Spectrum Analysis of Received Signals in Passive Radar

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***Abstract*—In this paper, we address the problem of estimating some signal spectrum characteristics in passive surveillant systems. The received signal is assumed to be completely unknown, i.e. no information about its spectrum is available. We estimate the spectrum of the received signal using a well known spectrum estimation method, Welch method. By some modifications the proposed method uses an adaptive threshold based on Constant False Alarm Rate (CFAR) processor to first detect the existence of signal spectrum, next estimates the carrier frequency of the received signal. We evaluate the method based on computer simulations and by RMSE measures. Numerical simulations demonstrate the capability of our method to estimate the begin and the end frequencies of a given chirp signal and also its bandwidth, accurately in low signal to noise ratios (SNR).**

***Index Terms*—Passive radar, Spectral analysis**

I. Introduction

Radar systems operate by radiating electromagnetic energy, are divided in two specific types: active radar and passive radar. In active radar, the transmitter transmits radio waves and the receiver uses the echo which returned from the reflecting objects to determine the properties of the target. The range, size, shape and location of target can be discerned by radar. Passive radar is a communications receiver that uses 'illuminators of opportunity' to measure the reflection from the environment and detects targets of interests. Passive radar is a receiver-only radar that can perform without transmitter hardware, hence it is undetectable. One of the most important issues in communications receiver including passive systems is perception and study the spectrum of radio waves in order to detect the target, extract the parameters of existing signals in environment and use of them. These operations in a perimeter surrounded by noise, interference and non-stationary signals requires to implement digital signal processing techniques.

Spectrum sensing involves making the spectrum of radio frequency (RF) observations and measuring power spectrum density (PSD) of received signals [1]. The problem of estimating the PSD of a wide-sense stationary random process is obtained by the Fourier transform of the auto-correlation sequences of received data. Therefore, estimating the PSD is equivalent to estimating the auto-correlation.

The methods for spectrum estimation are categorized into two classes: classical or non-parametric methods and non-classical or parametric methods [2]. Parametric methods are based on using a model for the process to estimate the power spectrum. In these approaches, initial information about the signal is required; hence they result exact estimation of parameter of signals and called high resolution methods. Since in passive radars, the received signal is completely unknown, so the parametric methods can not be used in passive systems. Classical or non-parametric methods begin by estimating the auto-correlation sequences. These methods include periodogram, modified periodogram, Bartlett method, Welch method and Blackman-Tuckey method.

The periodogram has an interesting interpretation in terms of filter banks [3]. Periodogram is proportional to the squared magnitude of the Fourier transform of the windowed signal. In the computational point of view, the periodogram is simple to evaluate, but this method can not produce an accurate estimation of the power spectrum, particularly for short data records. The variance of this method is independent to the length of data. When the amount of data increases, the variance does not decrease also the resolution is inversely proportional to the amount of data. The goal of modified method of periodogram is to modify the resolution to make it more exact periodogram, [4].

Modified periodogram is proposed to reduce the variance of periodogram by applying other windows instead of rectangular window. It offers no benefit in terms of reducing the variance just makes a trade of between spectral resolution (dependent to main lobe width of window) and spectral masking (dependent to side lobe amplitude of window) [4].

Bartlett's method considers estimating the power spectrum of a random process by periodogram averaging. In this method, the received signal is partitioned into K non-overlapping sequences of length L and the length of received signal is equal to $N = KL$. The variance will be proportional inversely to the number of segments and resolution is proportional to the number of segments. Therefore, the Barttlet's method allows to trade a reduction in spectral resolution for a reduction in variance by simply changing the values of K and L [4].

Welch method proposes two modifications to Barttlet's method. The first is to allow the sequences of signal to overlap and the second is to allow a data window to be apply to each sequence thereby producing a set of modified periodoram that are averaged. It is possible to increase the number and/or length of the sequences that are averaged by allowing the sequences to overlap, thereby trading a reduction in variance for a reduction in resolution. In this method, the resolution depends on the data window. Also the computational requirement increases in portion with *K*.

The methods of Barttlet and Welch are designed to reduce the variance of the periodogram by averaging periodogram and modified periodogram, respectively. Blackman-Tuckey method decreases the variance of the periodogram by taking the Fourier transform of a consistent estimate of the autocorrelation sequences [4].

Since we don't have any information about received signal in passive radars, for estimating the begin frequency of chirp signal and its bandwidth in these receivers, we face with detection problem. Therefore, in this paper for estimating the parameters of spectrum of signals we use one-point pre-detector and then compare the whole samples of received signals spectrum with an adaptive threshold.

In section II, we express a short review of Welch method. Since Welch method has better resolution and variance versus other methods of periodogram, we prefer to use the Welch method. In section III, one-point pre-detector will be introduced. In section III-A1, we will discuss about the adaptive threshold and estimating the begin frequency, end frequency and the bandwidth of received signal. In section IV, the resolution and RMSE of Welch method will be discussed. At the end, the concluding remarks are listed in section V.

II. Welch Spectral Estimator

In this section, we look at the Welch's method of periodogram averaging. As discussed before, the periodogram obtains an estimate of the spectrum of a random process $x(n)$ based on *N* samples of selected $x(n)$. The periodogram is proportional to the squared magnitude of the Fourier transform of the windowed signal [5].

$$P_{per}(e^{j\omega}) = \frac{1}{N}|\sum_{n=0}^{N-1} x(n)e^{-jn\omega}|^2 \tag{1}$$

In Welch method, the received signal is divided into the sequences that have overlap with each other. The sequences have offset by *D* points and the length of each sequence is *L*. The ith sequence is as following:

$$x_i(n) = x(n+iD), n = 0, 1, ..., L-1 \tag{2}$$

Amount of overlap between $x_i(n)$ and $x_{i+1}(n)$ is *L-D* points. If *K* sequences cover the entire *N* data points, then

$$N = L + D(K-1) \tag{3}$$

The estimation which produced with Welch's method in terms of $x(n)$ is written as follows:

$$P_W(e^{j\omega}) = \frac{1}{KLU}\sum_{i=0}^{K-1}|\sum_{n=0}^{L-1} w(n)x(n+iD)e^{-jn\omega}|^2 \tag{4}$$

$$U = \frac{1}{L}\sum_{n=0}^{L-1}|w(n)|^2 \tag{5}$$

where $w(n)$ is a window function that the resolution in Welch depends on it. The typical amount of overlap in Welch method is 50%, that gets average of more sequences for a given amount of data. It causes to decrease the variance of the estimation but the computational requirements will increase [4].

III. One-point pre-detector

In the passive surveillant systems, the blind detection is used since we do not have any information about the received signal. Before using the pre-detector algorithm, the received signal spectrum is estimated by the method that introduced in section II. The spectrum is compared with a threshold adapted to the environment, finally the samples of the spectrum that are greater than the threshold will be known as detected samples. If px_k is the kth spectrum samples of the received signal in the passive radar, in using one-point pre-detector we face a hypothesis testing problem as follows:

$$\begin{cases} H_0 : px_k = pw_k \\ H_1 : px_k = ps_k + pw_k \end{cases} \tag{6}$$

where pw_k is the *k*th spectrum sample of noise and ps_k is the *k*th spectrum sample of the existent signal in the environment. The distribution of the noise is supposed to be Gaussian with unknown variance and the received pulse is unknown. When the spectrum of noise is estimated by Welch method, the distribution of the spectrum of noise will change to Chi-square. The pre-detector problem is in the following form:

$$px_k \lessgtr_{H_1}^{H_0} T \tag{7}$$

where *T* is a pre-detector threshold that will be computed in the next section. The final detection is implemented with 0 and 1 sequences of primary pre-detector. In this study, the criterion of pulse spectrum detection is based on the existence of consecutive "1" and ignoring the existence of "0" between two "1". With the use of this algorithm, the spectrum of received pulse will be identified and the features are extracted.

A. Threshold Determination and Characteristics Estimation

1) Threshold Determination: As discussed in the previous section, there is not any information about the received signal in passive radar. Therefore, the characteristics determination of the existent signal spectrum, is required to use the blind detection that can be performed by an adaptive threshold. The CFAR is one of the common methods in active radars that computes the adaptive threshold. With some changes, we generalize and adapt the CFAR techniques to the passive radars. Indeed, this paper uses the modified CFAR algorithm for passive radar.

Two groups of data is used in the CFAR processor. The first group is the data that are collected from the cell under test (CUT) which searchs the existence or non-existence of target. The second group is the data which are collected from the reference cells that are spatially close to the CUT. The CFAR

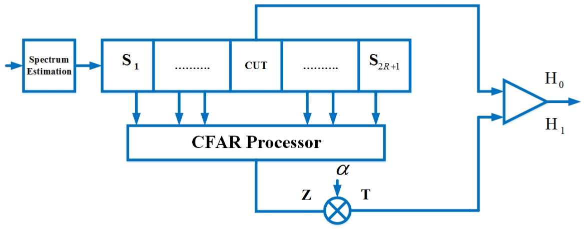

Fig. 1. CFAR detector Block diagram

processing window contains the leading reference window, CUT and lagging reference window. The block diagram of a typical CFAR processor is shown in figure 1: The existence of a target in the CUT can be tested as follows:

$$CUT \lessgtr_{H_1}^{H_0} T = \alpha Z \tag{8}$$

where α is a threshold coefficient and Z is a random variable depended on the reference cells. If the samples of spectrum are larger than the threshold, the main spectrum of signal is detected. In CFAR algorithm, there are different processes on leading and lagging reference windows. In this study, the SO-CFAR algorithm has been used to choose desired threshold for carrier frequency and bandwidth estimation of existent signal in the environment.

In each level of process, the number of cells in the CFAR processor window is assumed to be *2R+1* that *R* is the number of cells in the leading and lagging reference windows. Most of the algorithms in the CFAR processing, are performed by averaging the cells in the leading and lagging windows. The sum of the cells in leading and lagging windows are shown in following forms respectively:

$$U = \sum_{i=1}^{R} s_i \tag{9}$$

$$V = \sum_{i=R+2}^{2R+1} s_i \tag{10}$$

According to them, two well known CA-CFAR [6] and SO-CFAR [7] tests are shown consequently in (11),(12)

$$Z = U + V \tag{11}$$

$$Z = min(U, V) \tag{12}$$

In presented scenarios in our paper, the prototype signals are chirp signals with $2, 4\mu sec$ pulse width and $40, 60$MHz bandwidth that are mixed with a Gaussian noise with unit variance. The SNR for both signals is supposed $10dB$ and the Welch method has been used to estimate the spectrum of received signal. The performance of these algorithms is shown in figure 2 and 3. In these figures, the spectrum of signals, adaptive threshold and detected samples are depicted. In figure 2 when the CA-CFAR window sweeps spectrum of signal, the adaptive threshold increases and the spectrum of signal can not be detected. Therefore, this algorithm can not be used in passive radar.

As be seen in 3, the SO-CFAR algorithm can just detect the begin and end point of spectrum signal, because when the leading reference window is outside of the signal spectrum and the lagging reference window is inside, the begin point can be detected. Similarly, the end point can be detected. But when both leading and lagging window sweep the spectrum of signal, threshold will increase and the principal spectrum can not be detected. Some modification in this algorithm can help us to detect the spectrum of unknown signal in passive radar. When the lagging window sweeps the original spectrum, the leading window must not be shifted into the original spectrum until the lagging window goes out of the spectrum of signal. By this technique, this algorithm can detect the target and estimate the parameters of signal spectrum such as begin frequency, end frequency and bandwidth of the received signal [8]. In figure 4 the modified SO-CFAR algorithm is shown that can detect the spectrum of signal.

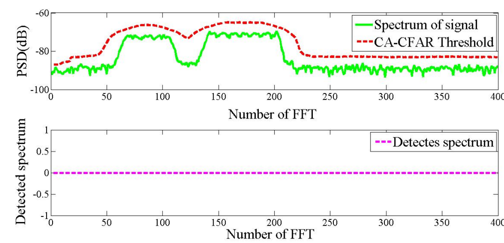

Fig. 2. Scenario for CA-CFAR by Welch method with rectangular window and overlap=50% in SNR1,2=10dB BW=40,60MHz

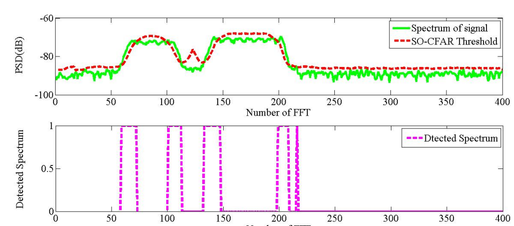

Fig. 3. Scenario for SO-CFAR by Welch method with rectangular window and overlap=50% in SNR1,2=10dB BW=40,60MHz

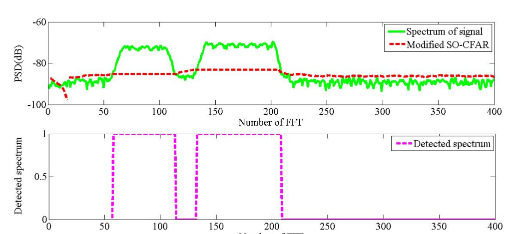

Fig. 4. Scenario for Modified SO-CFAR by Welch method with rectangular window and overlap=50% in SNR1,2=10dB BW=40,60MHz

2) Estimation of the Characteristics of Signal Spectrum: Since we are using the modified SO-CFAR algorithm in passive radar, estimation of characteristics of signal is faced with detection. In this algorithm, when two consecutive spectrum samples are larger than the threshold, the begin frequency of signal can be estimated. After some continuous consequtive detections of spectrum, when two sequential samples of spectrum are less than threshold, the end frequency of signal will be estimated. The bandwidth of signal is obtained by deducing the end and begin frequencies of signal. Moreover the carrier frequency can be computed by sum of begin frequency and half of bandwidth.

IV. Simulation Results

In this section, the performance of modified SO-CFAR algorithm is shown. This algorithm is investigated for Welch method in different situations. The performance of this algorithm is evaluated by resolution and RMSE measure.
The pulses that have been used are chirp signals with $2, 4\mu sec$ pulse width and $40, 60$MHz bandwidth. In all the simulations, the probability of false alarm is 0.01 in one-point pre-detector, the length of CFAR processor window is 35 samples, the sampling frequency is considered 1800MHz and the number of FFT is 2048.
If the detection algorithm could detect the chirp signals separately, the criterion of resolution is defined as the minimum distance between the end frequency of prior chirp signal and the begin frequency of next chirp signal. The RMSE is root minimum square error that has been computed the RMSE of begin frequency, end frequency and bandwidth of detected signal.
In this study, different windows with different overlaps for Welch method has been used and compared. The windows that have been studied are rectangular, Hanning, Hamming, Kaiser, Chebyshev and Taylor windows. There are different factors that affect on the resolution and RMSE such as the relative side lobe attenuation, the bandwidth of main lobe of window and the amount of overlap.

In order to achieve a valid content of resolution, this scenario is iterated 100 times. In the Welch method by rectangular window with -13dB side lobe attenuation and 50% overlap, the detector could detect the spectrum of two signals individually. The obtained resolution is 17MHz with 98% probability. The resolution of this procedure in 25% overlap is 16MHz with probability 98%, and in 12.5% overlap is 15MHz with probability 90%. It can be resulted that as much as the amount of overlap decreases, the resolution reduces that is desirable while the variance increases. As previously mentioned, the variance is proportional inversely to the number of segments. Decrement in amount of overlap makes the reduction in the number of segments and so the variance increases. On the other hand, increment in the variance results duplication in detection. Thus, there is a trade off between the resolution and variance. Another factor that affects on resolution is side lobe attenuation. As side lobe attenuation in different windows

TABLE I
Compare the resolution in different windows

window	side lobe attenuation	resolution		
		overlap=50%	overlap=25%	overlap=12.5%)
Rectangular	-13.3 dB	17MHz	16MHz	15MHz
Hanning	-31.5 dB	15MHz	10MHz	4MHz
Hamming	-42.7 dB	15MHz	10MHz	5MHz
Kaiser , $\beta = 0.5$	-13.6 dB	16MHz	15MHz	14MHz
Kaiser , $\beta = 4.5$	-33.3 dB	14MHz	11MHz	5MHz
Kaiser , $\beta = 7$	-51.2 dB	14MHz	10MHz	2MHz
Chebyshev	-100 dB	14MHz	11MHz	1MHz
Taylor	-30.3 dB	15MHz	14MHz	10MHz

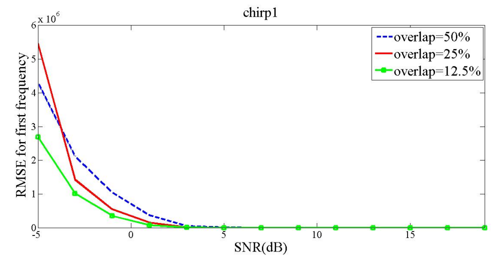

Fig. 5. Compare begin frequency RMSE for different overlap of rectangular window in Welch method

gets larger, the resolution improves. For instance, when we use Hanning window that its side lobe attenuation is about -31dB with 12.5% overlap and 99% probability the resolution becomes 4MHz. Although using the window with greater side lobe attenuation can make the resolution better, the algorithm faces with duplication. The Chebyshev window with -100dB side lobe attenuation can achieve best resolution about 1MHz but has duplication problem. It is a trade off between the side lobe attenuation and duplication, so the Hanning window with -31dB peak to side lobe can be the best choice. In table I the performance of this algorithm in different windows is displayed.

As the overlap increases, the RMSE will decrease. This claim can be proven in figure 5. we can see that in overlap=12.5%, the RMSE is minimum value. The most important factor in the RMSE curves is the bandwidth of main lobe of windows that is used for spectrum estimation. The minimum error is achieved by minimum bandwidth of the main lobe. For example, when we use rectangular window in Welch method, although it has just $-13dB$ side lobe attenuation but its main lobe is narrow; so its RMSE for begin frequency, end frequency and bandwidth is least. In the following the RMSE curves for begin frequency, end frequency and bandwidth of signal are demonstrated by Welch method with overlap=12.5%.

In figure 6, 7 and 8 we can observe the use of rectangular window and Kaiser window with $\beta = 0.5$ in Welch method (that results the minimum RMSE corresponding to the narrowest main lobe bandwidth). In Kaiser window, the parameter of β determines side lobe attenuation. Increment

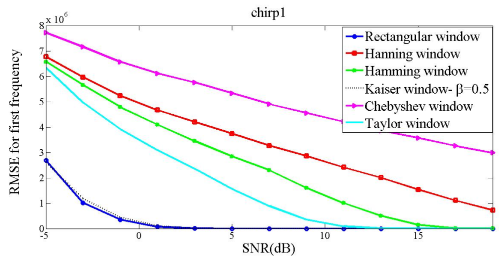

Fig. 6. Compare begin frequency RMSE for different windows in Welch method and overlap 12.5 %

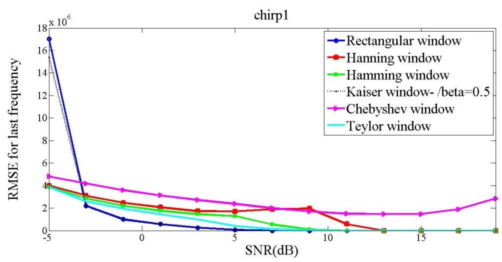

Fig. 7. Compare end frequency RMSE for different windows in Welch method and overlap 12.5 %

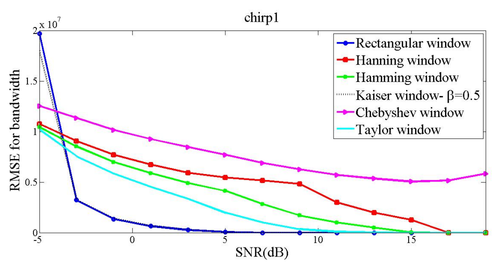

Fig. 8. Compare bandwidth RMSE for different windows in Welch method and overlap 12.5 %

in β makes the peak to side lobe amplitude increases and the increment in side lobe attenuation will increase the main lobe bandwidth. This increases the content of RMSE that is not desirable.

V. Conclusion

In this paper, the well known spectral estimation methods, was studied. In order to estimate the parameters of signal in frequency domain such as begin frequency, end frequency and the bandwidth of signal, we proposed a modified detection algorithm. The performance of this algorithm was investigated with resolution and RMSE in Welch method. We Showed that the type of window and amount of overlap in Welch method are effective in resolution. The Hanning window with 12.5% overlap and 4MHz has the best resolution.The RMSE depends on the main lobe bandwidth of the window. In this method, optimum RMSE belonged to the rectangular window with the narrowest main lobe bandwidth and 12.5% overlap. The precision in $SNR = 3$dB for estimating the begin frequency, end frequency and bandwidth was consecuitivly about 7KHZ, 277KHZ and 284KHZ.

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