Iterative Symbol Synchronization for Bandwidth Efficient Burst Transmission

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Abstract—**In this paper, a new symbol timing recovery is proposed that is suitable for burst transmission scheme. It is assumed that the channel is an Additive White Gaussian Noise one and the transmitted data is unknown at the receiver. The contribution of the paper is exploiting Gardner Timing Error Detection (TED) algorithm for None-Data-Aided (NDA) timing delay estimation in an iterative manner. It is shown that the estimated delay also maximize the log-likelihood function. The algorithm is suitable for different types of linear modulations such as PSK, QAM. Simulation results confirm the algorithm convergence to the maximum of the likelihood function and a good performance in terms of Mean Square Error (MSE). The proposed timing recovery algorithm is bandwidth efficient which does not claim any pilot symbols and proceeds without using preamble. The algorithm is capable of recovering all symbols of a short burst transmission and is practically easy to be implemented.**

Keywords-symbol timing recovery; burst mode; Gardner timing error detection

I. INTRODUCTION

Any practical receiver has to estimate synchronization parameters in order to perform accurate detection of received signal. Timing recovery or timing synchronization is a major task in all the digital communication systems. At the receiver, the timing information is used to decide when the incoming signal has to be sampled and what the transmitted symbols are. Extensive work on timing recovery methods has been done in the literature, see [1] for more details.

Symbol timing recovery may be done through an error tracking synchronizers. The general structure of a error tracking synchronizer is based on a closed loop feedback, which is controlled by an error generated from Timing Error Detector circuit. This structure finally brings the system to a tracking mode. The transition between initial state and tracking mode is called acquisition. In a case of burst communications, the main drawback of this approach is that: an accurate acquisition would be time consuming process so that the major part of the burst remains unrecovered during achieving a stable equilibrium point, or even in the case of short burst that the number of symbols per burst is so small, acquisition may need more symbols than received burst sequence's symbols. In this case we can make the acquisition more agile but performance of system prone to severe degradation. As a result, applying error tracking synchronizers for a burst sequence, is not reasonable.

Iterative symbol timing recovery during the last decade has been an attractive approach for timing recovery [2-4]. This is mainly due to the fact that it provides a numerical solution for the conventional means of unknown parameter estimation, Maximum Likelihood (ML) approach. As we will see later, the complexity of a direct computation based on ML criteria is inevitable [5], [6]. Also the general frame work of the iterative synchronization prevents symbol loss, which is a prevalent problem in error tracking synchronizers. Based on iterative structure first, timing delay is estimated then, the estimated delay is used by sampler to justify its sampling time and to generate new samples. This process proceeds iteratively.

In accordance with the above statements, the main idea introduced in this paper is to derive a practical timing recovery algorithm easy to implement, useful for burst communication and suitable for linear modulations. In order to improve spectral efficiency, timing synchronization for burst sequence is done through NDA estimator [7], [8]. Following this idea, Gardner TED is used which is able to exploit timing information contained in the received burst sequence independent of carrier and phase offset [9], [10]. This information is then delivered to the sampling block to resample the burst sequence. As a consequence the symbol timing error moves towards decreasing and all symbols of the burst can be recovered.

The rest of the paper is organized as follows; the general mathematical expression relating timing estimator to the ML solution is given in section II. And finally, we assess the performance of the proposed algorithm and analyze it in section III.

II. ALGORITHM DERIVATION

A. Signal model

Consider a burst transmission scheme where a sequence of bits is mapped onto a special constellation. Each complex symbol of the constellation, a_k is shaped by pulse shaping filter $h(t)$ to produce a signal of K symbols. The resulting transmitted signal is:

$$x(t) = \sum_{k=0}^{K-1} a_k h(t - kT) \tag{1}$$

In traditional Communication system, conventional corruption to the transmitted signal during passing through channel is unknown timing delay and additive white Gaussian noise. So the received signal can be modeled as:

$$r(t) = x(t-\tau) + w(t)$$
$$r(t) = \sum_{k=0}^{K-1} a_k h(t - kT - \tau) + w(t) \tag{2}$$

Where τ is an unknown timing delay and $w(t)$ is additive white Gaussian noise with pass band two-sided power spectral density $N_0/2$ and $h(t)$ is a root raised cosine filter.

B. Likelihood function derivation

Suppose **r** is a vector of sampled received signal $r(t)$. Clearly the optimal receiver detects the transmitted symbols based on maximizing $p(a_k|\mathbf{r})$. In the absence of unknown synchronization parameter, **r** contains sufficient information to straight forward computation of maximum likelihood function. However in practical receivers the presence of one or more unknown parameter causes a complex computation of optimal symbol detection. In our problem, the unknown synchronization parameter is τ. To explain the above statement precisely, suppose:

$$\Lambda(\mathbf{r}|\mathbf{a},\tau) = \underset{a\,,\tau}{\arg\max}\; p(\mathbf{r}|\mathbf{a},\tau) \tag{3}$$

Where $\Lambda(\mathbf{r}|\mathbf{a},\tau)$ denotes for likelihood function of observed data **r** conditioned on **a** (vector of transmitted symbols) and τ. According to the ML criteria, maximizing of $\Lambda(\mathbf{r}|\mathbf{a},\tau)$ with respect to each parameter **a** or τ impose a joint detection and estimation of **a** and τ. As far as symbol detection is concerned, ML function dependency to unknown synchronization parameter, can be removed by marginalizing ML function [1], [7].

$$\Lambda(\mathbf{r}|\mathbf{a}) = E_\tau\{\Lambda(\mathbf{r}|\mathbf{a},\tau)\} \tag{4}$$

$$\Lambda(\mathbf{r}|\mathbf{a}) = \int_\tau \Lambda(\mathbf{r}|\mathbf{a},\tau)\, f(\tau)\, d\tau \tag{5}$$

To avoid the challenging computation of integral over unknown synchronization parameter in (5) and without loss of generality, τ is considered as a static variable, which is unknown but deterministic. Therefore, there is no useful probabilistic information except that they are in a given region. In this case $\Lambda(\mathbf{r}|\mathbf{a})$ can be written as [1]:

$$\Lambda(\mathbf{r}|\mathbf{a}) = \Lambda(\mathbf{r}|\mathbf{a}\,, \tau = \hat{\tau}) \tag{6}$$

Where $\hat{\tau}$ is an accurate estimation of timing synchronization parameter that can be achieved in turn through following equation:

$$\hat{\tau} = \underset{\tau}{\arg\max}\, p(\mathbf{r}|\mathbf{a},\tau) \tag{7}$$

Generally, the optimum receiver has to estimate timing delay based on maximizing the joint likelihood function $p(\mathbf{r}|\mathbf{a},\tau)$ and then select the sequence **a** with the largest likelihood. The conventional approach of estimating τ through (7) is to maximize $p(\mathbf{r}|\mathbf{a},\tau)$ with respect to τ for each possible **a.** Although there is finite number of possible sequence, but this exhaustive search method impose a lot of computation and is not practical especially when the length of the burst increases. To cope with this limitation iterative algorithm is applied.

C. Problem formulation

Conditional probability density function of received $r(t)$ signal is:

$$p(r(t)|\mathbf{a},\tau) = C \exp\left(\frac{-1}{2\sigma^2}\int_{T_0}\left|r(t) - \sum_k a_k h(t-kT-\tau)\right|^2 dt \right. \tag{8}$$

Where T_0 is the observation interval. The constant values in (6) can be dropped in likelihood function. Without any loss of generality we can maximize the log likelihood function $\Lambda_L(r(t)|\mathbf{a},\tau)$ instead of $\Lambda(r(t)|\mathbf{a},\tau)$. The log-likelihood function is:

$$\Lambda_L(r(t)|\mathbf{a},\tau) = \frac{-1}{2\sigma^2}\int_{T_0}\left|r(t) - \sum_k a_k h(t-kT-\tau)\right|^2 dt \tag{9}$$

Expanding (9), the terms $\int_{T_0}|r(t)|^2\,dt$ and $\int_{T_0}|\sum_k a_k h(t-kT-\tau)|^2\,dt$ are independent of τ and can be dropped [6]. By a little manipulation we come to the following equation:

$$\Lambda_L(r(t)|\mathbf{a},\tau) = \sum_{k=0}^{K-1} Re\left\{a_k^* \int_{T_0} r(t) h^*(t-kT-\tau) dt\right\} \tag{10}$$

As it was mentioned, maximizing (10) with respect to τ for each possible transmitted sequence, is practically intractable. In order to maximize ML function, one may resort to Gardner timing delay estimation. Suppose $\hat{\tau}$ is the estimated timing delay, achieved through Gardner TED. It is proved that $\hat{\tau}$ is also the solution of maximizing (10) [11].

According to Gardner TED input signal should be sampled at the rate of twice the symbol rate. Using the estimated timing delay, the corresponding samples generate in $\frac{iT}{2} + \hat{\tau}$ timing position. For sampled data by the consideration of Nyquist theorem; the integration can be replaced by summation.

$$\Lambda_L(\mathbf{r}|\mathbf{a},\tau) = \sum_{k=0}^{K-1} Re\left\{ a_k^* \sum_{\frac{iT}{2}\in T_0} r\left(\frac{iT}{2}+\hat{\tau}\right).h^*\left(\frac{iT}{2}+\hat{\tau}-kT-\tau\right)\right\} \quad (11)$$

Eq. (11) is interpreted as matched filter output [6]. Clearly timing error is proportional to gradient of likelihood function with respect to τ i.e.:

$$e \propto \left\{-\frac{\partial\Lambda_L(\mathbf{r}|\mathbf{a},\tau)}{\partial\tau}\Big|\tau = \hat{\tau}\right\} \quad (12)$$

$$= \sum_{k=0}^{K-1} Re\left\{ a_k^* \sum_{\frac{iT}{2}\in T_0} r\left(\frac{iT}{2}+\hat{\tau}\right).h'^*\left(\frac{iT}{2}-kT\right)\right\} \quad (13)$$

Where,

$$h'(t) = \frac{d}{dt}\,h(t)$$

In (13) summation goes through all symbols of a received burst sequence. For each individual symbol the generated timing error is:

$$e_i = Re\{r\left(\frac{iT}{2}+\hat{\tau}\right)\sum_k a_k^*\ h'^*\left(\frac{iT}{2}-kT\right)\} \quad (14)$$

It has been proved in [10] that for the pulse shaping filters such as square-root raised cosine, the timing error only exists for odd i, if m equivalently represents $\frac{i+1}{2}$, (14) can be written as:

$$e_m = Re\{\ r\left(\left(m-\frac{1}{2}\right)T+\hat{\tau}\right)(a_m^* - a_{m-1}^*)\ \} \quad (15)$$

Replacing a_m^* by $r^*(mT+\hat{\tau})$, which is a suboptimal estimation of a_m^* results to exact Gardner Timing Error Detector.

$$e_m = Re\{r\left(\left(m-\tfrac{1}{2}\right)T+\hat{\tau}\right)(r^*(mT+\hat{\tau}) - r^*((m-1)T+\hat{\tau}))\} \quad (16)$$

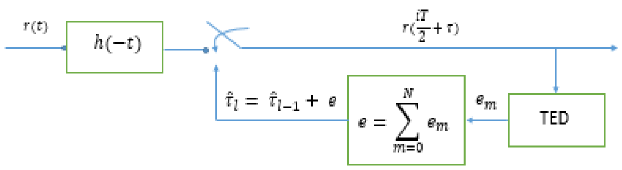

Figure.1 simple block diagram of proposed algorithm

Total timing error is achieved by making summation over all symbols of a burst.

$$e = \sum_{mT\in T_0} e_m \quad (17)$$

And timing delay estimation is update each time trough following equation:

$$\hat{\tau}_l = \hat{\tau}_{l-1} + e \quad (18)$$

Where l represents the iteration number. So, approximation of Gardner timing error for each symbol and make summation over all symbols of a burst consequently maximize the likelihood function. This procedure can be summarized in the following steps:

- First initializing $l = 1$ and $\hat{\tau}_0 = 0$ for prior applying Gardner algorithm on captured samples of a matched filter output to generate $\hat{\tau}_l$.
- Substituting last value of $\hat{\tau}_l$ in (11), to correct the sampling time and produce new samples.
- Using (16-18) to update $\hat{\tau}_l$ by the captured samples of pervious section.
- Check for convergence, if $e < \varepsilon$ and $\epsilon \cong 0$, the output is the latest estimation of modulation sequence, otherwise repeat the procedure from second step.

A simple block diagram of the system is shown in fig.1.

III. SIMULATION RESULTS

To illustrate the applicability of the introduced system, the simulation results are presented in this section. System's performance is analyzed in terms of mean squared error (MSE) which is averaged over 1000 Monte Carlo iterations. Simulations are carried out with linear modulations such as: BPSK, QPSK, 8PSK, 16QAM and 32QAM. The roll-off factor and the burst length are α and N respectively. Timing recovery initially starts by $\tau = 0.5T$ offset, which is the worst case in our problem. The up-sampling factor is set to 10 and new symbols are obtained via linear interpolation at the end of each iteration. The synchronizer operates with no need of prior carrier and phase synchronization and without any pilot symbols as a preamble.

Fig.2 displays the mean square error of proposed algorithm in terms of SNR per bit (E_b/N_0) for different linear modulation types. Burst length and α are equal to 100 and 0.5. As expected, performance improves as E_b/N_0 increases. Another result which is pointed out from this figure is that the performance of the algorithm gets better for increasing modulation order. This is mainly due to this fact that reliable timing error in Gardner's method usually would be obtained for the symbols of different amplitude (mostly for zero crossing symbols).

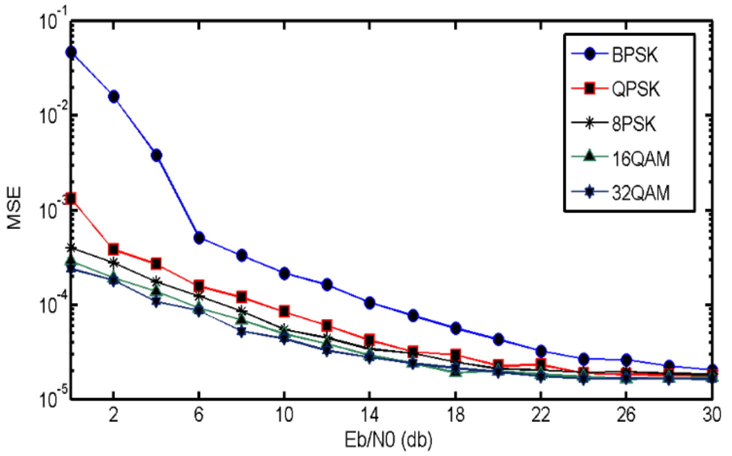

Fig.2 MSE vs E_b/N_0, $\alpha = 0.5, N = 100$

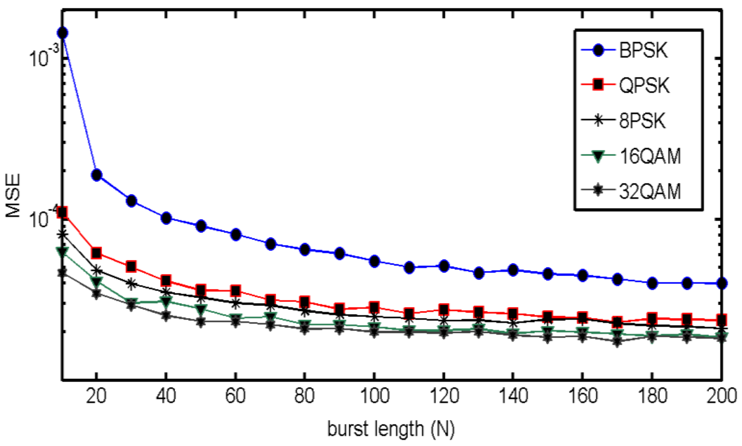

Fig.3 MSE vs N $\alpha = 0.5, E_b/N_0 = 20$ db

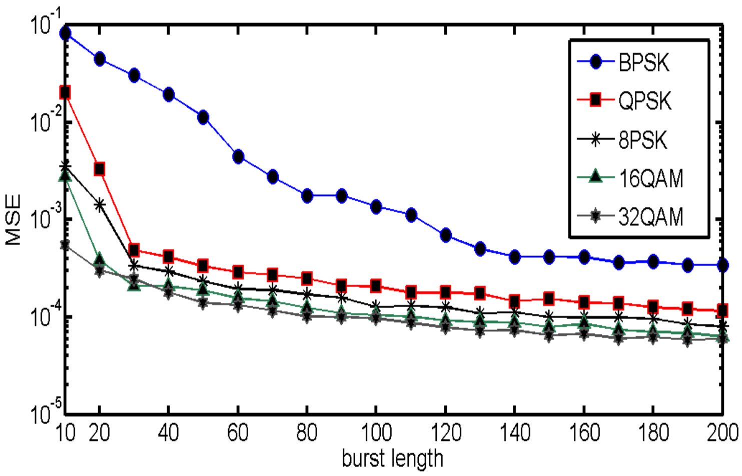

Fig.4 MSE vs N $\alpha = 0.5, E_b/N_0 = 7db$

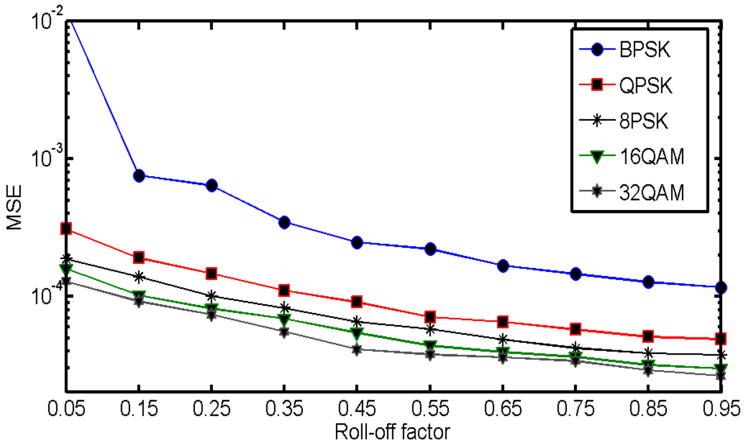

Fig.5 MSE vs Roll-Off $N = 100, E_b/N_0 = 10$

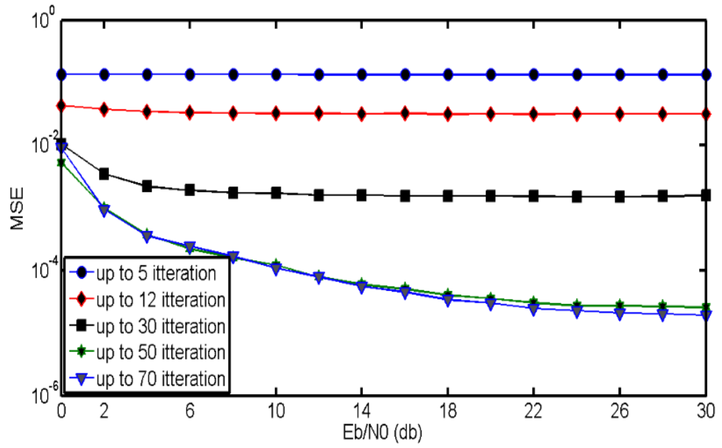

Fig.6 MSE vs E_b/N_0, for different iteration number, $N = 100$ $\alpha = 0.35$, modulation type: QPSK and limited iteration number.

As the modulation order increases the probability of dissimilar successive symbols increases. Therefore, more valid timing errors would be generated that improve the system performance.

Fig.3 and 4 present the MSE as a function of burst length (N) for $\alpha = 0.5$ and two different values of E_b/N_0. In fig.3 E_b/N_0 is set to 20db to evaluate system's performance in moderate SNRs, in order to have a better evaluation in low SNRs fig.4 is depicted for$E_b/N_0 = 7$db. Accordingly, the figures show that the estimation error decreases as the burst length increases, this can be interpreted as an AWGN noise cancellation effect, which is caused by summation. In other words, as the burst length grows up, noise summation tends to zero and consequently estimation gets better. However, simulations confirm the algorithm's capability of timing recovery with ultimately warranting its convergence to the maximum of likelihood function, even for burst length as short as 10 symbols.

In fig.5 the MSE versus the different values of roll-off factor is plotted. Burst length and E_b/N_0 are set to 100 and $10db$.

Roll-off factor starts from 0.05 increase 0.1 in each steps and ends to 0.95. As predicted. This figure also represents the diminishing trend of MSE for increasing α.

The produced result in Fig.6 shows the MSE in term of E_b/N_0, for the case of QPSK signal with roll-off factor and burst length correspond to α = 0.35 and N =100. Simulations have been done under the restricted iteration number. Clearly the most performance gain can be obtained for values of iteration number of 50 or greater than 50.

IV. CONCLUSION

In this paper a maximum likelihood based iterative timing recovery algorithm is presented that is suitable for burst transmission scheme. The algorithm benefits of a NDA TED estimator to update the time delay estimation and consequently generate the new sequence. Other advantages of the algorithm are the insensitive timing recovery of carrier frequency and initial phase offset, and lower sample rate needs for timing recovery. This algorithm leads to bandwidth efficient timing delay estimation which is able to recover all symbols of a burst regardless of any preamble.

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