= M_{21} = m_2(l_{c2}^2 + l_1 l_{c2}cos(q_2)) + I_2 \\ M_{22} &= m_2 l_{c2}^2 + I_2\end{aligned}$$

Also, matrix C is stated as

$$C(q,\dot{q}) = \begin{bmatrix}h\dot{q}_2 & h\dot{q}_2 + h\dot{q}_1\\ -h\dot{q}_1 & 0\end{bmatrix}$$

where

$$h = -m_2 l_1 l_{c2} sin(q_2).$$

In addition, the gravity vector is expressed as

$$G = \begin{bmatrix} (m_1 l_{c1} + m_2 l_1) gcos(q_1) + m_2 l_{c2} gcos(q_1 + q_2) \\ m_2 l_{c2} gcos(q_1 + q_2) \end{bmatrix}$$

It is easy to verify the two first properties explained in Section II. The third property is also satisfied with the following regressor.

$$Y(q, \dot{q}, \dot{q}, \ddot{q}) = \begin{bmatrix} \ddot{q}_1 & Y_{12} & \ddot{q}_2 & gcos(q_1) & gcos(q_1+q_2) \\ 0 & Y_{22} & \ddot{q}_1 + \ddot{q}_2 & 0 & gcos(q_1+q_2) \end{bmatrix}$$

in which,

$$Y_{12} = cos(q_2)(2\ddot{q}_1 + \ddot{q}_2) - sin(q_2)\dot{q}_2\dot{q}_2 - sin(q_2)\dot{q}_1\dot{q}_2 - sin(q_2)q$$

$$Y_{22} = cos(q_2)\ddot{q}_1 + sin(q_2)\dot{q}_1\dot{q}_1$$

$$\Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} = \begin{bmatrix} m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2) + I_1 + I_2 \\ m_2 l_1 l_{c2} \\ m_2 l_{c2}^2 + I_2 \\ m_1 l_{c1} + m_2 l_1 \\ m_2 l_2 \end{bmatrix} \quad (24)$$

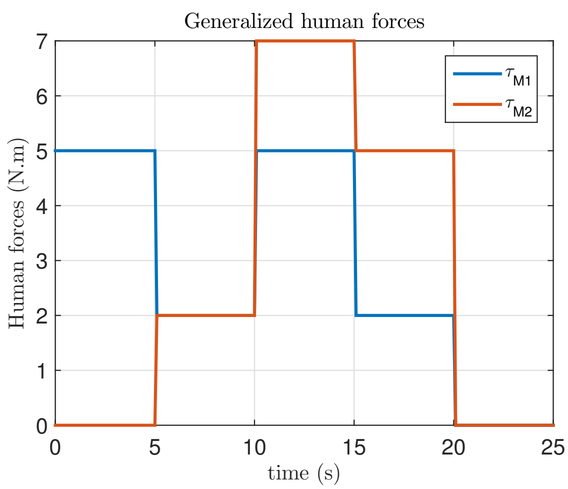
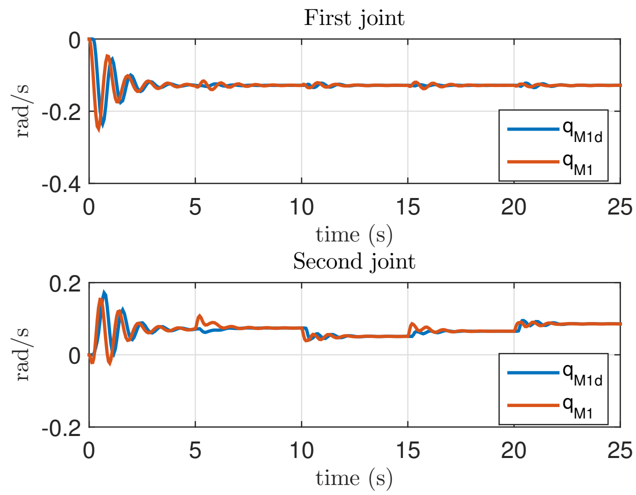


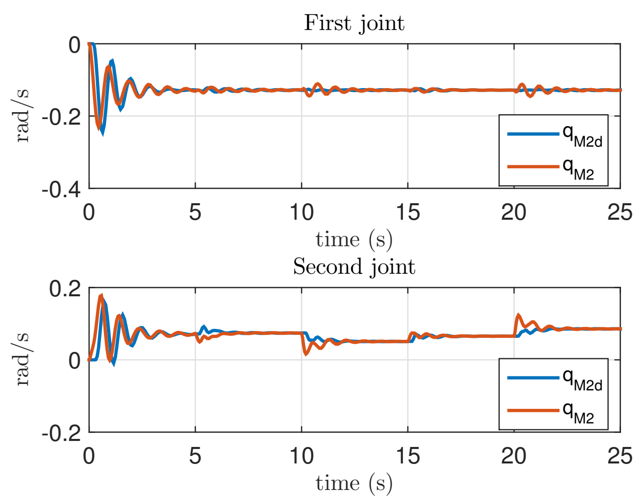
Fig. 3: Torques exerted by the human operators

In the simulations, we set $l_1 = 1m$ and $l_2 = 1m$ and $m_1 = m_2 = 1kg$ for all three manipulators. The initial conditions are considered to be $\ddot{q}_i(0) = \dot{q}_i(0) = q_i(0) = 0$. Besides we take $\Lambda = I$ and $\alpha_{jj} = 0.5$ for $i, j = 1, 2, 3, i \neq j$. Moreover, the controlers' gains are $K_p = I$ and $K_d = 3I$, the adaptation gain is set as $\Gamma = I$, and the time-delays in all channels are set to be $0.2s$.

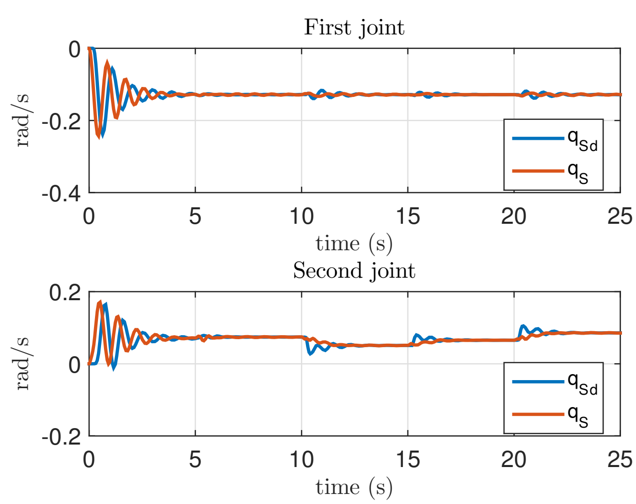
The torques exerted by the human operators are shown in Fig. 3. It is supposed that each operator exerts similar torques to the two joints but the torques exerted by operator #1 are different with that of operator #2. The desired and real values of the three manipulators are shown in Fig. 4 and the tracking errors are shown in Fig. 5. As the results show, the values of tracking errors are considerable at the beggining moments of simulation as compared to the next moments. Similar to the general adaptive controllers, its interpretation is that the vector of unknown parameters need some time to converge which



(a) Master #1 Manipulator



(b) Master #2 Manipulator



(c) Slave Manipulator

Fig. 4: The desired and real values of joint positions for each of the manipulators

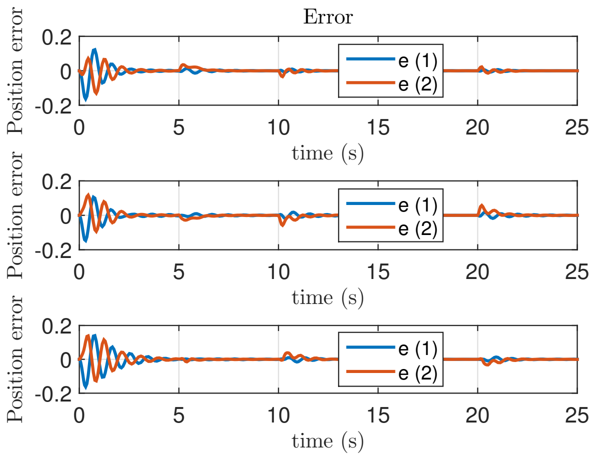


Fig. 5: Tracking error for each of the manipulators

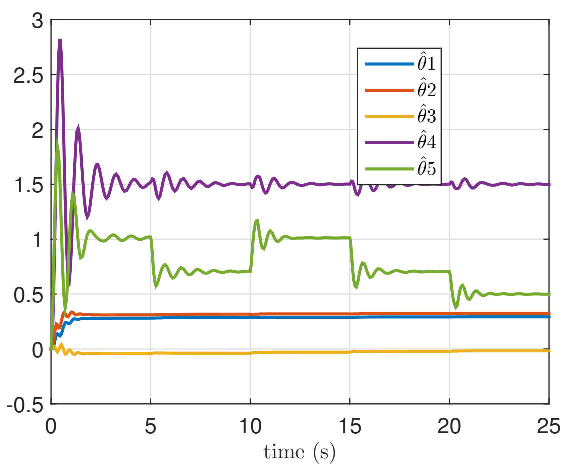


Fig. 6: Parameter estimation of master #1 Manipulator

cause tracking errors to get larg values. For better clarification, the parameter estimation of master #1 Manipulator is shown in Fig. 6. Apparently, the estimated parameters get larger variations at the beginning of simulation. Besides, as evident from the results, after passing the transient time of adaptive control, the real values of joint parameters track the desired values with satisfactory performance.

V. Conclusions

This paper proposes an adaptive controller for position synchronization of dual user teleoperation systems. The stability of the closed-loop system is verified and the convergence of tracking error is guaranteed in the presence of time delay and parametric uncertainty. Through simulation results, the effectiveness of the proposed approach in the sense of stability and performance is demonstrated.

The proposed controller can just tolerate dynaimc uncertainties in the parameters. In addition to the studied kind of uncertainty, there are other kinds of uncertainties such as kinematic and unstructured uncertainities that are widspread in the real robotic applications. Our next step is to address these kinds of uncertainties and to consider variations in time delay and dominance factors.

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