Outage Probability Improvement of a Two-Way Relay Network with MABC DF Protocol by Joint Relay Selection and Optimal Power Allocation

Alireza Hasani[1], Vahid Tabataba Vakili[2]
Department of Electrical and Electronic Engineering
Iran University of Science and Technology (IUST)
Tehran, Iran, 1684613114
Email: [1]alireza_hasani@elec.iust.ac.ir, [2]vakily@iust.ac.ir

Abstract— **In this paper we focus on improving the performance of a two-way relay network consisting of two end nodes and multiple relays. It is assumed that the system utilizes multiple access broadcast as the transmission protocol and decode and forward as the relaying protocol. The performance metric of the system is outage probability and we try to improve it by creating diversity gain and coding gain. Our approach is relay selection that yields diversity gain, jointed with optimal power allocation that results in coding gain. Four methods for joint relay selection and power allocation are introduced. Among them, three techniques known as OPA-OS, OPA-MMS and OPA-HS result in full diversity gain (equal to number of relays) and 1.5 dB coding gain in the outage probability. One remaining technique i.e. OPA-MSS although provides no diversity gain, but can yield coding gain in lower SNRs.**

Keywords-component; Two-way relay network; multiple access broadcast; decode and forward; power allocation; relay selection

I. INTRODUCTION

Cooperative (or relay) networks are promising to present several benefits in wireless networks such as transmission range extension, spatial diversity and channel fading combating. Among various topologies considered so far for relay networks, Two-Way Relay Networks (TWRNs) are of special importance, since they make bidirectional information flow possible among the two transceivers.

Generally, in a simple three-node relay network similar to Fig. 1, comprising of half-duplex transceivers, operating in a single frequency band, a and b need four time-slots to exchange their data: I) $a \rightarrow r$; II) $r \rightarrow b$; III) $b \rightarrow r$; IV) $r \rightarrow a$. By exploiting the idea of network coding, one can reduce the number of time-slots and therefore increase the spectral efficiency (transmitting more bits in less time). Multiple Access Broadcast (MABC) is one of the common transmission protocols that utilizes the network coding and enables terminals a and b to exchange data in two time-slots: I) $a \rightarrow r \leftarrow b$; II) $a \leftarrow r \rightarrow b$. We have explained the details of this protocol in section II.A. In addition, several relaying protocols such as Decode and Forward (DF), Amplify and Forward (AF) and Compress and Forward (CF) have been proposed that specify how relays forward the received signal [1-5].

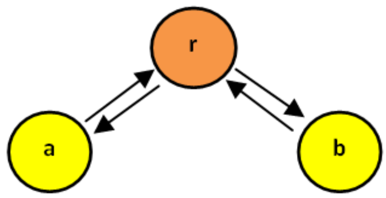

Figure 1. A three-node relay network, with half-duplex transceivers operating in single frequency band.

A vast amount of researches on relay networks have been devoted to performance improvement. The performance of a relay network can be explained in terms of metrics such as outage probability and/or error probability. For instance, one could devise a method increasing the slope of the outage/error probability curve and hence obtaining diversity gain in the system. Alternatively, one may be able to shift error/outage probability curves towards lower SNRs which is equivalent to achieving coding gain.

The methods by which the performance of a relay network is improved are different. Optimal resource (power, rate, etc.) allocation to the nodes and relay selection in multiple-relay networks are of important examples. In this paper, we have chosen outage probability as the performance metric and proposed joint relay selection and power allocation methods to obtain both diversity and coding gain.

Optimal Power Allocation (OPA) and/or relay selection in DF TWRNs form the basis of many previous works [6-17]. Specifically, in [6-8, 17] OPA has been discussed while [9-14] spotlight the relay selection. References [15, 16], on the other hand discuss the joint relay selection and power allocation in such networks.

The author in [14] considers a TWRN with MABC DF protocol consisting of two terminals and a cluster of relays; then introduces several techniques for relay selection with the target of achieving diversity gain in outage probability. This work assumes that the terminals and the selected relay transmit with equal power and the data rate of terminals are also the same (data rate fairness condition). On the other hand, we dealt with the issue of OPA in a MABC DF single-relay TWRN in [17]. To be specific, we derived a closed-form mathematical expression for the outage probability under the most general system model than former works, and then proposed an algorithm to find transmit power values that minimizes the outage probability and provides coding gain.

In this paper, we extend the idea of [17] to the case of a multiple-relay TWRN and propose joint relay selection and power allocation schemes, so that we can observe both diversity gain and coding gain simultaneously in the outage probability of the system.

The remainder of this paper is organized as follows. In section II, the system model that we study is exactly described and the concept of performance metric i.e. outage probability defined. In section III, we introduce our joint relay selection and power allocation techniques. The simulation and numerical results are included in section IV. Finally, we conclude the paper in section V.

II. System Model

In this section, we first describe our system model and then its performance metric.

A. System Model

The system model on which our approach is based, is the same as in [14]. This model is depicted in Fig. 2, representing a TWRN comprising of two terminals a and b and K relays. Terminals a and b aim to transmit independent messages w_a and $w_b$to each other via relays. It is assumed that there is no direct link between a and b and the information exchange is only possible by the help of relays. All the nodes are single-antenna half-duplex transceivers. Terminals a and b transmit with data rate R_a and R_b bits per channel use (BPCH) respectively, i.e. $w_a \in \{0,1,\cdots,\lfloor 2^{R_a} \rfloor - 1\}$ and $w_b \in \{0,1,\cdots,\lfloor 2^{R_b} \rfloor - 1\}$.

All the transmissions of network occur in a single frequency band. The protocol for information exchange is MABC DF which splits the transmission process into two phases. During the first phase, called MAC (Figure 2. Fig. 2-a), terminals a and b transmit their messages to the relays by forming a conventional MAC channel [14]. The transmit power of terminals a and b are represented by P_a and P_b respectively. Each relay attempts to extract messages w_a and w_b by Maximum Likelihood (ML) decoding [18]. In the second phase of transmission, called BC (Fig. 2-b), one of the relays is selected among others and assists terminals in exchanging data. We will explain about the relay selection scheme in section III. The selected relay, r', produces $w_{r'}$ by combining messages w_a and w_b using DF-JM network coding scheme [18]. DF-JM network coding is appropriate for the case that the messages to be combined are not necessarily of the same length, i.e. $R_a \neq R_b$. r' broadcasts $w_{r'}$ with transmit power $P_{r'}$. Terminals a and b exploit ML decoding scheme to extract $w_{r'}$ and then perform self-interference cancellation to recover the intended message. (Terminal a whishes to recover w_b and terminal b wishes to recover w_a from $w_{r'}$). Since the selected relay has only the role of assistant and no benefit acquires, we impose the constraint $P_{r'} \leq P_a, P_b$, so that it consumes less power than the terminals. The duration of MAC and BC phase in the protocol is assumed to be equal.

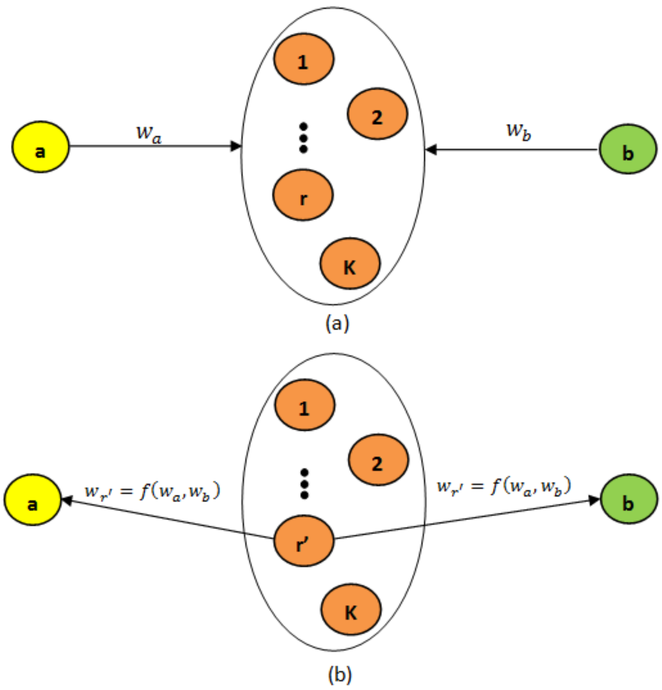

Figure 2. Steps of information exchange in the TWRN with MABC DF protocol. (a) MAC phase; (b) BC Phase.

It is assumed that all nodes can receive messages properly, without any error, since we have chosen outage probability as the performance metric in our work and error probability is not the subject of it.

The physical channels between each pair of nodes are subject to the block fading and modeled as a circularly symmetric and complex Guassian random variables with zero-mean and unit variance. Furthermore, different links are statistically independent of each other. Channel coefficient between terminal $i \in \{a,b\}$ and relay $r \in S = \{1,\ldots,K\}$ is represented by h_{ir} . All the channels are assumed to be reciprocal i.e. $h_{ir} = h_{ri}$. Additive noise is also considered in the receiving signals at the nodes which is modeled as a zero-mean circularly symmetric complex Guassian random variable with variance σ^2. Channel coefficients remain constant during one interval of transmission (one MAC plus BC phase), but may vary for the next interval independently. Necessary bits are also included in the MAC phase signals that make relays capable of estimating their own channels. However terminals a and b are not required to have knowledge of their own channel.

B. Performance Metric

For each communications network we can specify a region called achievable rate region. This is the region that determines what the data rate value of different nodes can be, in the system. If at least one of the nodes transmits with data rate value, outside of the achievable rate region, then an outage will occur. For the system described above, the achievable rate region is determined by the constraints [19]

$$R_a \leq \frac{1}{2} min \left\{ \log_2 \left(1 + P_a \frac{\left|h_{ar'}\right|^2}{\sigma^2} \right), \log_2 \left(1 + P_{r'} \frac{\left|h_{br'}\right|^2}{\sigma^2} \right) \right\}, (1)$$

$$R_b \leq \frac{1}{2} min \left\{ \log_2 \left(1 + P_b \frac{\left|h_{br'}\right|^2}{\sigma^2} \right), \log_2 \left(1 + P_{r'} \frac{\left|h_{ar'}\right|^2}{\sigma^2} \right) \right\}, (2)$$

$$R_a + R_a \leq \frac{1}{2} \log_2 \left(1 + P_a \frac{\left|h_{ar'}\right|^2}{\sigma^2} + P_b \frac{\left|h_{br'}\right|^2}{\sigma^2} \right). \qquad (3)$$

If the parameters R_a, R_b, P_a, P_b and $P_{r'}$are determined so that at least one of the constraints in (1)-(3) is not satisfied, then an outage will occur. We have evaluated, in our previous work [17], the outage probability and obtained a closed form mathematical expression for it. The reader may refer to (16)-(21) in [17] that specify the outage probability of the system. It should be recalled that these equations determine the outage probability of the system under the constraint $P_{r'} \leq P_a, P_b$. (We discussed the reasonability of this constraint in previous section).

III. RELAY SELECTION AND POWER ALLOCATION

The author in [14] has proposed four relay selection schemes for a system model described in part II-A, with the exception that he has assumed equal power allocation to the nodes ($P_a = P_b = P_r$) and data rate fairness ($R_a = R_b$). these relay selection schemes have set diversity gain in outage probability as the target. On the other hand, optimal power allocation is the main subject of our recent work [17]. We have proposed a power allocation technique in which transmit power of terminals a and b and the selected relay r' are adjusted such that outage probability is minimized and coding gain is obtained.

Now we improve relay selection schemes of [14] by employing our power allocation technique and devise new methods for joint relay selection and power allocation in a TWRN with MABC DF protocol. As a result of this improvement, diversity gain and coding gain simultaneously appear in outage probability and therefore a better quality of data transmission is achieved.

In all subsequent techniques, P_a^*, P_b^* and P_r^* are optimized power values according to the algorithm described in [17]. As we have clarified in [17] the power allocation scheme is unrelated to the exact channel coefficient values and only the statistical properties of channel are important.

The improved relay selection schemes according to the OPA is as follows.

A. *Optimal Power Allocation-Nonselection (OPA-NS)*

In this method, no selection is made among the relays or equivalently one of the K relays is randomly selected and therefore no diversity gain is expected from this scheme. It has been proved in [14] that diversity gain for this technique is equal to one ($d_{OPA-NS} = 1$).

We will use this scheme as a reference for comparison between proposed schemes in simulations (section IV).

B. *Optimal Power Allocation-Optimal Selection (OPA-OS)*

In each transmission period, the relays of the set S that can satisfy the constraints (1)-(3) are considered in the set S' which is obviously a subset of S (S' may be empty, namely no relay can satisfy these constraints). If S' is not empty, then one relay of this set will be selected randomly, otherwise (if $S'$is empty) an outage will occur. The selection technique can be represented as

$$r' = r \; with \; r \in S' \; and \; Pr\{r' = r\} = \frac{1}{|S'|}, \qquad (4)$$

where $Pr\{.\}$ is the probability of happening a random event and $|S|$ is the cardinality of S.

This selection scheme results in full diversity gain, i.e. $d_{OPA-OS} = K$ as demonstrated in [14].

The main challenge of this method is its complexity. Specifically, a central management node is needed to analyze all the relays and select one of them that has the specified conditions.

C. *Optimal Power Allocation-Max-Min Selection (OPA-MMS)*

By employing OPA technique, the MMS method of [14] is modified as

$$r' = arg \max_{r \in S} min \left[P_a^* \frac{|h_{ar}|^2}{\sigma^2}, P_r^* \frac{|h_{br}|^2}{\sigma^2}, P_b^* \frac{|h_{br}|^2}{\sigma^2}, P_r^* \frac{|h_{ar}|^2}{\sigma^2} \right]. (5)$$

The philosophy behind this selection is that it tries to maximize the point-to-point transmission rate constraints (1)-(2). Specifically, it selects the relay that its minimum instantaneous SNR is higher than others. Max-min selection also results in full diversity gain, i.e. $d_{OPA-MMS} = K$ [14].

The main advantage of max-min selection over optimal selection is its reduced complexity. All the relays that receive w_a and w_b, calculate their own term $min \left[P_a^* \frac{|h_{ar}|^2}{\sigma^2}, P_r^* \frac{|h_{br}|^2}{\sigma^2}, P_b^* \frac{|h_{br}|^2}{\sigma^2}, P_r^* \frac{|h_{ar}|^2}{\sigma^2} \right]$ and wait for a time interval inversely proportional to this term. As a result, the relay that has the largest value, is the one that transmits before others and also informs the remaining relays not to transmit.

D. *Optimal Power Allocation-Max-Sum Selection (OPA-MSS)*

By employing OPA technique, the MSS method of [14] is modified as

$$r' = arg \max_{r \in S} \left[P_a^* \frac{|h_{ar}|^2}{\sigma^2} + P_b^* \frac{|h_{br}|^2}{\sigma^2} \right]. \qquad (6)$$

To be specific, max-sum selection focuses on the sum-rate constraint (3) attempting to select the relay that has maximum sum-rate.

In contrast to OS and MMS, MSS fails to provide diversity gain at high SNRs. It has been confirmed in [14] that MSS is only efficient for low SNRs and $d_{MSS} \to 1$ as $SNR \to \infty$.

Implementing the MSS method resembles much to the MMS method. The only difference is that the relays must calculate the term $P_a^* \frac{|h_{ar}|^2}{\sigma^2} + P_b^* \frac{|h_{br}|^2}{\sigma^2}$ and set a timer inversely proportional to this term.

E. Optimal Power Allocation-Hybrid Selection (OPA-HS)

The HS method utilizes both MSS that prepares coding gain at low SNRs and MMS that provides diversity gain at high SNRs. By employing OPA technique, the HS method of [14] can be modified as:

If $R_a + R_b \le \frac{1}{2}\log_2\left(1 + P_a^* \frac{|h_{ar}|^2}{\sigma^2} + P_b^* \frac{|h_{br}|^2}{\sigma^2}\right)$, then

$$r' = arg\,\max_{r \in S} min\left[P_a^* \frac{|h_{ar}|^2}{\sigma^2}, P_r^* \frac{|h_{br}|^2}{\sigma^2}, P_b^* \frac{|h_{br}|^2}{\sigma^2}, P_r^* \frac{|h_{ar}|^2}{\sigma^2}\right] ,(7)$$

otherwise

$$r' = arg\,\max_{r \in S}\left[P_a^* \frac{|h_{ar}|^2}{\sigma^2} + P_b^* \frac{|h_{br}|^2}{\sigma^2}\right]. \quad (8)$$

HS method provides full diversity gain, i.e. $d_{OPA-HS} = K$ [14].

IV. SIMULATION AND NUMERICAL RESULTS

We have conducted various Monte Carlo simulations to demonstrate the effectiveness of our proposed joint relay selection and power allocation techniques. The results of these simulations have been depicted in Figure 3. Fig. 3-12. All the figures illustrate curves of outage probability versus $SNR = \frac{P_{sum}}{\sigma^2}$ (measured in dB) where $P_{sum} = P_a + P_b + P_{r'}$ is the total transmit power of active nodes. We have assumed $\sigma = 1$ (unit noise variance) in simulations which results in $SNR = P_{sum}$.

In all these figures, the curves specified by white circles (EPA, NS) represent the outage probability in the case of equal power allocation, i.e. $P_a = P_b = P_{r'} = \frac{P_{sum}}{3}$ and the nonselection method.

In the curves specified by white squares, OPA has not yet utilized and therefore $P_a = P_b = P_{r'} = \frac{P_{sum}}{3}$. However different relay selection schemes of [14] have been employed. As it is apparent of Fig. 3-6,9,10 using OS, MMS and HS schemes result in diversity gain in outage probability, but MSS scheme (Fig. 7.8) does not provide diversity gain and only coding gain is obtained in low SNRs.

In the curves specified by white triangles, the joint relay selection and power allocation methods, explained in section III, have been simulated. It is observed that for OPA-OS and OPA-MMS (Fig. 3-6), about 1.5 dB coding gain is obtained when OPA is carried out.

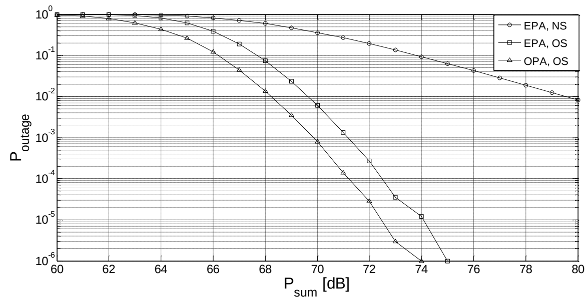

Figure 3. Outage probability versus SNR for a MABC DF TWRN with K=5, $R_a = 7$, $R_a = 4$ BPCH and OPA-OS method.

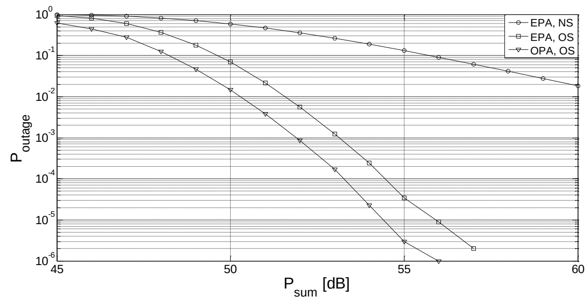

Figure 4. Outage probability versus SNR for a MABC DF TWRN with K=5, $R_a = 4$, $R_a = 4$ BPCH and OPA-OS method.

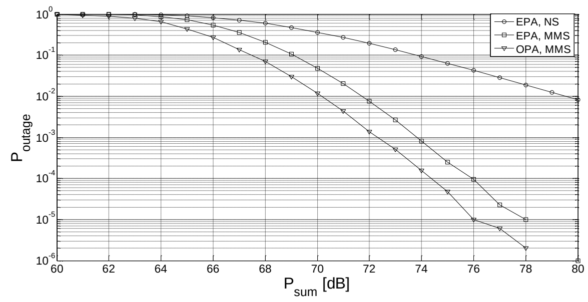

Figure 5. Outage probability versus SNR for a MABC DF TWRN with K=5, $R_a = 7$, $R_a = 4$ BPCH and OPA-MMS method.

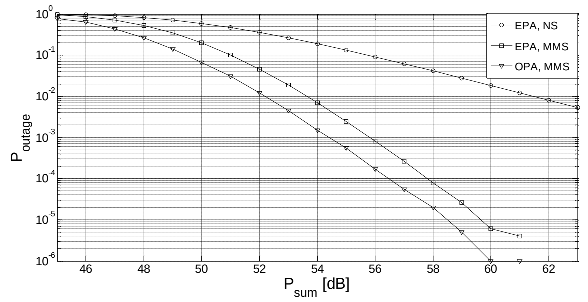

Figure 6. Outage probability versus SNR for a MABC DF TWRN with K=5, $R_a = 4$, $R_a = 4$ BPCH and OPA-MMS method.

Fig. 7,8 show that the OPA is effective as far as the MSS is effective. In other words as $SNR \to \infty$, the diversity gain becomes 1 and therefore the outage probability curve becomes parallel with the case of NS.

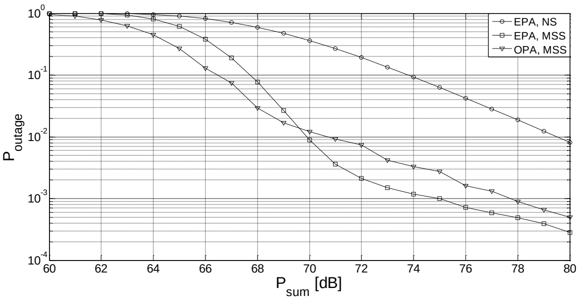

Figure 7. Outage probability versus SNR for a MABC DF TWRN with K=5, $R_a = 7$, $R_a = 4$ BPCH and OPA-MSS method.

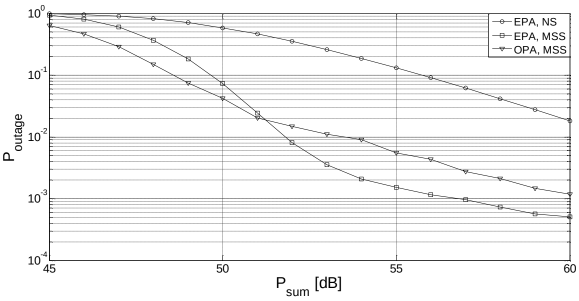

Figure 8. Outage probability versus SNR for a MABC DF TWRN with K=5, $R_a = 4$, $R_a = 4$ BPCH and OPA-MSS method.

By combining MMS and MSS in OPA-HS method, the coding gain of OPA- MMS for high SNRs and OPA-MSS for low SNRs become available simultaneously, as depicted in Fig. 9,10.

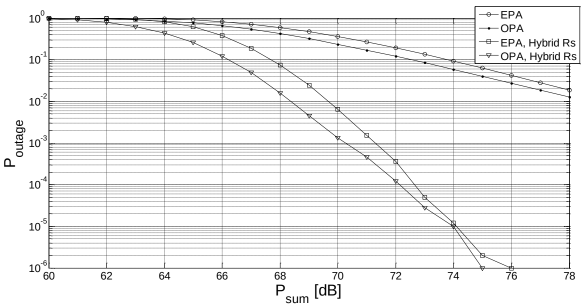

Figure 9. Outage probability versus SNR for a MABC DF TWRN with K=5, $R_a = 7$, $R_a = 4$ BPCH and OPA-HS method.

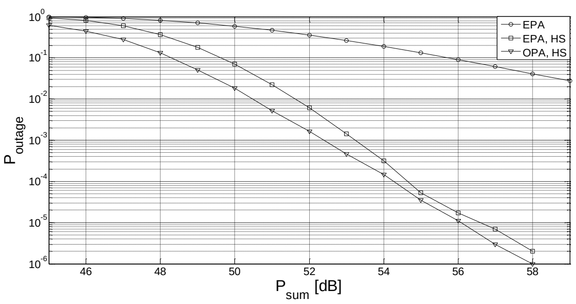

Figure 10. Outage probability versus SNR for a MABC DF TWRN with K=5, $R_a = 4$, $R_a = 4$ BPCH and OPA-HS method.

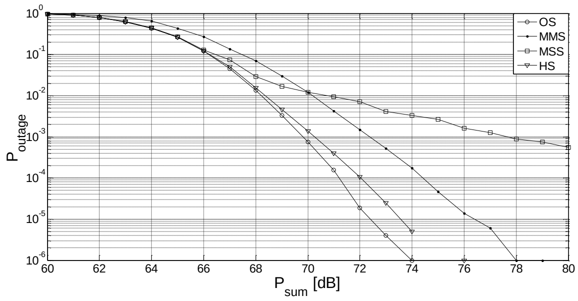

Figure 11. Outage probability versus SNR for a MABC DF TWRN with K=5, $R_a = 7$, $R_a = 4$ BPCH, comparison between OPA-OS, OPA-MMS, OPA-MSS and OPA-HS methods.

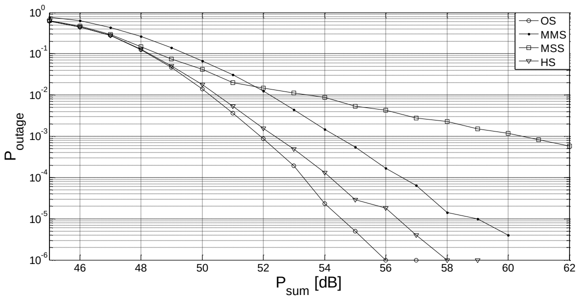

Figure 12. Outage probability versus SNR for a MABC DF TWRN with K=5, $R_a = 4$, $R_a = 4$ BPCH, comparison between OPA-OS, OPA-MMS, OPA-MSS and OPA-HS methods.

Finally, we have plotted all four methods in Fig. 11,12 in order to compare with each other.

V. Conclusion

In this paper, a TWRN with MABC DF protocol was studied. The system comprises two terminals that wish to exchange their messages with the help of one relay among all K ones. We then proposed four joint relay selection and power allocation schemes to improve the outage probability of the system. Three of these schemes, i.e. OPA-OS, OPA-MMS and OPA-HS provide full diversity gain (equal to K) and about 1.5 dB coding gain. The one remaining scheme, i.e. OPA-MSS is also effective in low SNRs although does not result in diversity gain. The OPA-OS is the optimal technique but most complicated in terms of implementation, while the OPA-MMS and OPA-HS are easy to implement in the system.

References

[1] T. M. Cover and A. E. Gamal, "Capacity theorems for the relay channel," *Information Theory, IEEE Transactions on*, vol. 25, pp. 572-584, 1979.

[2] J. N. Laneman, G. W. Wornell, and D. N. Tse, "An efficient protocol for realizing cooperative diversity in wireless networks," in *Information Theory, 2001. Proceedings. 2001 IEEE International Symposium on*, 2001, p. 294.

[3] J. N. Laneman, D. N. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *Information Theory, IEEE Transactions on,* vol. 50, pp. 3062-3080, 2004.

[4] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *Information Theory, IEEE Transactions on,* vol. 51, pp. 3037-3063, 2005.

[5] S. J. Kim, N. Devroye, P. Mitran, and V. Tarokh, "Comparison of bi-directional relaying protocols," in *Sarnoff Symposium, 2008 IEEE*, 2008, pp. 1-5.

[6] W. Shin, N. Lee, J. B. Lim, and C. Shin, "An optimal transmit power allocation for the two-way relay channel using physical-layer network coding," in *Communications Workshops, 2009. ICC Workshops 2009. IEEE International Conference on*, 2009, pp. 1-6.

[7] M. Zhao, Y. Zhou, D. Ren, and Y. Yang, "A minimum power consumption scheme for two-way relay with physical-layer network coding," in *Network Infrastructure and Digital Content, 2010 2nd IEEE International Conference on*, 2010, pp. 704-708.

[8] M. Pischella and D. Le Ruyet, "Optimal power allocation for the two-way relay channel with data rate fairness," *Communications Letters, IEEE,* vol. 15, pp. 959-961, 2011.

[9] T. J. Oechtering and H. Boche, "Bidirectional regenerative half-duplex relaying using relay selection," *Wireless Communications, IEEE Transactions on,* vol. 7, pp. 1879-1888, 2008.

[10] Y. Li, R. H. Louie, and B. Vucetic, "Relay selection with network coding in two-way relay channels," *Vehicular Technology, IEEE Transactions on,* vol. 59, pp. 4489-4499, 2010.

[11] Q. You, Y. Li, and Z. Che, "Joint relay selection and network coding using decode-and-forward protocol in two-way relay channels," in *Global Telecommunications Conference (GLOBECOM 2010), 2010 IEEE*, 2010, pp. 1-6.

[12] A. Hyadi, M. Benjillali, and M.-S. Alouini, "Outage performance of two-way DF relaying systems with a new relay selection metric," in *Wireless Communications and Networking Conference (WCNC), 2012 IEEE*, 2012, pp. 570-574.

[13] Y. Jeon, Y.-T. Kim, C. Song, Y.-O. Park, and I. Lee, "Diversity Analysis of Minimum Distance Based Relay Selection Schemes for Two-way Relaying Systems with Physical Network Coding," in *Vehicular Technology Conference (VTC Fall), 2012 IEEE*, 2012, pp. 1-5.

[14] I. Krikidis, "Relay selection for two-way relay channels with MABC DF: A diversity perspective," *Vehicular Technology, IEEE Transactions on,* vol. 59, pp. 4620-4628, 2010.

[15] T. P. Do, J. S. Wang, I. Song, and Y. H. Kim, "Joint relay selection and power allocation for two-way relaying with physical layer network coding," *Communications Letters, IEEE,* vol. 17, pp. 301-304, 2013.

[16] L. Pang, Y. Zhang, J. Li, Y. Ma, and J. Wang, "Power allocation and relay selection for two-way relaying systems by exploiting physical-layer network coding," *Vehicular Technology, IEEE Transactions on,* vol. 63, pp. 2723-2730, 2014.

[17] A. Hasani and V. T. Vakili, "Power Allocation in Two-Way Relay Networks with MABC DF Protocol and No Instantaneous Channel State Information," *Wireless Personal Communications,* pp. 1-19.

[18] Z. Chen, H. Liu, and W. Wang, "A novel decoding-and-forward scheme with joint modulation for two-way relay channel," *Communications Letters, IEEE,* vol. 14, pp. 1149-1151, 2010.

[19] Z. Chen, H. Liu, and W. Wang, "On the optimization of decode-and-forward schemes for two-way asymmetric relaying," in *Communications (ICC), 2011 IEEE International Conference on*, 2011, pp. 1-5.