Beam Focusing Using Two-dimensional Graphene-based Meta-reflect-array

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***Abstract*—A highly efficient two-dimensional meta-reflect-array manipulating the phase and amplitude of far-infrared electromagnetic wave is proposed. The reflect array consists of patterned graphene nano-disks reside on a subwavelenght thick optical cavity to enhance the interaction of electromagnetic wave. It is shown that the reflected wave from each unit cell can almost cover the 2π range phase shift by changing the radius of the graphene nano-disks while at the same time maximizes its amplitude. Proposing a transmission line model for the proposed structure, we realize a meta lens as a flat optics functionality.**

***Index Terms*—Graphene, Metasurfaces, Reflect array, Beam manipulating, Lenses**

I. Introduction

Wavefront shaping of light has been of great interest before the discovery of Maxwell equations, as lenses have been used to bend, magnify and focus the light since the dawn of civilization [1]. Conventional dielectric lenses were realized by phase accumulation along the optical path, so that the emerging wave front was steered in the desired manner. However, their significant thickness compared to the wavelength of operation besides their non-negligible reflection losses put a limit on the integration of such lenses into nano-photonic systems [2].

The desire to overcome the above mentioned challenges of conventional dielectric lenses has stimulated the concept of frequency selective surfaces (FSS) which can eventually determine the direction, amplitude and polarization of electromagnetic waves over a thin and deeply subwavelength surface. FSSs are planar or curved periodic structures consisting of arranged patches or apertures of conducting elements on a dielectric substrate. The elements shape, size, thickness and spacing together with the substrate parameters determine the behaviour of a FSS over a specific bandwidth [3].

In a different approach, the efforts to go beyond the limitations of conventional dielectric lenses have led to large interest in artificial engineered composite materials called metamaterials. Due to their ability to control light-matter interactions and light propagation in unusual ways, metamaterials have been the focus of attention from physicists over the past decade [4]. Despite the advantages of bulk metamaterials, their fabrication remains a challenge for current technology. Therefore, by combining the FSSs concept and metamaterials properties, a new concept of ultrathin metamaterials known as metasurfaces has been emerged [5].

Metasurfaces known as two-dimensional (2D) metamaterials with periodic or aperiodic elements provide us an unprecedented opportunity to control the interaction of electromagnetic waves between two mediums in unusual ways [6]. Recent studies in metasurfaces have shown that the generalized Snell's laws of refraction obtained by imparting a controlled gradient of phase discontinuity over a single surface. Therefore, by tuning the phase profile, both reflected and transmitted waves can be tailored to the desired directions instead of those predicted by the conventional Snell's law [5].

So far, most of presented metasurfaces are metallic-based structures. At optical frequencies, metallic-based structures with different geometries support surface plasmon modes acting as antennas in FSS which can determine the amplitude and the phase of transmitted or reflected waves. However, the plasmonic response of nobel metals becomes weak as approaching THz regime and longer wavelengths. This restricts the use of noble metals in many THz applications [7]. Graphene, a thin layer of carbon atoms has been known as a proper alternative candidate for plasmonics at THz band. In fact, exceptional electrical and mechanical properties of graphene such as high mobility of electrons, low resistivity, high optical transparency, tunable conductivity and extreme mode confinement has made it a substantial substitution for nobel metals in the field of plasmonic metamaterials [8]. Very recently, one dimensional array of graphene strips has been demonstrated as building blocks of metasurfaces instead of metallic-based structures [9]. However, there is not any previous report on two-dimensional (2D) array of graphene nano-disks. Actually, 2D array configuration of graphene strips offers us additional degrees of freedom in the design leading to important advantages for light steering such as 3D focusing, polarization-independent functionalities and strong polarization conversion.

In this paper, we investigate the first 2D reflect-array of graphene nano-disks. It is shown that the phase of the reflected electromagnetic wave can span over almost from π to $-\pi$ by varying the radii of the graphene disks while the amplitude of the reflectivity is sufficiently high. A reflective focusing lens by means of the proposed structure is also demonstrated.

II. Derivation of Equivalant Circuit

The schematic diagram of the proposed structure is illustrated in Fig. 1 (a) consisting of graphene nano-disk arrays as the top layer , a dielectric spacing layer in the middle with the fixed relative permittivity of $\varepsilon_s = 2.25$ and the thickness of t_d which is assumed to be a quarter-operating

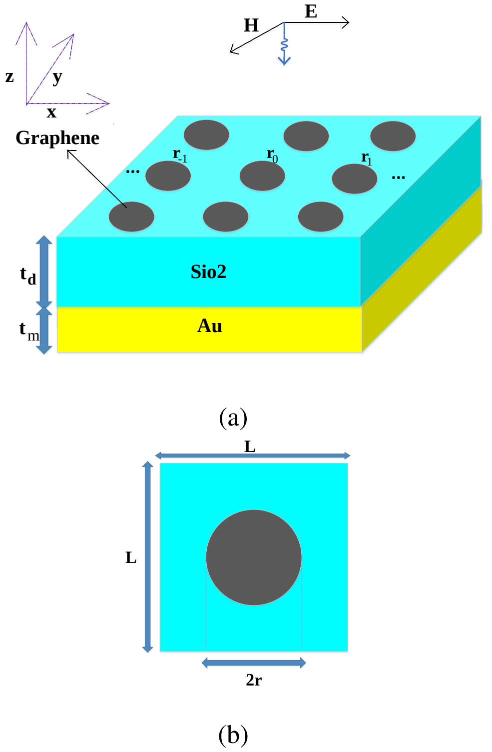


Fig. 1. (a) The proposed meta-reflect-array consisting of a middle layer of SiO_2 spacer sandwiched between a top layer of graphene nano-disks and a bottom layer of Au ground plate. (b) A unit cell of periodic array of graphene nano-disks.

wavelength and an optically thick metallic film placed at the bottom to close the transmission channel. It should be noted that the graphene can be modeled as an extremely thin surface characterized by a two-dimensional surface conductivity. The surface conductivity of graphene can be expressed by using Kubo's formula as follows [10]

$$\sigma_g = i\frac{e^2K_BT}{\pi\hbar^2(\omega+i\tau^{-1})}[\frac{\mu_c}{K_BT}+2ln(exp(-\frac{\mu_c}{K_BT})+1)] \\ +i\frac{e^2}{4\pi\hbar^2}ln[\frac{2|\mu_c|-\hbar(\omega+i\tau^{-1})}{2|\mu_c|+\hbar(\omega+i\tau^{-1})}] \quad (1)$$

where K_B is the Boltzman constant, $\hbar$ is the reduced Planck constant, T is temperature in Kelvin, μ_c is Fermi energy of graphene and τ is the relaxation time. Throughout this article, we fix $T = 300\ K$, $\tau = 1\ ps$ [11]. In the finite element method (FEM) based on a full-wave electromagentic solver (Ansoft's HFSS), we apply an impedance boundary condition that assigns the conductivity to a single sheet instead of a film with finite thickness. Accordingly, we can overcome the meshing difficulty, save memory, and shorten the simulation time.

The equivalent circuit of the proposed structure is shown in Fig. 2. The Au film exhibiting reflecting property is represented by the admittance $Y_{Au} \approx \infty$ .The homogenous mediums, namely the dielectric spacer and the free space are modeled with transmission lines whose wave admittances are

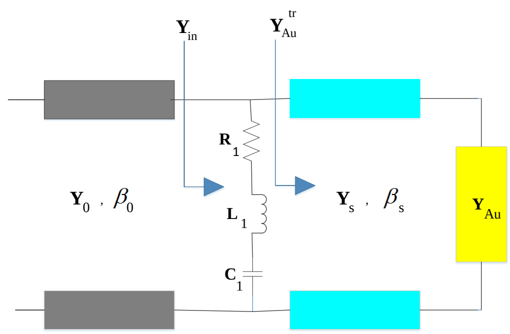


Fig. 2. Transmission line model of the proposed meta-reflect-array under normal incidence.

$Y_s = \sqrt{\mu_0/\varepsilon_0\varepsilon_s}$ and $Y_0 = \sqrt{\mu_0/\varepsilon_0}$, respectively. Furthermore, the metasurfaces are modeled with infinite number of shunt R-L-C admittances as proposed in [12]. To simplify the design procedure, we just consider one series R-L-C branch following an insignificant effect on the accuracy of the model [13],

$$R_1 = \frac{L^2K_1}{\pi^2S_1^2}Re\{\sigma_g^{-1}\}$$
$$L_1 = \frac{L^2K_1}{\pi^2S_1^2}\frac{Im\{\sigma_g^{-1}\}}{\omega}$$
$$C_1 = \frac{\pi^2S_1^2}{L^2K_1}\frac{\varepsilon_{eff}}{q_{11}} \quad (2)$$

where ε_{eff} is the effective permittivity [12] and q_{11}, K_1 and S_1 can be calculated from Table II, Eq. 29 and Eq. 30 of [12], respectively. Once the circuit parameters are calculated, the input admittance of the circuit can be easily obtained as[14]

$$Y_{in} = \frac{1}{R_1 + j\omega L_1 + \frac{1}{j\omega C_1}} + Y_{Au}^{tr} \quad (3)$$

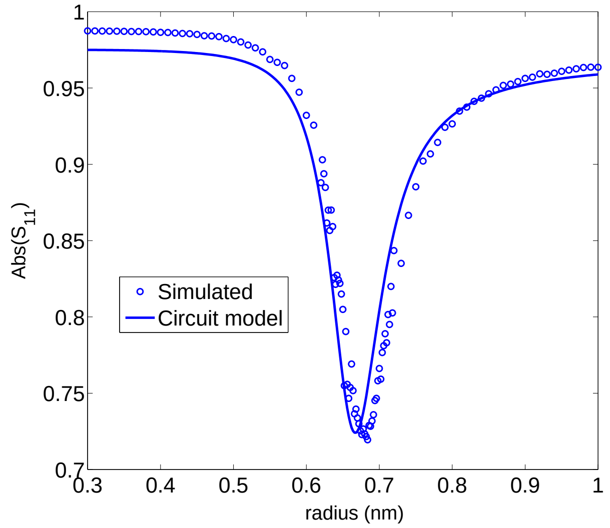
$$Y_{Au}^{tr} = Y_s\frac{Y_{Au}+jY_s tan\beta_s d}{Y_s + jY_{Au}tan\beta_s d}|_{Y(Au)\to\infty} = -jY_s cos\beta_s d \quad (4)$$

The reflection of the structure, S_{11}, can then be readily derived by

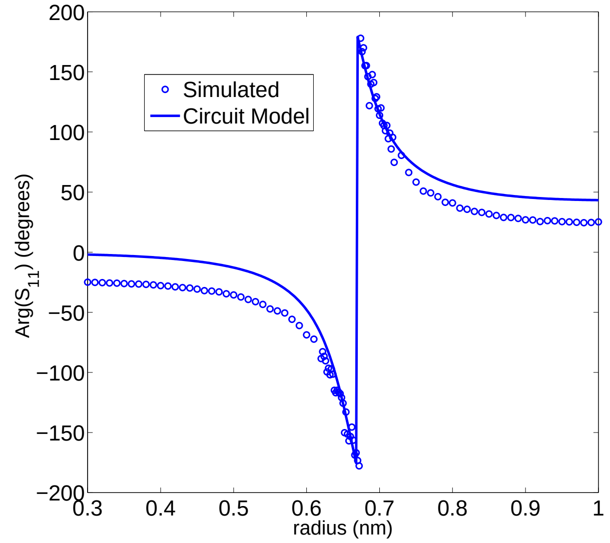
$$S_{11} = \frac{Y_{in}+Y_0}{Y_{in}-Y_0} \quad (5)$$

III. Structure Design and Simulation Results

To design the structure, instead of modeling the entire array, we first investigate a unit cell of the structure. Our goal is to achieve high reflection amplitude (> 0.7), and simultaneously, full control over the phase of the reflected wave. By appropriate tuning of the shunt R-L-C admittance, the aforementioned criteria can be satisfied. The admittance can be easily tuned by changing the radii of the graphene nano-diks. To manipulate the amplitude and the phase of the reflected wave efficiently, the admittance should follow smooth variation in a proper range around the resonance frequency.



(a)



(b)

Fig. 3. Reflection amplitude and phase versus radius of graphene nano-disks using circuit model and full-wave simulations (a) amplitude (b) phase.

Setting the operating frequency to be 11.1 THz, the Fermi energy of the graphene strips and the period of the array should be $\mu_c = 0.8\ eV$ and $L = 3\ \mu m$, respectively to achieve the aforementioned resonance behaviour.

The amplitude and phase of S_{11} versus the radius of the graphene nono-disk are depicted in Fig. 3 (a) and Fig. 3 (b), respectively. The results are obtained by the circuit model demonstrated in the previous section and full-wave simulations. The full-wave simulations are carried out by the Ansoft's HFSS.

As it is observed in Fig. 3, the reflectivity of the proposed structure is larger than 71.9% and the phase smoothly spans over from almost $-\pi$ to π with different graphene nano-disk radii. The finite loss of graphene restricts the phase shift range. In the lossless case, the phase can completely spans over the 2π range. These results affirm that we can achieve high reflectivity larger than 71.9% and almost a continuous 2π phase modulation by changing the radii of the graphene nano-diks. This property enables design of new planar optical devices to steer reflected electromagnetic wave with high efficiency.

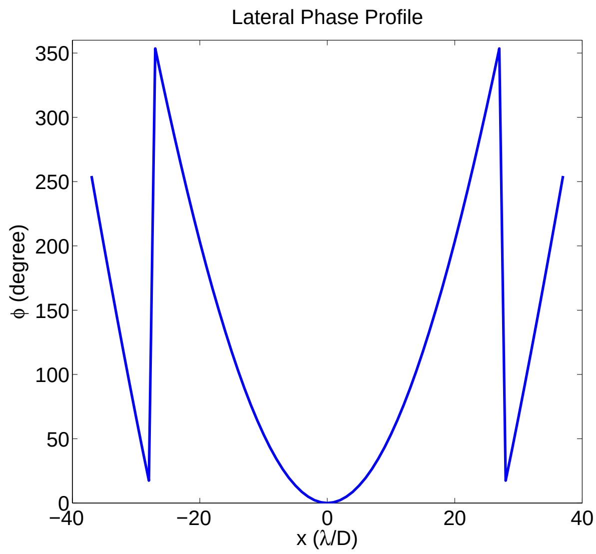


Fig. 4. Quadratic phase function to concentrate the incident wave at $F = 60\ \mu m$.

According to geometrical optics, to realize different optics functionalities such as beam focusing, anomalous bending and splitting, the reflected phase profile should be modulated properly. In this work, we aim to demonstrate a reflective focusing lens by means of the proposed configuration. To design such a lens, we should make sure that the reflected electromagnetic wave from each graphene nano-diks has either the same phase or with 2π multiplied by an arbitrary integer when propagating to the focal point. It means that the metasurface should carry a phase profile such that the reflected electromagnetic waves from the metasurface interfere constructively with each other at the focal point leading to compensation of the propagation phase between graphene nano-disks and the focal point. For a specific focal length F, the phase shift distribution $\phi(x)$ along the transverse direction should satisfy [15]

$$\phi(x) = \frac{2\pi}{\lambda}(\sqrt{F^2 + x^2} - F) \qquad (6)$$

where F is the focal length and λ is the wavelength.

In our study, to demonstrate the performance of the proposed reflect-array, 37 graphene-nanodisks have been simulated with a focal length of $F = 60\ \mu m$ and $x = mL$ with L equals to the periodicity of the graphene nano-disks and $m = 0, \pm1, \pm2, ...$ The phase distribution is calculated based on Eq. 6 and is depicted in Fig. 4. From the results presented in

Table I
REQUIRED RADII OF GRAPHENE NANO-DISKS TO FOCUS REFLECTED LIGHT

n	0	±1	±2	±3	±4	±5	±6	±7	±8	±9	±10	±11	±12	±13	±14	±15	±16	±17	±18
r_n (μm)	0.98	0.96	0.91	0.86	0.84	0.8	0.78	0.76	0.75	0.74	0.73	0.72	0.71	0.7	0.69	0.69	0.68	0.67	0.66

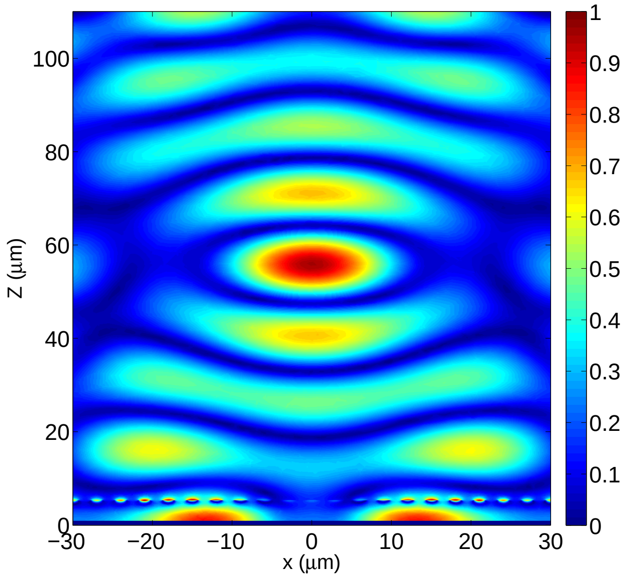


Fig. 5. The reflected magnetic field distribution (H_z) of the proposed structure at the frequency of $f = 11.1\ THz$.

Fig. 3 (b), we can determine the radius of each graphene nano-disks along the x axis to match the phase profile calculated by Eq. 6. The calculated radii of graphene nano-disks are summarized in Table I.

The intensity of magnetic field distribution for the designed lens is depicted in Fig. 5. An incident field at the frequency of $f = 11.1\ THz$ is launched from the top of the structure, and perpendicularly impinges upon the graphene nano-disks. From this figure, we can obtain that the reflected electromagnetic wave is efficiently focused. The focal point is at 60 μm , which agrees very well with the original design.

IV. CONCLUSION

In this paper, we introduced a two dimensional meta-reflect-array based on simple nano-disk geometries to efficiently focus the THz electromagnetic reflected wave. The structure consists of two dimensional patterned graphene nano-disks coupled with a subwavelength-thick optical cavity which substantially increases the interaction of electromagnetic wave with graphene metasurface. Using the equivalent circuit model, we showed that by proper tuning the radii of graphene nano-disks, such a configuration enables us to achieve almost 2π phases modulation of the reflected wave while at the same time maximizes its amplitude. By employing full-wave simulation, a reflective lens by means of the proposed structure was also successfully demonstrated. One can also do further research to implement other optics functionalities such as anomalous reflection by a well-designed configuration.

REFERENCES

[1] D. Brewster, "On an account of a rock-crystal lens and decomposed glass found in Niniveh", Die Fortschritte der Physik, Vol. 355, 1852.
[2] N. Yu, P. Genevet, F. Aieta, M. A. Kats, R. Blanchard, G. Aoust, J.-P. Tetienne, Z. Gaburro, and F. Capasso, "Flat optics: controlling wavefronts with optical antenna metasurfaces", IEEE Journal of Selected Topics in Quantum Electronics, Vol. 19, pp. 4700423-4700423, 2013.
[3] B. A. Munk, *Frequency Selective Surfaces: Theory and Design*, Wiley, New York, 2000.
[4] N. Engheta and R. W. Ziolkowski, *Metamaterials: physics and engineering explorations*, John Wiley & Sons, 2006.
[5] H. Mosallaei and K. Sarabandi, "A one-layer ultra-thin meta-surface absorber", IEEE Antennas and Propagation Society International Symposium, vol. 1, pp. 615-618, 2005.
[6] A. V. Kildishev, A. Boltasseva, and V. M. Shalaev, "Planar photonics with metasurfaces", Science, Vol. 339, p. 1232009, 2013.
[7] Maier, S. A, *Plasmonics: fundamentals and applications.*, Springer Science. 2007.
[8] A. N. Grigorenko, M. Polini and K. S. Novoselov, "Graphene Plasmonics", Nature Photonics, Vol. 6, 2012.
[9] Z. Li, K. Yao, F. Xia, S. Shen, J. Tian and Y. Liu, "Graphene Plasmonic Metasurfaces to Steer Infrared Light", Scintific Reports, Vol. 5, p. 12423, 2015.
[10] G. W. Hanson, "Dyadic greens functions and guided surface waves for a surface conductivity model of graphene", Journal of Applied Physics, vol. 103, 064302, 2008.
[11] A. Vakil and N. Engheta, "Transformation optics using graphene," Science, vol. 332, pp. 1291-1294, 2011.
[12] S. Barzegar-Parizi, B. Rejaei and Amin Khavasi, "Analytical Circuit Model for Periodic Arrays of Graphene Disks", IEEE Journal of Quantum Electronics, vol. 51, No. 9, 2015.
[13] Khavasi, Amin. "Design of ultra-broadband graphene absorber using circuit theory." JOSA B, vol. 32, no. 9, pp. 1941-1946, 2015.
[14] D. M. Pozar, *Microwave engineering,* John Wiley & Sons, 2009.
[15] J. Cheng and H. Mosallaei, "Optical metasurfaces for beam scanning in space," Optics letters, vol. 39, pp. 2719-2722 2014.