Unknown Premise Variables Observer-Based Controller Design for T-S Fuzzy Systems with Input Saturation

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Abstract— **In this paper, we consider the problem of observer-based controller design for nonlinear systems which can be represented by Takagi-Sugeno (T-S) fuzzy systems. Two practical restrictions have been considered to cover a more general problem. First, we suppose that the premise variables of the T-S model are unmeasurable, which permits one to utilize the proposed method in more practical systems. Second, actuator saturation is considered as a physical limitation and the controller is designed subject to this restriction. Sufficient conditions for the existence of such a controller are derived in terms of linear matrix inequalities (LMIs). The effectiveness of developed technique is shown through a numerical example.**

Keywords-T-S fuzzy system, observer-based controller, input saturation, unknown premise variable, PDC controller, linear matrix inequality (LMI).

I. INTRODUCTION

In the last decades, nonlinear control systems based on T-S fuzzy models have attracted lots of attentions [1, 2]. Nowadays, T-S fuzzy model has an important role in control engineering, neural networks, signal processing, artificial intelligence, robotics, data processing. In such models, the nonlinear systems are represented by nonlinear weighted summation of number of linear time-invariant models, and then this model can be used in analysis, stability and the design of controllers.

Usually, controller design in a T-S fuzzy system is based on PDC scheme which is a nonlinear controller constructed by nonlinear weighted summation of number of linear gains. Moreover, common quadratic Lyapunov function has been used to investigate the stability of many T-S fuzzy systems.

A usual limitation in many practical applications is that only the nonlinear T-S fuzzy system outputs are available for controller design and directly measuring all the system states is difficult or expensive. In these systems, output feedback T-S fuzzy controller were considered. Static and dynamic output feedback were studied in [3, 4, 5] and the fuzzy observer-based control feedback has been investigated in [6]. The fuzzy observers are designed to estimate the system states and the estimated states are employed for state-feedback or output feedback control of nonlinear systems.

The premise variables of the fuzzy observer may be considered to depend on the observer state variables (it is denoted as case B in [7]). Although, the designed controller in this case is more applicable practically, its design procedure is much more complex than the one in the case that premise variables are measurable (case A), since the separation principle is not applicable in case B.

In [8] and [9], the design of observer-based controller for a class of continuous-time nonlinear systems presented by T-S model with unmeasurable premise variable were considered. In [10], to overcome the hardness of measuring premise variable of a T-S fuzzy system, a fault detection and prediction scheme were designed for a class of fuzzy systems with unmeasurable premise variables and external disturbances.

Another limitation in the practical applications is the existence of saturation. Saturation is a nonlinear term in many dynamical systems which can exist in different parts of control system such as actuators, sensors and controllers. Because of physical limitation of the devices, actuators saturation is very destructive in practical control systems. Ignoring saturation can lead to performance degradation and even instability of closed-loop systems. Considering the actuator saturation in the controller design for a nonlinear T-S fuzzy system has been investigated in the literature [11, 12, 13]. For instance; in [13], the problem of the fuzzy model-based control of an overhead crane with the input delay and actuator saturation has been investigated. In [14], a method for T–S fuzzy model with input saturation for state feedback controller design and optimizing H_∞ performance bound has been proposed.

In some approaches, observer-based T-S fuzzy design with actuator saturation have been investigated. In [15], fault tolerant saturated control problem for discrete-time T-S fuzzy systems with delay is studied. Sufficient conditions of stabilization based on a fuzzy observer are presented. The observer is then used in fault detection, fault localization and controller reconfiguration to maintain asymptotic stability of the system. In [16], the fault tolerant control scheme was proposed for near space vehicles

(NSVs) with system uncertainty, unknown external disturbance, actuator faults and input saturation based on the sliding mode control. Considering input saturation, a compensated term was constructed in the control law. The stability of the closed-loop system was proved and all closed-loop signals were uniformly ultimately bounded via Lyapunov analysis. To the best of our knowledge, observer-based controller design for unknown premise variables T-S fuzzy systems with input saturation has not been considered yet in the literature.

In this paper, we propose a new method for observer-based controller design for T-S fuzzy system with input saturation and unknown premise variables. To consider the input saturation constraint, we extend the proposed method in [14] to the observer-based control of unknown premise variables T-S fuzzy systems which was given in [17]. The design conditions will be converted to LMIs using Finsler's lemma. It will be shown that the proposed observer-based controller stabilizes the saturated T-S fuzzy system in a pre-defined region of the system states. A simulation example will be given to verify the effectiveness of the proposed method.

The paper is organized as follows: In Section II, T-S fuzzy model and observer-based fuzzy controller are defined, and some preliminaries are presented. In Section III, an observer-based controller is designed for a T-S fuzzy system with input saturation and the stability of the closed loop system is proven. Simulation results are presented in Section IV. Finally, in section V some concluding remarks are given.

Notation: In this paper, $\tilde{A}_{ij}$ equals to $A_i - A_j$, $H(A)$ stands for ($A + A^T$) and the following notations will be used:

$$A_\mu = \sum_{i=1}^{r} h_i(\mu) A_i \quad , \tag{1}$$

$$A_{\mu\hat{\mu}} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\mu) h_j(\hat{\mu}) A_{ij} \ , \tag{2}$$

$$\overline{A}_{\mu\hat{\mu}} = A_\mu - A_{\hat{\mu}} . \tag{3}$$

II. PRELIMINARIES

A. T-S Fuzzy model

The T-S fuzzy model that was suggested by Takagi-Sugeno, represented by if-then rules that describe the input-output relation of system. In general, the T-S fuzzy system is described as follow:

$$\begin{aligned} \dot{x}(t) &= A_\mu x(t) + B_{2\mu} u(t) \\ y(t) &= C_{2\mu} x(t) \end{aligned} \tag{4}$$

where $x(t) \in R^n$, $u(t) \in R^m$, and $y(t) \in R^p$ is the state vector, control input, and output vector, respectively. A_{ij} is the fuzzy set, r is the number of model rules and $\mu_1 \sim \mu_p$ are known premise variables that can be functions of state variables $\mu = [\mu_1 \ \mu_2 \ \ldots \mu_p]$ and:

$$h_i(\mu) = w_i(\mu) \Big/ \sum_{i=1}^{r} w_i(\mu) , \tag{5}$$

with:

$$w_i(\mu) = \prod_{j=1}^{p} M_{ij}(\mu_j) \tag{6}$$

B. Observer model with saturated input

The observer design is a very important problem in control systems. Since in many practical nonlinear control systems, state variables are often unavailable, output feedback or observer-based control is necessary and has absorbed and attracted researchers, [18, 19].

In this paper the fuzzy state observer for T-S fuzzy model with saturated input is formulated as follows:

$$\dot{\hat{x}}(t) = A_{\hat{\mu}} \hat{x}(t) + B_{2\hat{\mu}} \overline{u}(t) + L_{\hat{\mu}} (y(t) - \hat{y}(t)) \tag{7}$$

$$\hat{y}(t) = C_{2\hat{\mu}} \hat{x}(t) \tag{8}$$

where $\hat{\mu}$ is the estimate value of premise variable μ and L_i 's are the observer gains. Using the state estimates, the following control law is used:

$$\overline{u} = sat(u) \tag{9}$$

where:

$$\overline{u} = \frac{1+\varepsilon}{2} u - \frac{1+\varepsilon}{2} u + \overline{u} \tag{10}$$

Define the new auxiliary signal $v(t)$ as follows:

$$v(t) = -\frac{1+\varepsilon}{2} u(t) + \overline{u}(t) \tag{11}$$

Thus:

$$\overline{u} = \frac{1+\varepsilon}{2} u + v(t) \tag{12}$$

In this paper, we use the following structure to construct the control signal:

$$u(t) = -Z_{\hat{\mu}} \hat{x}(t) \tag{13}$$

where Z_i 's are the controller gains to be designed. Notice that the saturation condition is defined by:

$$sat(u) = \begin{cases} u_{\min} & if \quad u < u_{\min} \\ u & if \quad u_{\min} < u < u_{\max} \\ u_{\max} & if \quad u > u_{\max} \end{cases} \tag{14}$$

where $u_{\lim}$ is the control input limit.

C. Closed loop system

The estimation error can be defined as $e = x(t) - \hat{x}(t)$. Then, the error dynamics of observer is obtained as:

$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t) \tag{15}$$

Or equivalently:

$$\dot{e} = A_{\mu} x(t) + B_{2\mu}\bar{u} - \dot{\hat{x}}(t) \tag{16}$$

By using (9) and substituting (7) in (16), we have:

$$\begin{aligned} \dot{e} &= (A_{\mu\hat{\mu}} - \frac{1+\varepsilon}{2} B_{2\mu\hat{\mu}} Z_{\hat{\mu}} - L_{\hat{\mu}} c_{2\mu\hat{\mu}})\hat{x} \\ &+ (A_{\mu} - L_{\hat{\mu}} c_{2\mu})e + B_{2\mu} v \end{aligned} \tag{17}$$

Thus, the closed-loop system becomes:

$$\dot{\theta}(t) = A_{cl}\theta(t) + \begin{bmatrix} B_{2\hat{\mu}} \\ B_{2\mu} \end{bmatrix} v(t) \tag{18}$$

where:

$$\theta(t) = \begin{bmatrix} \hat{x}(t) \\ e(t) \end{bmatrix} \tag{19}$$

and:

$$A_{cl} = \begin{bmatrix} A_{\hat{\mu}} - \frac{1+\varepsilon}{2} B_{2\hat{\mu}} Z_{\hat{\mu}} + L_{\hat{\mu}} C_{2\mu\hat{\mu}} & L_{\hat{\mu}} C_{2\mu} \\ A_{\mu\hat{\mu}} - \frac{1+\varepsilon}{2} B_{2\mu\hat{\mu}} Z_{\hat{\mu}} - L_{\hat{\mu}} C_{2\mu\hat{\mu}} & A_{\hat{\mu}} - L_{\hat{\mu}} C_{2\mu} \end{bmatrix} \tag{20}$$

D. Useful lemmas

The following lemmas have been used in the proof of the main result of this paper:

Lemma 1 [14]: For saturation constraint defined by (14), as long as $|u(t)| \le \frac{u_{\lim}}{2}$, we have:

$$\left\| \bar{u}(t) - \frac{1+\varepsilon}{2} u(t) \right\| \le \frac{1-\varepsilon}{2} \|u(t)\| \tag{21}$$

and hence:

$$\left[\bar{u}(t) - \frac{1+\varepsilon}{2} u(t)\right]^T \left[\bar{u}(t) - \frac{1+\varepsilon}{2} u(t)\right] \le (\frac{1-\varepsilon}{2})^2 u^T(t) u(t) \tag{22}$$

where $0 < \varepsilon < 1$.

Lemma 2 [7]: For any matrices or vector X and Y with appropriate dimensions, we have:

$$X^T Y + Y^T X \le \zeta X^T X + \zeta^{-1} Y^T Y \tag{23}$$

where $\zeta > 0$ is any scalar.

III. Main results

In this section, we offer a novel method for observer-based controller design of T-S fuzzy system with input saturation based on a common quadratic Lyapunov function. The design conditions will be formulated as LMIs. Moreover, we will compute an approximation of the region of attraction based on the proposed approach.

Theorem 1. Given positive scalar design parameters δ_1 and δ_2, the observer-based closed-loop system (18) is asymptotically stable if there exist symmetric positive-definite matrices Q_1, Q_2 and matrices $S_1, S_2^{ijk}, S_3^{ijk}, S_6^{j}, M_k, S_4^{ijk}$ and S_5^{ijk} ($i, j, k = 1, \ldots, r$) such that following LMI condition holds:

$$\Pi_{jk}^{i} < 0 \tag{24}$$

$$\begin{bmatrix} Q_1 & M_k^T \\ M_k & (\frac{u_{\lim}}{\varepsilon})^2 \rho^{-1} I \end{bmatrix} \ge 0, \; k = 1, \ldots, r \tag{25}$$

where for $C_{2i} = C_{2k}$:

$$\Pi_{jk}^{i} = H\left\{\begin{bmatrix} A_j Q_1 - \frac{1+\varepsilon}{2} B_{2j} M_k + \zeta^{-1} B_{2j} B_{2j}^T & \zeta^{-1} B_{2j} B_{2j}^T + D_3^j & \delta_1 D_3^j & M_k^T \\ \tilde{A}_{ij} Q_1 - \frac{1+\varepsilon}{2} \tilde{B}_{2ij} M_k + \zeta^{-1} B_{2i} B_{2j}^T & A_i Q_2 - D_3^j + \zeta^{-1} B_{2i} B_{2i}^T & -\delta_1 D_3^j & 0 \\ 0 & Q_2 - T_i D_1^{ijk} & -\delta_1 T_i D_1^{ijk} & 0 \\ M_k & 0 & 0 & -\zeta^{-1}(\frac{2}{1-\varepsilon})^2 I \end{bmatrix}\right\} \tag{26}$$

and for $C_{2i} \ne C_{2k}$:

$$\Pi_{jk}^{i} = H\left\{\begin{bmatrix} A_j Q_1 - \frac{1+\varepsilon}{2} B_{2j} M_k + \zeta^{-1} B_{2j} B_{2j}^T & \zeta^{-1} B_{2j} B_{2j}^T + D_3^j & \delta_1 D_3^j & \delta_2 D_3^j & M_k^T \\ \tilde{A}_{ij} Q_1 - \frac{1+\varepsilon}{2} \tilde{B}_{2ij} M_k + \zeta^{-1} B_{2i} B_{2j}^T & A_i Q_2 - D_3^j + \zeta^{-1} B_{2i} B_{2i}^T & -\delta_1 D_3^j & -\delta_2 D_3^j & 0 \\ Q_1 & Q_2 - T_i D_1^{ijk} & -\delta_1 T_i D_1^{ijk} & 0 & 0 \\ -(T_k D_2^{ijk} + Q_1) & 0 & 0 & -\delta_2 T_k D_2^{ijk} & 0 \\ M_k & 0 & 0 & 0 & -\zeta^{-1}(\frac{2}{1-\varepsilon})^2 I \end{bmatrix}\right\} \tag{27}$$

with:

$$D_1^{ijk} = \begin{bmatrix} S_1 & 0 \\ S_2^{ijk} & S_3^{ijk} \end{bmatrix}, \tag{28}$$

$$D_2^{ijk} = \begin{bmatrix} S_1 & 0 \\ S_4^{ijk} & S_5^{ijk} \end{bmatrix}, \tag{29}$$

$$D_3^{j} = \begin{bmatrix} S_6^j & 0 \end{bmatrix} \tag{30}$$

The stabilizing control feedback and the observer gains in (7) and (13) are given by:

$$\tilde{Z}_k = M_k Q^{-1} \tag{31}$$

where:

$$\tilde{Z}_k \triangleq \begin{bmatrix} -Z_k & 0 \end{bmatrix} \tag{32}$$

and:

$$L_j = S_6^j S_1^{-1}. \tag{33}$$

On the other hand T_i 's $(i = 1, \ldots, r)$ satisfying

$$C_{2i} T_i = \begin{bmatrix} I & 0 \end{bmatrix} \tag{34}$$

Proof: Consider the common Lyapunov function candidate as follows:

$$V(\theta(t)) = \theta^T(t) P \theta(t) \tag{35}$$

where $P=\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$ with $P_1>0$ and $P_2>0$.

The derivative of the Lyapunov function is as follow:

$$\dot{V}(\theta(t))=\dot{\theta}^T(t)P\theta(t)+\theta^T(t)P\dot{\theta}(t) \tag{36}$$

According to (18) we have :

$$\begin{aligned} &\dot{V}(\theta(t)) = \\ &\theta^T(t)\left(A_{cl}{}^T P+PA_{cl}\right)\theta(t)+v^T(t)\begin{bmatrix} B_{2\hat{\mu}} \\ B_{2\mu} \end{bmatrix}^T P\theta(t) \\ &+\theta^T(t)P\begin{bmatrix} B_{2\hat{\mu}} \\ B_{2\mu} \end{bmatrix}v(t) \end{aligned} \tag{37}$$

Now by using lemmas 1 and 2, it results as:

$$\begin{aligned} \dot{V} \le\ & \theta^T(t)(A_{cl}^T P+PA_{cl}+\zeta^{-1}P^T\begin{bmatrix} B_{2\hat{\mu}} \\ B_{2\mu} \end{bmatrix}\begin{bmatrix} B_{2\hat{\mu}} \\ B_{2\mu} \end{bmatrix}^T P)\theta(t) \\ &+\hat{x}^T(\zeta(\frac{1-\varepsilon}{2})^2 Z_{\hat{\mu}}^T Z_{\hat{\mu}})\hat{x} \end{aligned} \tag{38}$$

By the way, according to (32) and Pre- and post-multiplying (38) by $Q \triangleq P^{-1}$ gives:

$$\begin{aligned} \dot{V} \le\ & \theta^T(t)((QA_{cl}^T+A_{cl}Q+\zeta^{-1}\begin{bmatrix} B_{2\hat{\mu}} \\ B_{2\mu} \end{bmatrix}\begin{bmatrix} B_{2\hat{\mu}} \\ B_{2\mu} \end{bmatrix}^T \\ &+\zeta(\frac{1-\varepsilon}{2})^2 Q\tilde{Z}_{\hat{\mu}}^T\tilde{Z}_{\hat{\mu}}Q))\theta(t) \end{aligned} \tag{39}$$

So for Lyapunov stability the following inequality must be hold:

$$(QA_{cl}^T+A_{cl}Q+\zeta^{-1}\begin{bmatrix} B_{2\hat{\mu}} \\ B_{2\mu} \end{bmatrix}\begin{bmatrix} B_{2\hat{\mu}} \\ B_{2\mu} \end{bmatrix}^T+\zeta(\frac{1-\varepsilon}{2})^2 Q\tilde{Z}_{\hat{\mu}}^T\tilde{Z}_{\hat{\mu}}Q) \le 0 \tag{40}$$

To drive LMI conditions, (40) holds if:

$$\begin{bmatrix} A_{cl}Q+QA_{cl}+\zeta^{-1}BB^{-1} & Q\tilde{Z}_k^T \\ \tilde{Z}_k Q & -\zeta^{-1}(\frac{2}{1-\varepsilon})^2 I \end{bmatrix} < 0 \tag{41}$$

To do that, let matrix Q be partitioned as $Q=diag(Q_1,Q_2)$, then $Q_1>0,\ Q_2>0$ therefor the inequality (41) can be rewritten as:

$$H\left\{\begin{bmatrix} A_{\hat{\mu}}Q_1-\frac{1+\varepsilon}{2}B_{2\hat{\mu}}M+L_{\hat{\mu}}C_{2\mu\hat{\mu}}Q_1+\zeta^{-1}B_{2\hat{\mu}}B_{2\hat{\mu}}^T & L_{\hat{\mu}}C_{2\mu}Q_2+\zeta^{-1}B_{2\hat{\mu}}B_{2\mu}^T & Q\tilde{Z}_k^T \\ A_{\mu\hat{\mu}}Q_1-\frac{1+\varepsilon}{2}B_{2\mu\hat{\mu}}M+L_{\hat{\mu}}C_{2\mu\hat{\mu}}Q_1+\zeta^{-1}B_{2\mu}B_{2\hat{\mu}}^T & A_{\mu}Q_2-L_{\hat{\mu}}C_{2\mu}Q_2+\zeta^{-1}B_{2\mu}B_{2\mu}^T & 0 \\ \tilde{Z}_k Q & 0 & -\zeta^{-1}(\frac{2}{1-\varepsilon})^2 I \end{bmatrix}\right\}<0 \tag{42}$$

For the sake of brevity, using the same procedure given in [17], it is not hard to show that (24) holds if (42) is satisfied.

On the other hand, the constraint $|u(t)| \le \frac{u_{\lim}}{2}$ can be written as:

$$\left|\sum_{k=1}^{r} h_k(\hat{\mu})Z_k\hat{x}(t)\right| \le \frac{u_{\lim}}{2} \tag{43}$$

We know that if $|Z_{\hat{\mu}}\hat{x}(t)| \le \frac{u_{\lim}}{2}$, then (43) holds.

Let $\Omega(k)=\left\{\hat{x}(t) \,\middle|\, |\hat{x}^T(t)Z_{\hat{\mu}}^T Z_{\hat{\mu}}\hat{x}(t)| \le (\frac{u_{\lim}}{2})^2\right\}$ then the equivalent condition for an ellipsoid $\Omega(P,\rho)=\left\{\theta^T(t)P\theta(t)\le\rho\right\}$ being a subset of $\Omega(k)$, i.e. $\Omega(P,\rho)\subset\Omega(k)$ is:

$$\tilde{Z}_k(\frac{P}{\rho})^{-1}\tilde{Z}_k^T \le (\frac{u_{\lim}}{\varepsilon})^2 \tag{44}$$

Now by using the Schur complement, LMI condition for (44) results in:

$$\begin{bmatrix} P & \tilde{Z}_k^T \\ \tilde{Z}_k & (\frac{u_{\lim}}{\varepsilon})^2\rho^{-1}I \end{bmatrix} \ge 0 \tag{45}$$

Pre-and post-multiplying (45) by $\begin{bmatrix} P^{-1} & 0 \\ 0 & I \end{bmatrix}$ gives:

$$\begin{bmatrix} P^{-1} & P^{-1}\tilde{Z}_k^T \\ \tilde{Z}_k P^{-1} & (\frac{u_{\lim}}{\varepsilon})^2\rho^{-1}I \end{bmatrix} \ge 0 \tag{46}$$

By $Q=P^{-1}$, (46) is equivalent to:

$$\begin{bmatrix} Q & Q\tilde{Z}_k^T \\ \tilde{Z}_k Q & (\frac{u_{\lim}}{\varepsilon})^2\rho^{-1}I \end{bmatrix} \ge 0 \tag{47}$$

It is easy to show that if (25) holds then (47) holds too. This completes the proof.□

Remark 1. It should be noted that an estimate of the region of attraction of the closed-loop system in which the observer-based controller proposed in Theorem 1 is utilized, can be calculated by $\Omega(P,\rho)=\left\{\theta^T(t)P\theta(t)\le\rho\right\}$ where $P=Q^{-1}$ is derived by solving the LMIs.

Remark 2. A possible solution T_i associated with the output matrix C_{2i} in (36) can be given by:

$$T_i=\left[C_{2i}^T(C_{2i}C_{2i}^T)^{-1} \quad C_{2i}^{\perp}\right] \tag{48}$$

where $C_{2i}^{\perp}$ is the orthogonal basis for the null space of C_{2i} with $C_{2i}C_{2i}^{\perp}=0$.

Remark 3. For the case that output matrices in (4) satisfy $C_{2i} = C_2$, $(i = 1,\ldots,r)$, in order to relax the proposed LMIs given in Theorem 1, one can solve the following alternative LMIs:

$$\Pi^i_{jj} < 0, \quad j = 1,\ldots,r, \tag{49}$$

$$\frac{2}{r-1}\Pi^i_{jj} + \Pi^i_{jk} + \Pi^i_{kj} < 0, \quad j,k = 1,\ldots,r, \quad j \neq k, \tag{50}$$

where Π^i_{jk} is defined in (27) and D_1 is defined as:

$$D_1 = \begin{bmatrix} S_1 & 0 \\ S_2 & S_3 \end{bmatrix}.$$

IV. SIMULATION EXAMPLE

Consider a 2-rule T-S fuzzy model with the following matrices [20]:

$$A_1 = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2.5 & 0 \\ -2.3 & -1 \end{bmatrix}, \quad B_{21} = B_{22} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$C_{21} = C_{22} = \begin{bmatrix} 10 & 2 \end{bmatrix}$$

In order to design observer-based controller for this T-S fuzzy system, we solve the LMIs of Theorem 1 by setting the design parameters $\delta_1 = 10^{-7}$, $\varepsilon = 0.99$, $\rho = 0.9$, $u\lim = 11.9$, initial conditions $x_0 = \begin{bmatrix} 1 & 2 & -1 & 0 \end{bmatrix}$,and according to (34) we have:

$$T = \begin{bmatrix} 1 & 2 \\ -4.5 & -10 \end{bmatrix}.$$

The grades of membership functions are defined as:

$$h_1(x(t)) = 0.5 + \arctan(x_2(t)) / \pi,$$

$$h_2(x(t)) = 1 - h_1(x(t))$$

where $x(t) = [x_1(t) \quad x_2(t)]^T$. Then, the feasible solutions using SeDuMi [21] are given by:

$$Z_1 = \begin{bmatrix} 13.761 & \text{-8.711e-07} \end{bmatrix}, Z_2 = \begin{bmatrix} 13.108 & \text{2.334e-06} \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 11.483 \\ -6.772 \end{bmatrix}, L_2 = \begin{bmatrix} 13.940 \\ -0.908 \end{bmatrix}$$

The close-loop system states of the system and their observed values using an observer-based controller with the abovementioned matrices are given in Fig. 1 and the errors between states and their obtained observer values are shown in Fig. 2. Moreover, in Fig. 3, the control signal with saturation and without this constraint are shown. u (dash lined) and $\bar{u}$ (solid lined) are represented for control signal without constraint and saturated control signal, respectively.

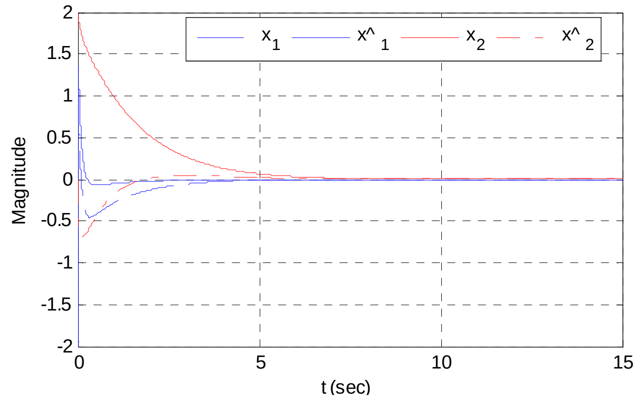

Figure 1. Closed-loop system states and their observed values

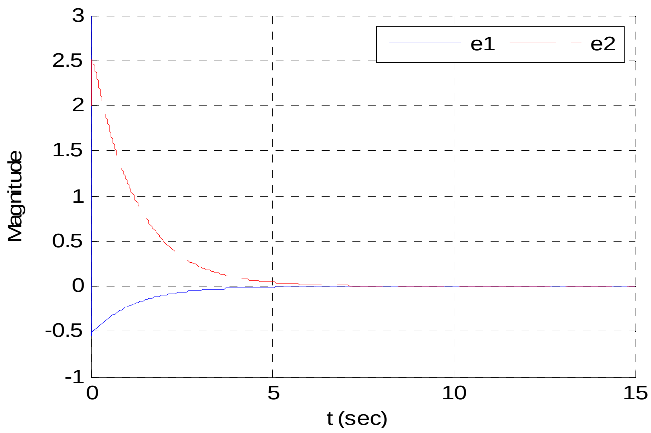

Figure 2. Error between states and their observed values

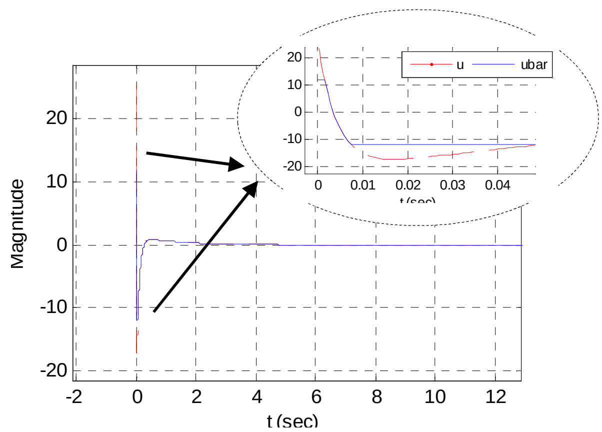

Figure 3. Control signal with saturation and control signal without saturation constraint

According to Figs. 1-3, it is clear that the proposed controller obtained from solving the LMIs of Theorem 1 successfully stabilizes the closed-loop system with the saturation constraint on the control signal. In fact, although the control signal cannot exceed the maximum value 11.9, the controller stabilizes the closed-loop system. It should be noted that if we did not consider the saturation constraint, the control signal a maximum value 30.231 would be obtained which is approximately two times more than the maximum permissible value for the actuator output.

V. CONCLUSIONS

In this paper, we proposed an observer based controller design for a class of nonlinear systems with input saturation. The nonlinear system was represented by a T-S fuzzy model in a pre-defined region. Moreover, it was supposed that premise variables were not measurable to be used in the structure of the observer-based controller. Then, by using the PDC scheme, an observer-based controller with merging input saturation constraint into the design process was proposed for this T-S fuzzy system. It was shown that the proposed observer-based controller stabilizes the saturated T-S fuzzy system in a pre-defined region of the system states. The conditions for the existence of such a controller were converted into some LMIs. Simulation results verified the efficiency of the proposed controller.

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