Filter-and-Forward Strategy for Multiple Access Relay Channel with Multi-antenna Relay

Fatemeh Bahadori, Bahareh Akhbari and Marjan Maleki
Faculty of Electrical Engineering
K.N.Toosi University of Technology
Email: f.bahadori@ee.kntu.ac.ir, akhbari@eetd.kntu.ac.ir, m.maleki@ee.kntu.ac.ir;

***Abstract*—In this paper, the filter-and-forward (FF) relay design for a multiple access relay channel (MARC) with a multi-antenna relay is considered. The relay uses the FF strategy, in which it is equipped with finite impulse response (FIR) filter to suppress the effect of inter-symbol interference (ISI) caused by the frequency selectivity of the channel. We consider two design criteria for optimizing the relay filter. The first criterion is the minimization of the multi-antenna relay transmit power subject to signal-to-interference-plus-noise ratio (SINR) constraints, and we solve this non-convex problem based on a convex relaxation. For the second criterion we formulate the problem of joint power allocation and relay filter design for the maximization of the worst received SINR subject to constraints on total transmit power of sources and the relay transmit power, and we solve it with an alternating optimization algorithm. Simulation results show that in the frequency selective fading case, the proposed FF approach can significantly improve the performance as compared to the commonly used amplify-and-forward (AF) relaying strategy. Also, providing simulation results, we investigate the effect of the number of relay antennas and relay filter length on the performance.**

***Index Terms*—filter and forward (FF); multiple access relay channel (MARC); multi-antenna relay; frequency selective channel; amplify and forward (AF).**

I. Introduction

Cooperative communication is an efficient technique to achieve cooperative diversity without equipping each user with multiple antennas [1]. The most important relaying strategies that have been proposed are amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) [2]- [3]. The AF strategy, in which the relay nodes send the phase-shifted and scaled version of the received signals to the destination, is one of the most popular relaying strategies [4]. However, the AF strategy is not efficient in reduction of significant amount of inter-symbol-interference (ISI) in frequency selective channels [4], which are more consistent with real communication models, especially in broadband communication systems.

To compensate the frequency selectivity of these channels, a new filter-and-forward (FF) approach has been proposed in [5]. According to the FF strategy, the received signal at the relay node is passed through a FIR filter and then transmitted to the destination. Most of the existing works on FF relaying strategy consider the equipped networks with multiple single-antenna relays [4]- [6]. In this paper, we consider a multiple access relay channel (MARC) with a multi-antenna relay. In the MARC, multiple sources (two sources in this paper) communicate with a single destination in the presence of a relay node [7]. The MARC with AF, DF, CF and compute and forward relay strategies are investigated in [8]- [11]. The problem that we deal with in this paper is the MARC with frequency selective fading channels. By using a new FF relaying approach we show that the performance can significantly be improved as compared to using the traditional AF relaying strategy.

As mentioned before, in our model we consider one multi-antenna relay rather than several single antenna relays, which is beneficial because of diversity gain of MIMO systems [12]. Furthermore, it is helpful in the MARC which we can gain the diversity with only one transmission from the relay node [13]- [14]. For the FF relay, we consider two design criteria for optimizing the relay filter: The first criterion is minimizing the multi-antenna relay transmit power and we provide a convenient algorithm to solve it. The second criterion is the maximization of the minimum received signal-to-interference-plus-noise ratio (SINR) and we solve it with an iterative alternating algorithm. This algorithm splits the main problem in two sub-problems, i.e., optimizing the relay filter to maximize the worst SINR for a given power allocation, and optimizing the power allocation at the sources to maximize the worst SINR for a given relay filter. The first sub-problem reduces to a semi-definite programming (SDP) problem, and the second one reduces to a linear programming (LP) problem. Providing simulation results we investigate the effect of the number of relay antennas and relay filter length on the performance of both optimization problems.

The remainder of this paper is organized as follows: In Section II we present the model of the MARC system with a multi-antenna relay. The optimization problem is provided in Section III. Section IV presents the simulation results and conclusions are drawn in Section V.

The following notations are used in this paper: The small and capital bold letters are used to denote vectors and matrices, respectively. All vectors are column vectors. The symbol $E(\cdot)$ denotes the expectation and $\otimes$ represents the Kronecker operator and $vec(\cdot)$ signifies the matrix vectorization operator. Also, $(\boldsymbol{A})^T$,$(\boldsymbol{A})^*$, $(\boldsymbol{A})^H$, $Tr(\boldsymbol{A})$ and $rank(\boldsymbol{A})$ denote transpose, conjugate, conjugate transpose, trace and the rank of the matrix $\boldsymbol{A}$ respectively. $\boldsymbol{I}_N$ and $\boldsymbol{0}_{M\times N}$ stand for the $N\times N$ identity matrix and $M\times N$ zero matrix, respectively.

II. System Model

Let us consider a full duplex multi access relay channel, composed of two source nodes denoted as A and B which

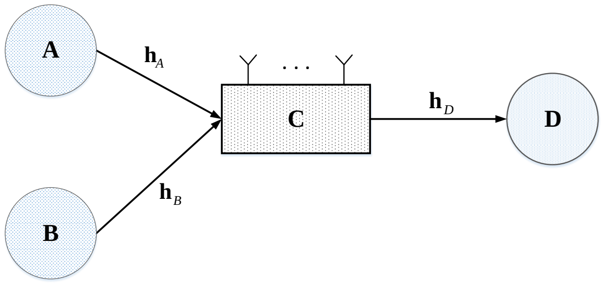

Fig. 1. The multiple access relay channel with a multi-antenna relay.

they communicate with destination node D with the help of the relay C, as shown in Fig. 1. We assume that the receiver perfectly knows the CSI. All nodes are single-antenna except the relay C that is equipped with R antennas. With the assumption of frequency selective channels between all nodes, the received signal at the relay at time sample n can be written as

$$\boldsymbol{r}_c(n) = \sum_{l=0}^{N-1} \boldsymbol{h}_{A,l} s_A(n-l) + \sum_{l=0}^{N-1} \boldsymbol{h}_{B,l} s_B(n-l) + \boldsymbol{n}(n) \tag{1}$$

where $\boldsymbol{n}(n) \triangleq [n(1),\ n(2),\ \dots,\ n(R)]^T$ is the $R \times 1$ vector of zero-mean complex white Gaussian noise with covariance matrix $\sigma^2 \boldsymbol{I}_R$ at relay station, $s_i(n)$ for $i \in \{A, B\}$ denotes the transmitted signal by source i and $\boldsymbol{h}_{i,l}, l = 0, \dots, N-1$ is the $R \times 1$ vector of l'th effective tap of channel impulse response between the source i and relay C. Also N is the channel length between the sources and the relay which is assumed to be identical for two channels. With the following definitions

$$\begin{aligned}
\boldsymbol{H}_A &\triangleq [\boldsymbol{h}_{A,0},\ \dots,\ \boldsymbol{h}_{A,N-1}]_{R\times N} \\
\boldsymbol{H}_B &\triangleq [\boldsymbol{h}_{B,0},\ \dots,\ \boldsymbol{h}_{B,N-1}]_{R\times N} \\
\boldsymbol{s}_A(n) &\triangleq [s_A(n),\ \dots s_A(n-N+1)]^T{}_{N\times 1} \\
\boldsymbol{s}_B(n) &\triangleq [s_B(n),\ \dots,\ s_B(n-N+1)]^T{}_{N\times 1}
\end{aligned}$$

The received signal vector at the relay can be rewritten as:

$$\boldsymbol{r}_c(n) = \boldsymbol{H}_A \boldsymbol{s}_A(n) + \boldsymbol{H}_B \boldsymbol{s}_B(n) + \boldsymbol{n}(n) \tag{2}$$

In order to compensate the effect of frequency selectivity of communication channels, the FF relay approach is used. Because of using the multi-antenna relay, the corresponding taps of filter between the reception and transmission antennas form $R \times R$ beamforming matrices. So, the transmitted signal by the relay is:

$$\begin{aligned}
\boldsymbol{t}_C(n) &= \sum_{l=0}^{N_w-1} \boldsymbol{W}_l \boldsymbol{r}_C(n-l) \\
&= \sum_{l=0}^{N_w-1} \Big(\boldsymbol{W}_l \boldsymbol{H}_A \boldsymbol{s}_A(n-l) + \boldsymbol{H}_B \boldsymbol{s}_B(n-l) \\
&\quad + \boldsymbol{n}(n-l) \Big)
\end{aligned} \tag{3}$$

where $\boldsymbol{W}_l$ is a $R \times R$ matrix, which the element in its m'th row and n'th column corresponds to the l'th tap of FIR filter between m'th receiving antenna and n'th transmitting antenna of the relay and N_w is the length of the FIR filter at the relay. For $i \in \{A, B\}$ and $l = 0, \dots, N_w - 1$ we define:

$$\begin{aligned}
\tilde{\boldsymbol{s}}_i(n) &\triangleq \Big[s_i(n),\ \dots,\ s_i\Big(n - (N + N_w - 2)\Big)\Big]^T{}_{(N+N_w-1)\times 1} \\
\boldsymbol{H}_{i,l} &\triangleq \Big[\underbrace{\mathbf{0}_{R\times 1} \dots \mathbf{0}_{R\times 1}}_{R\times l}\ \underbrace{\boldsymbol{H}_i}_{R\times N}\ \underbrace{\mathbf{0}_{R\times 1} \dots \mathbf{0}_{R\times 1}}_{R\times (N_w-l-1)} \Big]_{R\times(N+N_w-1)} \\
\boldsymbol{\Xi}_i &\triangleq \Big[\boldsymbol{H}_{i,0}^T, \dots, \boldsymbol{H}_{i,(N_w-1)}^T \Big]^T{}_{RN_w\times(N+N_w-1)} \\
\tilde{\boldsymbol{n}}(n) &\triangleq \Big[\boldsymbol{n}^T(n),\ \dots,\ \boldsymbol{n}^T\Big(n - (N_w - 1)\Big)\Big]^T{}_{RN_w\times 1} \\
\tilde{\boldsymbol{W}} &\triangleq \Big[\boldsymbol{W}_0{}^T,\ \dots,\ \boldsymbol{W}_{N_w-1}{}^T \Big]^T{}_{RN_w\times R}
\end{aligned}$$

With these definitions, the transmitted signal by the relay can be rewritten as:

$$\begin{aligned}
\boldsymbol{t}_C(n) &= \tilde{\boldsymbol{W}}^T \boldsymbol{\Xi}_A \tilde{\boldsymbol{s}}_A(n) + \tilde{\boldsymbol{W}}^T \boldsymbol{\Xi}_B \tilde{\boldsymbol{s}}_B(n) \\
&\quad + \tilde{\boldsymbol{W}}^T \tilde{\boldsymbol{n}}(n)
\end{aligned} \tag{4}$$

The received signal at the destination is given by

$$\begin{aligned}
y_D(n) &= \sum_{l=0}^{N_s-1} \boldsymbol{h}_{D,l}^T \boldsymbol{t}_C(n-l) + n_D(n) \\
&= \sum_{l=0}^{N_s-1} \boldsymbol{h}_{D,l}{}^T \Big(\tilde{\boldsymbol{W}}^T \boldsymbol{\Xi}_A \tilde{\boldsymbol{s}}_A(n-l) + \tilde{\boldsymbol{W}}^T \boldsymbol{\Xi}_B \tilde{\boldsymbol{s}}_B(n-l) \\
&\quad + \tilde{\boldsymbol{W}}^T \tilde{\boldsymbol{n}}(n-l) \Big) + n_D(n)
\end{aligned} \tag{5}$$

where $\boldsymbol{h}_{D,l}$ is $R \times 1$ vector of the l'th tap of the impulse response of the channel between the relay and destination. N_s is the channel length between the relay and destination and $n_D(n)$ is an additive Gaussian noise with variance $\sigma_d{}^2$ at the receiver. By defining

$$\begin{aligned}
\boldsymbol{D}_l &\triangleq \boldsymbol{I}_{RN_w} \otimes \boldsymbol{h}_{D,l} \quad , \quad \boldsymbol{v} \triangleq vec\Big(\tilde{\boldsymbol{W}}^T\Big) \\
\breve{\boldsymbol{s}}_i(n) &\triangleq \Big[s_i(n),\ \dots,\ s_i\Big(n - (N + N_s + N_w - 3)\Big)\Big]^T{}_{(N+N_s+N_w-2)\times 1} \\
\breve{\boldsymbol{\Xi}}_{i,l} &\triangleq \Big[\underbrace{\mathbf{0}_{RN_w\times 1} \dots \mathbf{0}_{RN_w\times 1}}_{RN_w\times l}\ \underbrace{\boldsymbol{\Xi}_i}_{RN_w\times(N+N_w-1)}\ \underbrace{\mathbf{0}_{RN_w\times 1} \dots \mathbf{0}_{RN_w\times 1}}_{RN_w\times(N_s-l-1)} \Big] \\
\hat{I}_l &\triangleq \Big[\underbrace{\mathbf{0}_{RN_w\times R} \dots \mathbf{0}_{RN_w\times R}}_{RN_w\times R}\ \underbrace{\boldsymbol{I}_{RN_w}}_{RN_w\times RN_w}\ \underbrace{\mathbf{0}_{RN_w\times R} \dots \mathbf{0}_{RN_w\times R}}_{RN_w\times R(N_s-l-1)} \Big] \\
\breve{\boldsymbol{H}}_i &\triangleq \Big[\boldsymbol{\Xi}_{i,0}^T \dots \boldsymbol{\Xi}_{i,N_s-1}^T \Big]^T{}_{RN_sN_w\times(N+N_s+N_w-2)} \\
\tilde{\boldsymbol{I}} &\triangleq \Big[\hat{I}_0^T,\ \dots,\ \hat{I}_{N-1}^T \Big]^T{}_{RN_sN_w\times R(N_s+N_w-1)} \\
\boldsymbol{D} &\triangleq \Big[\boldsymbol{D}_0,\ \dots,\ \boldsymbol{D}_{N_s-1} \Big]_{R^2N_w\times RN_sN_w} \\
\breve{\boldsymbol{n}}(n) &\triangleq \Big[\boldsymbol{n}^T(n),\ \dots,\ \boldsymbol{n}^T\Big(n - (N_s + N_w - 2)\Big)\Big]^T{}_{R(N_s+N_w-1)\times 1}
\end{aligned}$$

we can rewrite (5) as

$$\begin{aligned}
y_D(n) &= \boldsymbol{v}^T \boldsymbol{D} \breve{\boldsymbol{H}}_A \breve{\boldsymbol{s}}_A(n) + \boldsymbol{v}^T \boldsymbol{D} \breve{\boldsymbol{H}}_B \breve{\boldsymbol{s}}_B(n) \\
&\quad + \boldsymbol{v}^T \boldsymbol{D} \tilde{\boldsymbol{I}} \breve{\boldsymbol{n}}(n) + n_D(n).
\end{aligned} \tag{6}$$

From the definition of $\breve{\boldsymbol{s}}_i(n)$, it can be seen that only the first element of $\breve{\boldsymbol{s}}_i$, i.e., $s_i(n)$ is the desired signal at time sample and the remaining elements which are defined as $\bar{\boldsymbol{s}}_i(n) \triangleq [s_i(n-1) \ldots s_i(n-(N+N_s+N_w-3))]^T$, can be regarded as ISI. Hence, the received signal at the destination can be decomposed into a signal part and ISI part as

$$\begin{aligned}\tilde{y}_D(n) = & \underbrace{\boldsymbol{v}^T\boldsymbol{D}\tilde{\boldsymbol{h}}_A s_A(n)}_{Desired\ signal\ of\ A} + \underbrace{\boldsymbol{v}^T\boldsymbol{D}\bar{\boldsymbol{H}}_A\bar{\boldsymbol{s}}_A(n)}_{ISI\ caused\ from\ A} \\ & + \underbrace{\boldsymbol{v}^T\boldsymbol{D}\tilde{\boldsymbol{h}}_B s_B(n)}_{Desired\ signal\ of\ B} + \underbrace{\boldsymbol{v}^T\boldsymbol{D}\bar{\boldsymbol{H}}_B\bar{\boldsymbol{s}}_B(n)}_{ISI\ caused\ from\ B} \\ & + \underbrace{\boldsymbol{v}^T\boldsymbol{D}\tilde{\boldsymbol{I}}\breve{\boldsymbol{n}}(n)+n_D(n)}_{Noise} \end{aligned} \quad (7)$$

where $\tilde{\boldsymbol{h}}_i$ for $i \in \{A,B\}$ is the first column of $\breve{\boldsymbol{H}}_i$ and the rest of the columns are shown with $\bar{\boldsymbol{H}}_i$. In the following, we consider two design criteria for optimizing the relay filter.

III. FF RELAY TRANSMIT POWER MINIMIZATION

We first consider the problem of designing the FF relay to minimize the relay transmit power subject to constraints on the QoS at the destination. This problem is formulated as follows

$$\begin{cases} \min\limits_{\boldsymbol{v}} & P_r \\ s.t. & SINR_i \geqslant \gamma, \qquad i=A,B \end{cases} \quad (8)$$

where for $i \in \{A,B\}$,$SINR_i$ is the SINR related to source i, and γ is the SINR threshold.

Based on (3), we can obtain the relay transmit power as

$$\begin{aligned} P_r &= Tr\left(E(\boldsymbol{t}_C\boldsymbol{t}_C^H)\right) \\ &= Tr\biggl(E\Bigl(\sum_{l=0}^{N_w-1}\boldsymbol{W}_l\boldsymbol{r}_C(n-l)\boldsymbol{r}_C^H(n-l)\boldsymbol{W}_l^H\Bigr)\biggr) \\ &= P_A Tr\left(\sum_{l=0}^{N_w-1}\boldsymbol{W}_l\boldsymbol{H}_A\boldsymbol{H}_A^H\boldsymbol{W}_l^H\right) + P_B Tr\biggl(\sum_{l=0}^{N_w-1}\boldsymbol{W}_l \\ &\quad \boldsymbol{H}_B\boldsymbol{H}_B^H\boldsymbol{W}_l^H\biggr) + \sigma^2 Tr\left(\sum_{l=0}^{N_w-1}\boldsymbol{W}_l\boldsymbol{W}_l^H\right) \\ &= P_A\sum_{l=0}^{N_w-1}\boldsymbol{v}_l^T\Bigl(\boldsymbol{I}_R\otimes\boldsymbol{H}_A\boldsymbol{H}_A^H\Bigr)\boldsymbol{v}_l^* + P_B\sum_{l=0}^{N_w-1}\boldsymbol{v}_l^T\Bigl(\boldsymbol{I}_R \\ &\quad \otimes\boldsymbol{H}_B\boldsymbol{H}_B^H\Bigr)\boldsymbol{v}_l^* + \sigma^2\sum_{l=0}^{N_w-1}\boldsymbol{v}_l^T\boldsymbol{v}_l^*{}_{R^2N_w\times 1} \end{aligned} \quad (9)$$

By defining $\boldsymbol{v}_l \triangleq vec(\boldsymbol{W}_l)$, $\boldsymbol{X}_i \triangleq \boldsymbol{I}_R\otimes\boldsymbol{H}_i\boldsymbol{H}_i^H$ for $i \in \{A,B\}$ and $\boldsymbol{v} \triangleq vec(\tilde{\boldsymbol{W}}^T) = \Bigl[\boldsymbol{v}_0^T \ldots \boldsymbol{v}_{N_w-1}^T\Bigr]^T_{R^2N_w\times 1}$ we can simplify (9) as

$$P_r = \boldsymbol{v}^T\Bigl(\boldsymbol{I}_{N_w}\otimes(P_A\boldsymbol{X}_A+P_B\boldsymbol{X}_B)+\sigma^2\boldsymbol{I}_{R^2N_w}\Bigr)\boldsymbol{v}^* \quad (10)$$

Using different components of the received signals in (7), P_S^i, P_I^i for $i \in \{A,B\}$ and P_N which are the power of the desired signal, the power of ISI caused by source i and the power of noise respectively, can be expressed as follows:

$$\begin{aligned} P_S^i &= Tr\left(E\left(\boldsymbol{v}^T\boldsymbol{D}\tilde{\boldsymbol{h}}_i\boldsymbol{s}_i(n)\boldsymbol{s}_i^H(n)\tilde{\boldsymbol{h}}_i^H\boldsymbol{D}^H\boldsymbol{v}^*\right)\right) \\ &= P_i Tr(\boldsymbol{v}^T\boldsymbol{D}\tilde{\boldsymbol{h}}_i\tilde{\boldsymbol{h}}_i^H\boldsymbol{D}^H\boldsymbol{v}^*) \\ P_I^i &= Tr\left(E\left(\left(\boldsymbol{v}^T\boldsymbol{D}\bar{\boldsymbol{H}}_i\bar{\boldsymbol{s}}_i(n)\right)\left(\boldsymbol{v}^T\boldsymbol{D}\bar{\boldsymbol{H}}_i\bar{\boldsymbol{s}}_i(n)\right)^H\right)\right) \\ &= P_i Tr\left(\boldsymbol{v}^T\boldsymbol{D}\bar{\boldsymbol{H}}_i\bar{\boldsymbol{H}}_i^H\boldsymbol{D}^H\boldsymbol{v}^*\right) \\ P_N &= \sigma^2 Tr(\boldsymbol{v}^T\boldsymbol{D}\tilde{\boldsymbol{I}}\tilde{\boldsymbol{I}}^H\boldsymbol{D}^H\boldsymbol{v}^*)+\sigma_d^2 \end{aligned} \quad (11)$$

Thus we can write the received SINR with respect to the sources A and B as

$$\begin{aligned} SINR_A &= \frac{P_A Tr(\boldsymbol{v}^T\boldsymbol{D}\tilde{\boldsymbol{h}}_A\tilde{\boldsymbol{h}}_A^H\boldsymbol{D}^H\boldsymbol{v}^*)}{Tr\left(\boldsymbol{v}^T\boldsymbol{Q}_A\boldsymbol{v}^*\right)+\sigma_d^2} \\ SINR_B &= \frac{P_B Tr(\boldsymbol{v}^T\boldsymbol{D}\tilde{\boldsymbol{h}}_B\tilde{\boldsymbol{h}}_B^H\boldsymbol{D}^H\boldsymbol{v}^*)}{Tr\left(\boldsymbol{v}^T\boldsymbol{Q}_B\boldsymbol{v}^*\right)+\sigma_d^2} \end{aligned} \quad (12)$$

where

$$\begin{aligned} \boldsymbol{Q}_A &\triangleq P_A\boldsymbol{D}\bar{\boldsymbol{H}}_A\bar{\boldsymbol{H}}_A^H\boldsymbol{D}^H+\sigma^2\boldsymbol{D}\tilde{\boldsymbol{I}}\tilde{\boldsymbol{I}}^H\boldsymbol{D}^H \\ \boldsymbol{Q}_B &\triangleq P_B\boldsymbol{D}\bar{\boldsymbol{H}}_B\bar{\boldsymbol{H}}_B^H\boldsymbol{D}^H+\sigma^2\boldsymbol{D}\tilde{\boldsymbol{I}}\tilde{\boldsymbol{I}}^H\boldsymbol{D}^H \end{aligned}$$

So by replacing (10) and (12) into (8), we can rewrite (8) as

$$\begin{cases} \min\limits_{\boldsymbol{v}} & \boldsymbol{v}^T\left(\boldsymbol{I}_{N_w}\otimes(P_A\boldsymbol{X}_A+P_B\boldsymbol{X}_B)+\sigma^2\boldsymbol{I}_{R^2N_w}\right)\boldsymbol{v}^* \\ \boldsymbol{s}.t. & \dfrac{P_A Tr(\boldsymbol{v}^T\boldsymbol{D}\tilde{\boldsymbol{h}}_A\tilde{\boldsymbol{h}}_A^H\boldsymbol{D}^H\boldsymbol{v}^*)}{Tr\left(\boldsymbol{v}^T\boldsymbol{Q}_A\boldsymbol{v}^*\right)+\sigma_d^2} \geqslant \gamma \\ & \dfrac{P_B Tr(\boldsymbol{v}^T\boldsymbol{D}\tilde{\boldsymbol{h}}_B\tilde{\boldsymbol{h}}_B^H\boldsymbol{D}^H\boldsymbol{v}^*)}{Tr\left(\boldsymbol{v}^T\boldsymbol{Q}_B\boldsymbol{v}^*\right)+\sigma_d^2} \geqslant \gamma \end{cases} \quad (13)$$

Note that this is a non-convex problem. Let us define $\boldsymbol{X} \triangleq \boldsymbol{v}^*\boldsymbol{v}^T$. Then, by using $Tr(\boldsymbol{ABC}) = Tr(\boldsymbol{CAB})$ and by relaxing the rank-one constraint, problem (13) is converted to (14), which is a SDP problem [15] as

$$\begin{cases} \min\limits_{\boldsymbol{X}} & Tr\left(\boldsymbol{X}\boldsymbol{Q}_P\right) \\ s.t. & Tr\Bigl((\boldsymbol{M}_A-\gamma\boldsymbol{Q}_A)\boldsymbol{X}\Bigr) \geq \sigma_d^2\gamma \\ & Tr\Bigl((\boldsymbol{M}_B-\gamma\boldsymbol{Q}_B)\boldsymbol{X}\Bigr) \geq \sigma_d^2\gamma \end{cases} \quad (14)$$

where

$$\begin{aligned} &\boldsymbol{Q}_P = \boldsymbol{I}_{N_w}\otimes(P_A\boldsymbol{X}_A+P_B\boldsymbol{X}_B)+\sigma^2\boldsymbol{I}_{R^2N_w}\ , \\ &\boldsymbol{M}_A = \boldsymbol{D}\tilde{\boldsymbol{h}}_A\tilde{\boldsymbol{h}}_A^H\boldsymbol{D}^H\ , \quad \boldsymbol{M}_B = \boldsymbol{v}^T\boldsymbol{D}\tilde{\boldsymbol{h}}_B\tilde{\boldsymbol{h}}_B^H\boldsymbol{D}^H. \end{aligned}$$

The optimal solution can be obtained by an interior-point method and by using CVX software package [16].

The optimal solution of problem (14) is the optimal solution of problem (8) if it has rank one, otherwise, randomization techniques can be employed to obtain a good approximation to the rank-one problem [17].

IV. Maximization of Worst SINR

In this section, we consider the FF relay design problem of maximizing the minimum received SINR at the destination subject to constraints on total transmit power of sources and relay transmit power. The optimization problem of joint sources power allocation and FF relay filter design is thus yielded as

$$\begin{cases} \max\limits_{\boldsymbol{v},P_A,P_B} & min\,(SINR_A, SINR_B) \\ s.t. & P_A + P_B \le P_{s,\max} \\ & P_r \le P_C \end{cases} \tag{15}$$

where for $i \in \{A, B\}$, $SINR_i$ is the SINR related to source i, P_i is the power at source i, $P_{s,max}$ is the maximum available transmit power of sources and P_C is the maximum available relay transmit power.

Because of the complicated non-convex nature of this problem, we use suboptimal alternating optimization algorithm [6]. Therefore, at the first step it is assumed that the allocated power of sources is given and the problem (16) is solved to optimize the relay filter for a given power allocation at the sources. In the second problem, with the given relay filter, the sources power allocation is optimized. These two problems are solved by an alternating algorithm.

The first sub-problem can be explicitly written as follows, in which due to the fixed values of P_A and P_B the first constraint is dropped.

$$\begin{cases} \max\limits_{\boldsymbol{v}} & min\,(SINR_A, SINR_B) \\ s.t. & P_r \le P_C \end{cases} \tag{16}$$

So by replacing (10) and (12) into (16), and introducing the slack variable t, the given max-min problem can be rewritten

$$\begin{cases} \max\limits_{\boldsymbol{v},t} & t \\ s.t. & \\ & \frac{P_A Tr(\boldsymbol{v}^T \boldsymbol{D}\tilde{\boldsymbol{h}}_A \tilde{\boldsymbol{h}}_A^H \boldsymbol{D}^H \boldsymbol{v}^*)}{Tr(\boldsymbol{v}^T \boldsymbol{Q}_A \boldsymbol{v}^*) + \sigma_d^2} \ge t \\ & \frac{P_B Tr(\boldsymbol{v}^T \boldsymbol{D}\tilde{\boldsymbol{h}}_B \tilde{\boldsymbol{h}}_B^H \boldsymbol{D}^H \boldsymbol{v}^*)}{Tr(\boldsymbol{v}^T \boldsymbol{Q}_B \boldsymbol{v}^*) + \sigma_d^2} \ge t \\ & P_A + P_B \le P_{s,\max} \\ & \boldsymbol{v}^T \boldsymbol{Q}_P \boldsymbol{v}^* \le P_C \end{cases} \tag{17}$$

Introducing a new matrix $\boldsymbol{X} \triangleq \boldsymbol{v}^* \boldsymbol{v}^T$ and deleting the rank-one constraint on $\boldsymbol{X}$, the relaxed problem can be denoted as

$$\begin{cases} \max\limits_{\boldsymbol{X},t} & t \\ s.t. & \\ & Tr\left(\boldsymbol{X}(P_A \boldsymbol{D}\tilde{\boldsymbol{h}}_A \tilde{\boldsymbol{h}}_A^H \boldsymbol{D}^H - t\boldsymbol{Q}_A)\right) \ge t\sigma_d^2 \\ & Tr\left(\boldsymbol{X}(P_B \boldsymbol{D}\tilde{\boldsymbol{h}}_B \tilde{\boldsymbol{h}}_B^H \boldsymbol{D}^H - t\boldsymbol{Q}_B)\right) \ge t\sigma_d^2 \\ & P_A + P_B \le P_{s,\max} \\ & Tr\,(\boldsymbol{X}\boldsymbol{Q}_P) \le P_C \\ & \boldsymbol{X} \ge 0 \end{cases} \tag{18}$$

Note that due to the variation of t , the relaxed optimization problem is quasi-convex [15], but it reduces to a SDP problem for given value of t. The solution of the quasi-convex optimization problem can be obtained through bisection search method [15]. So we have

$$\begin{cases} Find & \boldsymbol{X} \\ s.t. & \\ & Tr\left(\boldsymbol{X}(P_A \boldsymbol{D}\tilde{\boldsymbol{h}}_A \tilde{\boldsymbol{h}}_A^H \boldsymbol{D}^H - t\boldsymbol{Q}_A)\right) \ge t\sigma_d^2 \\ & Tr\left(\boldsymbol{X}(P_B \boldsymbol{D}\tilde{\boldsymbol{h}}_B \tilde{\boldsymbol{h}}_B^H \boldsymbol{D}^H - t\boldsymbol{Q}_B)\right) \ge t\sigma_d^2 \\ & P_A + P_B \le P_{s,\max} \\ & Tr\,(\boldsymbol{X}\boldsymbol{Q}_P) \le P_C \\ & \boldsymbol{X} \ge 0 \end{cases} \tag{19}$$

Assume the optimal value of problem (18), t_{op} lies in the interval $[t_l\,;\,t_u]$. The feasibility problem (19) is feasible for $t \le t_{op}$, and is infeasible for $t > t_{op}$. Thus, we can briefly describe the procedure of the bisection search in the *Algorithm 1*. In this Algorithm, ε is the allowed error tolerance. After applying bisection method, if the rank of the matrix $\boldsymbol{X}$ that is obtained by solving the relaxed optimization problem (19) is not one, we apply randomization techniques for generating the rank-one solution.

In the second step of the alternating algorithm, we need to optimize the allocated source powers P_A and P_B , for the given relay vector $\boldsymbol{v}$, which is obtained from the previous step. Thus, this sub-problem can be written as

$$\begin{cases} \max\limits_{P_A,P_B} & min\,(SINR_A, SINR_B) \\ s.t. & P_A + P_B \le P_{s,max} \\ & P_r \le P_C \\ & SINR_A \ge t_0\ ,\ SINR_B \ge t_0 \end{cases} \tag{20}$$

where t_0 is the minimum allowed value for the worst received SINR of sources at the destination. By introducing the slack variable t and using (10) and (12), the sources power allocation problem in (20) can be rewritten as a LP optimization problem [15] denoted in (21) and can be solved by a standard convex optimization solver, like CVX.

$$\begin{cases} \max\limits_{P_A,P_B,t} & t \\ s.t. & \\ & Tr\left(\boldsymbol{X}(P_A \boldsymbol{D}\tilde{\boldsymbol{h}}_A \tilde{\boldsymbol{h}}_A^H \boldsymbol{D}^H - t\boldsymbol{Q}_A)\right) \ge t \\ & Tr\left(\boldsymbol{X}(P_B \boldsymbol{D}\tilde{\boldsymbol{h}}_B \tilde{\boldsymbol{h}}_B^H \boldsymbol{D}^H - t\boldsymbol{Q}_B)\right) \ge t \\ & P_A + P_B \le P_{s,max} \\ & Tr\,(\boldsymbol{X}\boldsymbol{Q}_P) \le P_C \\ & t \ge t_0 \end{cases} \tag{21}$$

TABLE I
DESCRIPTION OF ALGORITHM 1

Step 1: Set $t = (t_l + t_u)/2$
Step 2: Solve problem (19). If it is feasible, then set $t_l = t$, otherwise $t_u = t$,
Step 3: Repeat the process until convergence criterion $t_u - t_u < \epsilon$ is satisfied.

TABLE II
DESCRIPTION OF ALGORITHM 2

Step 1: Initialize P_A and P_B.
Step 2: Solve problem (16) with Algorithm 1.
Step 3: Set the minimum allowed t_{op} for the worst SINR in problem (20) as the maximum value t_{op} obtained from Step 2.
Step 4: For the given $\boldsymbol{v}$ and t_{op} from Steps 2 and 3, solve problem (20) to obtain new P_A and P_B.
Step 5: Go to Step 2. Here, set t_l of problem (16) as the solution to problem (21) in Step 4.
Step 6: Repeat Steps 2 to 5 until the termination criterion $|t_0 - t_l| < \varepsilon$ is satisfied.

Now, for the joint optimization of the sources power allocation and relay filter design to maximize the worst SINR, we describe the procedure of the alternating algorithm by combining problems (16) and (20) in *Algorithm 2*.

V. Simulation Results

In our simulations, we consider the MARC with a multi-antenna relay that uses FF strategy. The communication channels between all nodes are quasi-static frequency selective with length $N = N_s = 3$. The channel impulse response coefficients are modeled as zero-mean complex Gaussian random variables with an exponential power delay profile [5] that is:

$$p(t) = \frac{1}{\sigma_t} \sum_{l=0}^{X-1} e^{-t/\sigma_t} \delta(t - lT_s) \tag{22}$$

where $X \in \{N, N_s\}$, T_s is the symbol duration and $\sigma_t = 2T_s$ is symbol duration and $\sigma_t = 2T_s$ represents the delay spread. The noise variances at the relay and destination nodes are 1dB.

We first investigate the performance of the FF relay design, provided in problem (8) which minimizes the relay transmit power subject to SINR constraints. Fig. 2 shows the minimum required transmit power of relay, versus minimum required SINR at the destination and for different lengths of the relay filter. It can be seen that by increasing the number of relay filter taps, ISI decreases and so, for a fixed value of SINR, the required relay transmit power reduces. Note that for the filter length $N_w = 1$, the FF protocol is equivalent to the AF and we see that the relay transmit power for the FF relay strategy is significantly reduced when compared with that required by the AF relay. Since we use a multi antenna relay, also we investigate the effect of number of antennas at the relay node for problem (8), in Fig. 2. As it can be seen, increasing the number of relay antennas improves the performance significantly, such that increasing the number of antennas from $R = 4$ to $R = 6$ improves the performance even more than increasing the filter length. For example, in the case of $N_w = 1$ and $R = 6$ minimum relay transmit power is less than that of the case $N_w = 4$ and $R = 4$.

This happens due to the fact that, by increasing the number of antennas at the relay, the number of independent paths increases and higher diversity gain can be obtained. We next investigate the performance of the joint sources power allocation and FF relay filter design, provided in problem

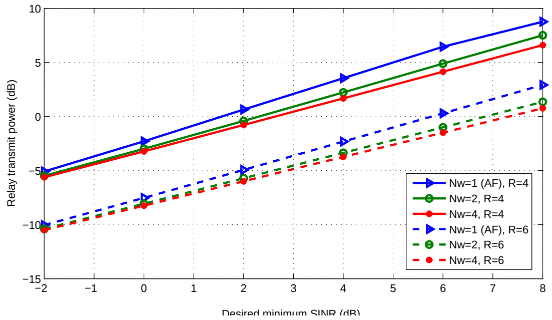

Fig. 2. Relay transmit power versus the desired minimum SINR for $R = 4$ and $R = 6$. ($P_A = P_B = 20\,dB$).

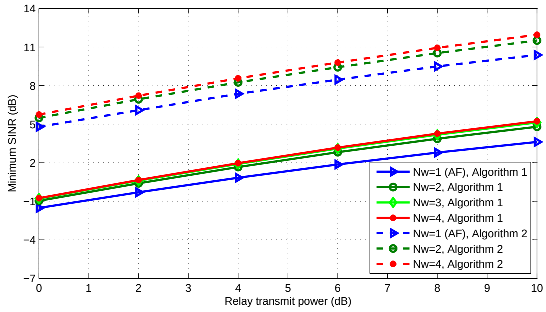

Fig. 3. The worst SINR versus the relay transmit power P_C based on *Algorithm 1* ($P_A = P_B = P_{s,\max}/2 = 20\,dB$) and *Algorithm 2*. ($R = 4$).

(15), to maximize the worst SINR subject to constraints on the transmit power of the sources and the relay. First, we consider only the relay filter optimization for a given equal source power allocation, i.e., $P_A = P_B = P_{s,\max}/2$, based on Algorithm 1. It is assumed that the relay is equipped with $R = 4$ antennas.

Fig. 3 shows the result. It can be seen that by increasing the number of the relay filter taps the value of SINR improves, because more amount of ISI can be canceled. As the previous case of relay transmit power minimization, the gain by the FF relay over the AF relay is significant. We next evaluate the performance of the joint sources power allocation and FF relay filter design which is obtained by Algorithm 2, and illustrated in Fig. 3. It can be seen that the joint optimization significantly improves the results of only designing the relay filter that is presented in Algorithm 1.

In Fig 4, we also investigate the effect of the number of employed antennas at the relay node for the second problem. Similar to the previous problem, improvement of the minimum SINR by increasing the number of relay antennas from 4 to 6 is obvious. We can see that in the case of $N_w = 1$ and $R = 6$, the minimum SINR is more than the case $N_w = 4$ and $R = 4$. Thus, in two problems we see that by adjusting the filter length and the number of antennas we can achieve a better performance.

VI. Conclusion

In this paper, we have studied a multiple access relay network with frequency selective channels and a multi-antenna relay. In order to reduce the effect of ISI, we have considered

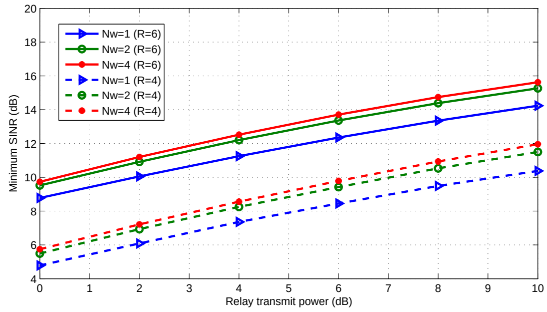

Fig. 4. The worst SINR versus the relay transmit power P_C, based on *Algorithm 2* for $R = 4$ and $R = 6$.

the FF relay approach. To design a FF relay, we have investigated two optimization problems which are minimizing the relay transmit power subject to the SINR constraints and maximizing the worst SINR subject to the transmit power constraints. An efficient solution based on the semi-definite relaxation and an alternating algorithm has been presented for solving these problems respectively. Simulation results have demonstrated the improvement of the performance by using the proposed FF relaying strategy as compared to the performance obtained by the commonly used AF protocol. Also we have shown that the number of relay antennas, is an important parameter in improving the performance.

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