Filter-and-Forward Relay Design for OFDM-based Multiple Access Relay Channel

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Abstract— **In this paper, we study the filter-and-forward (FF) relaying strategy for a multiple access relay channel (MARC) based on orthogonal frequency division multiplexing (OFDM) transmission. To reduce OFDM demodulation and redemodulation complexity at the relay, the FF-relay directly filters the incoming signal with a finite impulse response (FIR) filter and forwards it to the destination. We intend to design the relay filter subject to ensuring quality of service (QoS) at the destination. We propose an optimization criterion that is the joint sources power allocation and relay filter design, to maximize the worst subcarrier channel signal-to-noise ratio (SNR) subject to the sources and the relay transmit power constraints. We employ an alternating optimization algorithm to find the relay filter and the allocated power to sources, for our problem that is non-convex. Simulation results show that the proposed FF approach has a better performance for OFDM based MARC in frequency selective channels in comparison to the amplify and forward (AF) strategy. Also we investigate the effect of the relay filter length on the system performance.**

Keywords-component; filter and forward (FF); multiple access relay channel (MARC); OFDM; amplify and forward (AF); frequency selective channel.

I. INTRODUCTION

Cooperative communication is an important technique that can extend the coverage of wireless networks and also can exploit cooperative spatial diversity of different users in the network by means of providing several copies of a signal which have independent channel gains [1]. To obtain cooperative diversity, different relaying strategies have been proposed. Amplify and forward (AF), where the relay transmits an amplified version of its received signal including noise to the destination, decode and forward (DF) where the relay decodes its received signal, encodes a new one and transmits it to the destination, compress and forward (CF) and other strategies have been widely used in relay networks [2], [3]. Among the relaying schemes, the AF strategy due to its simplicity is suitable for cheap relay deployment and is more popular. Recently, in the case of frequency selective channels there have been some efforts to extend the AF relaying strategy to another scheme called filter and forward (FF) scheme [4]-[6], since AF is not efficient in suppressing the significant amount of inter-symbol interference (ISI) [1]. FF strategy is a linear filtering relaying scheme in which the incoming signal to the relay is filtered with a finite impulse response (FIR) filter [5].

On the other hand, multiple access relay channel (MARC) is used to well model a cooperation in multi-user networks, where multiple sources (generally considers two sources) communicate to a destination with the help of a relay node [7]. Since there are different relaying strategies, different scenarios of MARC such as using AF, DF and network coding schemes have been studied [8] - [10].

In this paper, we consider using FF relaying strategy in the MARC with frequency selective channels. In frequency selective channels another scheme that is used to compensate the effects of frequency selectivity of the channel is orthogonal frequency division multiplexing (OFDM) transmission [11]. OFDM is a type of multi-carrier modulation in which the subcarriers of the corresponding subchannels are mutually orthogonal [11]. So by using OFDM transmission in frequency selective channels, the signals encounter flat fading subchannels. Thus, in this paper we investigate OFDM transmission for MARC system model (with two sources). Our considered model can be regarded as the generalization of the investigated model in [5] that is a three-node relay channel. We use direct FF relaying for OFDM transmission that the received signal at the relay passes through a FIR filter at the chip rate of OFDM modulation and is then directly forwarded to the destination. Therefore there is not any need for OFDM demodulation/remodulation at the relay.

In this paper, we consider a criterion to design the FF relay for OFDM systems. We formulate the problem of joint relay filter design and sources power allocation for the worst subcarrier signal-to-noise ratio (SNR) maximization. The constraints are on the sources and the relay transmit powers. The resulted optimization problem is non-convex that we solve it with an efficient iterative algorithm which includes two sub-problems. The first problem is designing the relay filter to maximize the worst subcarrier SNR for a given allocated power, and the second is the power allocation optimization at the sources to maximize the worst subcarrier SNR for a given relay filter. Providing simulation results we investigate the effect of the relay filter length on the performance of the system. Also we show that the FF approach has a better performance for OFDM based MARC in frequency selective channels in comparison with AF scheme.

Notations: In this paper, vectors and matrices are written in bold letters and capital letters denote matrices. All vectors are vertical. Superscripts $(.)^T$, $(.)^*$ and $(.)^H$ denote transpose, conjugate and Hermitian transpose, respectively. $Vec(.)$ signifies the matrix vectorization operator and $\otimes$ denotes the Kronecker operator. $Tr\,(.)$ and $rank\,(.)$ stand for the trace and the rank of matrix, respectively. The symbol $E\,(.)$ represents the expectation operation. $\mathbf{x} \sim CN\,(\mu,\Sigma)$ means that $\mathbf{x}$ is complex circularly symmetric Gaussian distributed with mean vector μ and covariance matrix Σ . I_N and $0_{N\times M}$ imply the $N\times N$ identity matrix and the $N\times M$ zero matrix, respectively. The notation Toeplitz $(\mathbf{f}^T, N)$ indicates a $N\times(N+L_f-1)$ Toeplitz matrix with N rows and $[\mathbf{f}^T, 0,...,0]$ as its first row vector, where $\mathbf{f}^T$ is a row vector of size L_f . Moreover, $diag[d_0,\cdots,d_{N-1}]$ denotes a diagonal matrix with diagonal elements $d_0,\cdots,d_{N-1}$.

The rest of the paper is organized as follows. Section II defines the system model. The FF relay design problem is formulated in Section III and the proposed algorithm for solving this problem is also described in Section III. Finally, simulation results and conclusion are provided in Sections VI and V, respectively.

II. SYSTEM MODEL

We consider a full duplex MARC, with two source nodes as A and B , the relay node R and the destination node D . All channels are frequency selective and we assume that the channel state information (CSI) of the sources to relay (SR) channels are known to the relay, and the relay also knows the relay to destination (RD) channel distribution. The sources and the destination employ OFDM modulation and demodulation respectively with N subcarriers. The OFDM symbol vector of size N at each source i for $i\in\{A,B\}$ is given by $\boldsymbol{s}_i \triangleq [s_i(N-1), s_i(N-2),...,s_i(0)]^T$, where each data symbol is assumed to be as $s_i[k] \sim CN\,(0,P_{s_i,k})$ for $k=0,1,...,N-1$. Then the N –point normalized inverse discrete Fourier transform (IDFT) yields the time domain signal vector at each source i for $i\in\{A,B\}$ that are given by

$$x_i[n] \triangleq 1/\sqrt{N}\sum_{k=0}^{N-1} s_i[n]e^{j(2\pi nk/N)} \quad n=0,1,...,N-1 \qquad (1)$$

where $1/\sqrt{N}$ is a scale factor. An alternative method that avoids ISI is to attach a cyclic prefix here with length L_{cp} to each block of N signal samples as

$$\tilde{x}_i[n] = \begin{cases} x_i[n], & n=0,1,\cdots,N-1 \\ x_i[N+n], & n=-1,-2,\cdots,-L_{CP} \end{cases} \qquad (2)$$

Then $\tilde{x}_i[n]$ is transmitted from the source i to the relay, and the received signal at the relay is given by

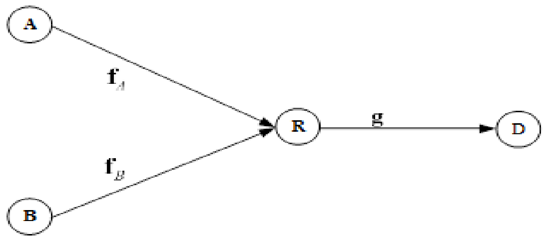

Figure 1. The multiple access relay channel with OFDM transmission.

$$y_r[n] = \sum_{l=0}^{L_A-1} f_{A,l}\tilde{x}_A[n-l] + \sum_{l=0}^{L_B-1} f_{B,l}\tilde{x}_B[n-l] + n_r[n] \qquad (3)$$

where $n_r[n]$ is the additive white Gaussian noise at the relay with $n_r[n] \sim CN\,(0,\sigma_r^2)$, $\mathbf{f}_i = [f_{i,0},\cdots,f_{i,L_i-1}]^T$ is the vector of channel impulse response between the source i ($i\in\{A,B\}$) and the relay. According to the FF strategy to compensate the effect of the frequency selectivity of communication channels, the received signal at the relay node is passed through a FIR filter and then is transmitted to the destination. Thus, the transmitted signal by the relay at time n is

$$x_t[n] = \sum_{l=0}^{L_r-1} r_l y_r[n-l] \qquad (4)$$

where $\mathbf{r} = [r_0,\cdots,r_{L_r-1}]^T$ are the taps of FIR filter with length L_r . Finally, the transmitted signal by the relay passes through RD channel with order of L_g and channel tap coefficient vector $\mathbf{g} = [g_0,\cdots,g_{L_g-1}]^T$. So, the received signal at the destination can be written as

$$y_d[n] = \sum_{l=0}^{L_g-1} g_l x_t[n-l] + n_d[n] \qquad (5)$$

where $n_d(n)$ is an additive Gaussian noise with variance σ_d^2 at the destination and g_l , $l=0,1,...,L_g-1$, are independent and identically distributed (i.i.d.) random variable according to $g_l \overset{iid}{\sim} CN\,(0,\sigma_g^2)$. So, each tap is independently Rayleigh faded and instead of the realization of each tap, its distribution is just known to the relay. We can rewrite (4) and (5) in the form of matrix as

$$\begin{aligned} \mathbf{x}_t &= \mathbf{R}\mathbf{F}_A\tilde{\mathbf{x}}_A + \mathbf{R}\mathbf{F}_B\tilde{\mathbf{x}}_B + \mathbf{R}\mathbf{n}_r \\ \mathbf{y}_d &= \mathbf{G}\mathbf{R}\mathbf{F}_A\tilde{\mathbf{x}}_A + \mathbf{G}\mathbf{R}\mathbf{F}_B\tilde{\mathbf{x}}_B + \mathbf{G}\mathbf{R}\mathbf{n}_r + \mathbf{n}_d \end{aligned} \qquad (6)$$

by defining the following vectors and matrices for $i\in\{A,B\}$:

$$\mathbf{y}_d = \left[y_d[N-1],\cdots,y_d[0]\right]^T$$

$$\mathbf{x}_t = \left[x_t[N-1],\cdots,x_t[0],x_t[-1],\cdots,x_t[-L_g+1]\right]^T$$

$$\tilde{\mathbf{x}}_i = \left[x_i[N-1]\cdots,x_i[0],\cdots,x_i[-L_g-L_r-L_i+3]\right]^T$$

$$\mathbf{n}_r = \left[n_r[N-1], \cdots, n_r[0], \cdots, n_r[-L_g - L_r + 2] \right]^T$$
$$\mathbf{F}_i = Toeplitz\,(\mathbf{f}_i^T, N + L_g + L_r - 2)$$
$$\mathbf{G} = Toeplitz\,(\mathbf{g}^T, N),$$
$$\mathbf{R} = Toeplitz\,(\mathbf{r}^T, N + L_g - 1)$$

We assume that $L_g + L_r + L_i - 3 \le L_{CP}$ for transmission without interference. Thus the discrete Fourier transform (DFT) of the received vector of size N at the destination is given by

$$\begin{aligned}\hat{\mathbf{y}}_d &= \mathbf{W}_N^H \mathbf{G}\mathbf{R}\mathbf{F}_A \tilde{\mathbf{x}}_A + \mathbf{W}_N^H \mathbf{G}\mathbf{R}\mathbf{F}_B \tilde{\mathbf{x}}_B + \mathbf{W}_N^H \mathbf{G}\mathbf{R}\mathbf{n}_r + \mathbf{W}_N^H \mathbf{n}_d \\ &= \mathbf{W}_N^H \mathbf{H}_{cA} \mathbf{W}_N \mathbf{s}_A + \mathbf{W}_N^H \mathbf{H}_{cB} \mathbf{W}_N \mathbf{s}_B + \mathbf{W}_N^H \mathbf{G}\mathbf{R}\mathbf{n}_r + \mathbf{W}_N^H \mathbf{n}_d \\ &= \mathbf{D}_A \mathbf{s}_A + \mathbf{D}_B \mathbf{s}_B + \mathbf{W}_N^H \mathbf{G}\mathbf{R}\mathbf{n}_r + \mathbf{W}_N^H \mathbf{n}_d \end{aligned} \tag{7}$$

where $\boldsymbol{W}_N^H$ is the normalized DFT matrix of size N, $\boldsymbol{H}_{ci}$ for $i \in \{A, B\}$ is a $N \times N$ circulant matrix that is generated from the Toeplitz filtering matrices $\boldsymbol{GRF}_i$, and:

$$\boldsymbol{D}_i = diag\left[d_{i,0}, ..., d_{i,N-1} \right] = \boldsymbol{W}_N^H \boldsymbol{H}_{ci} \boldsymbol{W}_N \tag{8}$$

To do relay filter optimization in Section III we must first drive the expression of the received signal in terms of relay tap coefficients. We need to know only the first row $\boldsymbol{h}_{ci}^T$ of $\boldsymbol{H}_{ci}$, to compute $\boldsymbol{D}_i$. Since $\boldsymbol{H}_{ci}$ is generated from the Toeplitz filtering matrix $\boldsymbol{GRF}_i$, so only $\tilde{\boldsymbol{g}}^T \boldsymbol{RF}_i$ (the first row of $\boldsymbol{GRF}_i$) is sufficient, where $\tilde{\mathbf{g}}^T = [\mathbf{g}^T, 0, \cdots, 0]^T_{1 \times (N + L_g - 1)}$ is the first row of $\boldsymbol{G}$. Then we have $\boldsymbol{h}_{ci}^T = \tilde{\boldsymbol{g}}^T \boldsymbol{RF}_i \boldsymbol{T}_i$ where $\boldsymbol{T}_i = \left[\boldsymbol{I}_N ; \boldsymbol{0}_{(L_g + L_r + L_i - 3) \times N} \right]$

Now the diagonal elements of $\boldsymbol{D}_i$ can be obtained as

$$\left[d_{i,0}, ..., d_{i,N-1} \right]^T = \boldsymbol{W}_N^H (\tilde{\boldsymbol{g}}^T \boldsymbol{RF}_i \boldsymbol{T}_i)^T \tag{9}$$

where $\sqrt{N}\boldsymbol{W}_N^H$ is the DFT matrix of size N. Finally, the k th subcarrier of the received signal at the destination is given by

$$\hat{y}_d[k] = \hat{y}_{d,S}[k] + \hat{y}_{d,N}[k] \tag{10}$$

with signal component as

$$\begin{aligned} \hat{y}_{d,S}[k] &\triangleq \hat{y}_{d,S_A}[k] + \hat{y}_{d,S_B}[k] \\ \hat{y}_{d,S_i}[k] &\triangleq \sqrt{N}\, \mathbf{w}_k^H \mathbf{T}_i^T \mathbf{F}_i^T \mathbf{R}^T \tilde{\mathbf{g}} s_i[k], \ i \in \{A, B\} \end{aligned} \tag{11}$$

where $\hat{y}_{d,S_i}$ is the signal component with respect to source i and $\mathbf{w}_k^H$ is the $k+1$ th row of $\boldsymbol{W}_N^H$. The noise component is

$$\hat{y}_{d,N}[k] = \mathbf{w}_k^H \mathbf{G}\mathbf{R}\mathbf{n}_r + \mathbf{w}_k^H \mathbf{n}_d \tag{12}$$

where $k = 0, 1, ..., N-1$.

III. MAXIMZATION OF THE WORST SUBCARRIER SNR

We consider the FF relay design problem for the MARC described in Section II. We consider the problem of maximizing the worst subcarrier SNR at the destination subject to sources and relay power constraints. The problem of optimizing the relay filter and the sources power allocation to maximize the worst subcarrier SNR is formulated as follows:

$$\begin{cases} \max\limits_{\substack{\mathbf{r}, \\ P_{s_A,0}, \cdots, P_{s_A,N-1}, \\ P_{s_B,0}, \cdots, P_{s_B,N-1}.}} \ \min\limits_{k \in \{0,1,\cdots,N-1\}} \ SNR_A^k, SNR_B^k \\ s.t. \qquad \sum\limits_{k=0}^{N-1} (P_{s_A,k} + P_{s_B,k}) \le P_{s,\max} \\ \qquad\qquad P_r \le P_{r.\max} \end{cases} \tag{13}$$

where $P_{s,max}$ is the maximum available total transmit power of sources and $P_{r,max}$ is the maximum available relay transmit power. To achieve the SNR of the k th subcarrier channel, first we express the received signal power at the destination in terms of the relay filter $\mathbf{r}$:

$$\mathrm{E}\left\{ \left| \hat{y}_{d,S}[k] \right|^2 \right\} = \mathrm{E}\left\{ \left| \hat{y}_{d,S_A}[k] \right|^2 \right\} + \mathrm{E}\left\{ \left| \hat{y}_{d,S_B}[k] \right|^2 \right\} \tag{14}$$

First we compute the received signal power at the destination corresponding to source A as

$$\begin{aligned} P_A^k &\triangleq \mathrm{E}\left\{ \left| \hat{y}_{d,S_A}[k] \right|^2 \right\} \\ &= N\,\mathrm{E}\left\{ \mathbf{w}_k^H \mathbf{T}_A^T \mathbf{F}_A^T \mathbf{R}^T \tilde{\mathbf{g}} \left| s_A[k] \right|^2 \tilde{\mathbf{g}}^H \mathbf{R}^* \mathbf{F}_A^* \mathbf{T}_A^* \mathbf{w}_k \right\} \\ &= N \mathbf{w}_k^H \mathbf{T}_A^T \mathbf{F}_A^T \mathbf{R}^T \mathrm{E}\left\{ \left| s_A[k] \right|^2 \tilde{\mathbf{g}}\tilde{\mathbf{g}}^H \right\} \mathbf{R}^* \mathbf{F}_A^* \mathbf{T}_A^* \mathbf{w}_k \\ &= N P_{s_A,k} \sigma_g^2 Tr\left(\mathbf{w}_k^H \mathbf{T}_A^T \mathbf{F}_A^T \mathbf{R}^T \tilde{\mathbf{I}}_{Lg} \mathbf{R}^* \mathbf{F}_A^* \mathbf{T}_A^* \mathbf{w}_k \right) \\ &= N P_{s_A,k} \sigma_g^2 Tr\left(\underbrace{\mathbf{F}_A^* \mathbf{T}_A^* \mathbf{w}_k \mathbf{w}_k^H \mathbf{T}_A^T \mathbf{F}_A^T}_{\triangleq \mathbf{K}_{A,k}} \mathbf{R}_{Lg}^T \mathbf{R}_{Lg}^* \right) \\ &= N P_{s_A,k} \sigma_g^2 \left[vec(\mathbf{R}_{Lg}^T) \right]^H \bar{\mathbf{K}}_{A,k} vec(\mathbf{R}_{Lg}^T) \\ &= N P_{s_A,k} \sigma_g^2 \left[vec(\mathbf{R}^T) \right]^H \tilde{\mathbf{K}}_{A,k} vec(\mathbf{R}^T) \\ &= N P_{s_A,k} \sigma_g^2 \mathbf{r}^H \mathbf{E} \tilde{\mathbf{K}}_{A,k} \mathbf{E}^H \mathbf{r}. \end{aligned} \tag{15}$$

where $P_{s_A,k}$ is the transmit power of source A, and

$$\tilde{\mathbf{I}}_{Lg} \triangleq \left[\mathbf{I}_{Lg} \quad \mathbf{0}_{Lg \times (N-1)}; \quad \mathbf{0}_{(N-1) \times Lg} \quad \mathbf{0}_{(N-1) \times (N-1)} \right]$$
$$\bar{\mathbf{K}}_{A,k} = \mathbf{I}_{Lg} \otimes \mathbf{K}_{A,k} \ ; \ \tilde{\mathbf{K}}_{A,k} = \tilde{\mathbf{I}}_{Lg} \otimes \tilde{\mathbf{K}}_{A,k}$$
$$\mathbf{R} = \left[\mathbf{R}_{Lg}; \quad \mathbf{R}_{N-1} \right], vec(\mathbf{R}^T) = \mathbf{E}^H \mathbf{r}$$
$$\mathbf{E} = [\mathbf{I}_{L_r}, \mathbf{0}_{L_r \times (N + L_g - 2)}, \mathbf{0}_{L_r \times 1}, \mathbf{I}_{L_r}, \mathbf{0}_{L_r \times (N + L_g - 3)} \\ \cdots, \mathbf{0}_{L_r \times (N + L_g - 2)}, \mathbf{I}_{L_r}]$$

Using similar techniques as above, we can derive the received signal power with respect to source B. We also can derive the received noise power as

$$P_N^k = \mathrm{E}\left\{\left|\hat{y}_{d,N}[k]\right|^2\right\}$$
$$= \sigma_r^2 tr\left(\mathbf{R}^H \underbrace{E\left\{\mathbf{G}^H \mathbf{w}_k \mathbf{w}_k^H \mathbf{G}\right\}}_{\triangleq \mathbf{M}_k} \mathbf{R}\right) + \sigma_d^2$$
$$= \sigma_r^2 \left[vec(\mathbf{R})\right]^H \tilde{\mathbf{M}}_k vec(\mathbf{R}) + \sigma_d^2$$
$$= \sigma_r^2 \mathbf{r}^H \mathbf{Q}\tilde{\mathbf{M}}_k \mathbf{Q}^H \mathbf{r} + \sigma_d^2 \quad (16)$$

where

$$\tilde{\mathbf{M}}_k = \mathbf{I}_{N+L_g+L_r-2} \otimes \mathbf{M}_k \quad ; vec(\mathbf{R}) = \mathbf{Q}^H \mathbf{r}$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{q}_1^T & \mathbf{q}_2^T & \cdots & \mathbf{q}_{N+L_g-1}^T & \mathbf{0}^T & \cdots & \cdots & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{q}_1^T & \mathbf{q}_2^T & \cdots & \mathbf{q}_{N+L_g-1}^T & \mathbf{0}^T & \cdots & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{0}^T & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \mathbf{0}^T \\ \mathbf{0}^T & \cdots & \cdots & \mathbf{0}^T & \mathbf{q}_1^T & \mathbf{q}_2^T & \cdots & \mathbf{q}_{N+L_g-1}^T \end{bmatrix}$$

and $\mathbf{q}_j^T$ is the j th row of $\mathbf{I}_{N+L_g-1}$.

Based on (15) an (16), the SNR of the k th subcarrier channel corresponding to sources A and B are expressed as

$$SNR_A^k = \frac{NP_{s_A,k}\sigma_g^2 \mathbf{r}^H \mathbf{E}\tilde{\mathbf{K}}_{A,k}\mathbf{E}^H \mathbf{r}}{\sigma_r^2 \mathbf{r}^H \mathbf{Q}\tilde{\mathbf{M}}_k \mathbf{Q}^H \mathbf{r} + \sigma_d^2}$$
$$SNR_B^k = \frac{NP_{s_B,k}\sigma_g^2 \mathbf{r}^H \mathbf{E}\tilde{\mathbf{K}}_{B,k}\mathbf{E}^H \mathbf{r}}{\sigma_r^2 \mathbf{r}^H \mathbf{Q}\tilde{\mathbf{M}}_k \mathbf{Q}^H \mathbf{r} + \sigma_d^2} \quad (17)$$

Based on (6), we can obtain the relay transmit power as

$$\mathrm{E}\{Tr(\mathbf{x}_t \mathbf{x}_t^H)\} = Tr(\mathbf{R}\mathbf{F}_A \underbrace{\mathrm{E}\{\tilde{\mathbf{x}}_A \tilde{\mathbf{x}}_A^H\}}_{\triangleq \sum_{\tilde{\mathbf{x}}_A}} \mathbf{F}_A^H \mathbf{R}^H) +$$
$$Tr(\mathbf{R}\mathbf{F}_B \underbrace{\mathrm{E}\{\tilde{\mathbf{x}}_B \tilde{\mathbf{x}}_B^H\}}_{\triangleq \sum_{\tilde{\mathbf{x}}_B}} \mathbf{F}_B^H \mathbf{R}^H) + Tr(\sigma_r^2 \mathbf{R}\mathbf{R}^H)$$
$$= Tr(\mathbf{R}\underbrace{(\mathbf{F}_A \textstyle\sum_{\tilde{\mathbf{x}}_A} \mathbf{F}_A^H + \mathbf{F}_B \sum_{\tilde{\mathbf{x}}_B} \mathbf{F}_B^H + \sigma_r^2 \mathbf{I})}_{\triangleq \Pi} \mathbf{R}^H)$$
$$= [vec(\mathbf{R}^H)]^H \tilde{\Pi} vec(\mathbf{R}^H) = \mathbf{r}^H \mathbf{E}\tilde{\Pi}\mathbf{E}^H \mathbf{r}^* = \mathbf{r}^H \mathbf{E}\tilde{\Pi}^* \mathbf{E}^H \mathbf{r}, \quad (18)$$

where $\tilde{\Pi} = \mathbf{I}_{N+L_g-1} \otimes \Pi$.

Now, based on (17) and (18), the optimization problem (13) is non-convex. So in order to avoid high computational complexity for solving a non-convex problem, we use a suboptimal alternating optimization algorithm [5]. This technique splits the main problem in two sub-problems. That is, first, it is assumed that the allocated power of sources is initialized and the problem (19) is considered to optimize the relay filter for a fixed power allocation at the sources. Then, using the relay filter obtained by solving problem (19), the sources power allocation is optimized. These two problems are solved in an alternating algorithm until the convergence is established. Let the first problem for $k = 0, 1, \cdots, N-1$ can be written as follows

$$\begin{cases} \max\limits_{\mathbf{r}} \quad \min\limits_{k \in \{0,1,\cdots,N-1\}} SNR_A^k, SNR_B^k \\ s.t. \quad P_r \le P_{r,\max} \end{cases} \quad (19)$$

By introducing the slack variable τ, the given max–min problem can be rewritten as

$$\begin{cases} \max\limits_{\mathbf{r}} \ \tau \\ s.t. \quad SNR_A^k \ge \tau, SNR_B^k \ge \tau \\ \qquad P_r \le P_{r,\max} \end{cases} \quad (20)$$

The above optimization problem is non convex and we convert it to a convex problem by a semi-definite relaxation (SDR) [12]. It is relaxed by dropping the rank-one constraint, which leads to the convex semi-definite programming problem (SDP) [12], as (21). By defining $\Omega \triangleq \mathbf{r}\mathbf{r}^H$ and

$$\Phi = \mathbf{E}\tilde{\Pi}^* \mathbf{E}^H$$
$$\Psi_N(k) = \sigma_r^2 \mathbf{Q}\tilde{\mathbf{M}}_k \mathbf{Q}^H$$
$$\Psi_{A,S}(k) = NP_{s_A,k}\sigma_g^2 \mathbf{E}\tilde{\mathbf{K}}_{A,k}\mathbf{E}^H$$
$$\Psi_{B,S}(k) = NP_{s_B,k}\sigma_g^2 \mathbf{E}\tilde{\mathbf{K}}_{B,k}\mathbf{E}^H$$

we have

$$\begin{cases} \max\limits_{\Omega} \ \tau \\ s.t. \quad tr([\Psi_{A,S}(k) - \tau\Psi_N(k)]\Omega) \ge \sigma_d^2 \tau \\ \qquad tr([\Psi_{B,S}(k) - \tau\Psi_N(k)]\Omega) \ge \sigma_d^2 \tau \\ \qquad tr(\Phi\Omega) \le P_{r,\max} \\ \qquad \Omega \ge 0 \end{cases} \quad (21)$$

Due to variation of τ, the relaxed optimization problem is quasi-convex [12] and can be obtained by solving its corresponding feasibility problem as

TABLE I. DESCRIPTION OF ALGORITHM 1.

***Algorithm 1*:** Choose some suitable interval subject to $\tau_{opt} \in (\tau_l, \tau_u)$

Step 1: Set $\tau = (\tau_l + \tau_u)/2$.

Step 2: Solve problem (22). If it is feasible, then set $\tau_l = \tau$, otherwise, $\tau_u = \tau$.

Step 3: Repeat the process until the criterion $\tau_u - \tau_l < \varepsilon$ is satisfied**.**

$$\begin{cases} Find\ \Omega \\ s.t. \quad tr([\Psi_{A,S}(k)-\tau\Psi_N(k)]\Omega) \ge \sigma_d^2\tau \\ \quad tr([\Psi_{B,S}(k)-\tau\Psi_N(k)]\Omega) \ge \sigma_d^2\tau \\ \quad tr(\Phi\Omega) \le P_{r,\max} \\ \quad \Omega \ge 0 \end{cases} \tag{22}$$

For the optimal value of problem (21) (that we call it τ_{opt}), problem (22) is feasible for $\tau \le \tau_{opt}$, and infeasible for $\tau > \tau_{opt}$. Thus, we solve it by a bisection search method [12] with the procedure described in Algorithm 1.

In Algorithm 1, ε is the allowed error tolerance. After applying bisection method, if the rank of matrix Ω that is obtained by solving the relaxed optimization problem (22) is not one, we apply randomization techniques for generating the rank-one solution [13].

In the second step of the alternating algorithm, we need to optimize the allocated sources powers $P_{s_A,k}$ and $P_{s_B,k}$, for the given relay filter vector $\mathbf{r}$, which is obtained from the previous step. Thus, this sub-problem can be written as

$$\begin{cases} \max\limits_{\substack{P_{s_A,0},\cdots,P_{s_A,N-1},\\ P_{s_B,0},\cdots,P_{s_B,N-1}.}} \ \min\limits_{k\in\{0,1,\cdots,N-1\}} \ SNR_A^k, SNR_B^k \\ s.t. \quad \sum_{k=0}^{N-1}(P_{s_A,k}+P_{s_B,k}) \le P_{s,\max} \\ \quad P_r \le P_{r.\max} \\ \quad SNR_A^k \ge \tau_0, SNR_B^k \ge \tau_0 \quad \forall k \end{cases} \tag{23}$$

Now, we first rewrite the relay power constraint in (18) as a linear form of $P_{s_i,k}$, $(i \in \{A,B\})$ as follows:

$$\mathbf{r}^H\mathbf{E}\tilde{\Pi}^*\mathbf{E}^H\mathbf{r} = \sum_{k=0}^{N-1}P_{s_A,k}Tr(\mathbf{q}_{k+1}\mathbf{q}_{k+1}{}^T\tilde{\mathbf{W}}_N^H\mathbf{F}_A^H\mathbf{R}^H\mathbf{R}\mathbf{F}_A\tilde{\mathbf{W}}_N) + \sum_{k=0}^{N-1}P_{s_B,k}Tr(\mathbf{q}_{k+1}\mathbf{q}_{k+1}{}^T\tilde{\mathbf{W}}_N^H\mathbf{F}_B^H\mathbf{R}^H\mathbf{R}\mathbf{F}_B\tilde{\mathbf{W}}_N) + Tr(\sigma_r^2\mathbf{R}\mathbf{R}^H) \tag{24}$$

where

$$\tilde{\mathbf{W}}_N = \left[\mathbf{w}_{N-1}, \mathbf{w}_{N-2}, \cdots, \mathbf{w}_0, \mathbf{w}_{N-1}, \cdots, \mathbf{w}_{N-L_g-L_r-L_f+3}\right]^T$$

and $\mathbf{w}_{k-1}^T$ denotes the k th row of the normalized IDFT matrix. Considering the relay power constraint as a linear form of $P_{s_i,k}$ (that is denoted in (24)), the optimization problem in (23) can be written as a linear programming (LP) problem [12] formulated as (25) by defining the following terms:

$$C_i(k) = Tr\left(\mathbf{q}_{k+1}\mathbf{q}_{k+1}{}^T\tilde{\mathbf{W}}_N^H\mathbf{F}_i^H\mathbf{R}^H\mathbf{R}\mathbf{F}_i\tilde{\mathbf{W}}_N\right), i \in \{A,B\}$$

$$C_2 = Tr\left(\sigma_r^2\mathbf{R}\mathbf{R}^H\right)$$

$$C_i'(k) = \frac{N\sigma_g^2\mathbf{r}^H\mathbf{E}\tilde{\mathbf{K}}_{i,k}\mathbf{E}^H\mathbf{r}}{\sigma_r^2\mathbf{r}^H\mathbf{Q}\tilde{\mathbf{M}}_k\mathbf{Q}^H\mathbf{r}+\sigma_d^2}$$

TABLE II. DESCRIPTION OF ALGORITHM 2.

***Algorithm 2*:**
Step 1: Initialize $P_{s_A,k}$ and $P_{s_B,k}$ for $k=0,1,...,N-1$.
Step 2: Solve problem (19) with Algorithm 1.
Step 3: Set the allowed minimum τ_0 for the worst subcarrier SNR in problem (23) as the maximum value τ_{opt} obtained from Step 2.
Step 4: For the given $\mathbf{r}$ and τ_{opt} from Steps 2 and 3, solve problem (25) to obtain new $P_{s_A,k}$ and $P_{s_B,k}$.
Step 5: Go to Step 2. Here, set τ_l of problem (19) as the solution to problem (25) in Step 4.
Step 6: Repeat Steps 2 to 5 until the termination criterion $\lvert\tau_0-\tau_l\rvert<\varepsilon$ is satisfied.

$$\begin{cases} \max\limits_{\substack{P_{s_A,0},\cdots,P_{s_A,N-1},\\ P_{s_B,0},\cdots,P_{s_B,N-1},\\ \tau}} \ \tau \\ s.t. \quad \sum_{k=0}^{N-1}P_{s_A,k}C_A(k)+P_{s_B,k}C_B(k)+C_2 \le P_{r,\max} \\ \quad P_{s_A,k}C_A'(k) \ge \tau \\ \quad P_{s_B,k}C_B'(k) \ge \tau \\ \quad \sum_{k=0}^{N-1}(P_{s_A,k}+P_{s_B,k}) \le P_{s,\max} \\ \quad \tau \ge \tau_0 \end{cases} \tag{25}$$

This LP problem can be solved by a standard convex optimization solver, like CVX [12]. Now we combine problems (19) and (23) in Algorithm 2 for the joint sources power allocation and relay filter design to maximize the worst subcarrier SNR.

IV. SIMULATION RESULTS

We have assumed a MARC with OFDM transmitters which the number of OFDM subcarriers are $N=32$. A relay uses FF strategy and the quasi-static frequency selective sources-to-relay and relay-to-destination channels are with the lengths $L_A = L_B = L_g = 3$. Each plotted value in each figure is obtained by taking the average over the 500 channel realizations. The channel impulse response coefficients are modeled as zero-mean complex Gaussian random variables with an exponential power delay profile [4] that is:

$$p(t) = \frac{1}{\sigma_t}\sum_{l=0}^{X-1}e^{-t/\sigma_t}\delta(t-lT_s) \tag{26}$$

where $X \in \{N, N_s\}$, T_s is the symbol duration and $\sigma_t = 2T_s$ represents the delay spread. The noise variances at the relay and the destination are 1 dB.

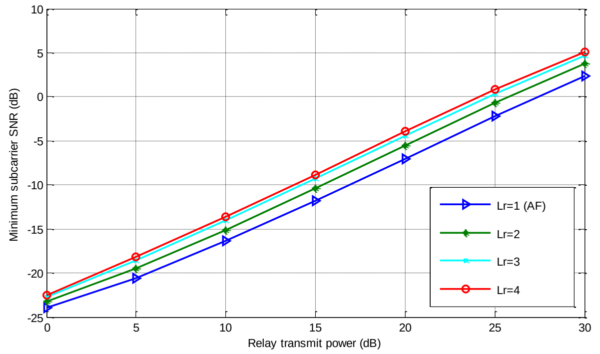

Figure 2. The worst subcarrier SNR versus maximum allowed relay transmit power,based on Algorithm 1 $(P_{s_A,k} = P_{s_B,k} = P_{s,\max} / N , P_{s_i,k} = 20dB)$

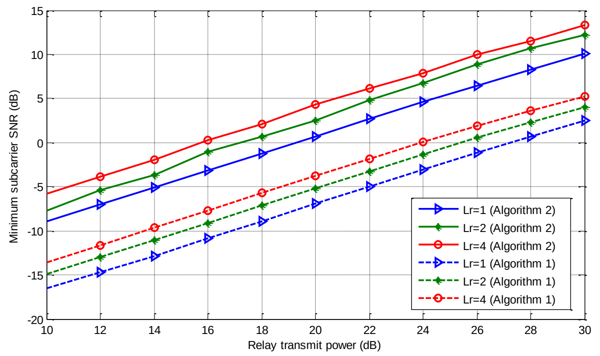

Figure 3. The worst subcarrier SNR versus maximum allowed relay transmit power,based on Algorithm1 $(P_{s_A,k} = P_{s_B,k} = P_{s,\max} / N , P_{s_i,k} = 20dB)$, and Algorithm 2.

We evaluate the performance of FF relay design to maximize the worst subcarrier SNR subject to some constraints on transmit powers of nodes, provided in problem (13). First, we consider the relay filter design only for a given equal sources power allocation, i.e. $P_{s_A,k} = P_{s_B,k} = P_{s,\max} / N$, based on Algorithm 1 that the results are shown in Fig 2. It can be seen that the obtained gain in terms of achievable SNR by using the FF relay is significantly more than that obtained by using the AF relay. Note that fore the length ($L_r = 1$) the FF relay is equivalent to the AF relay. The performance is also improved by increasing the number of the relay filter taps.

We next (as shown in Fig. 3) evaluate the performance of the joint FF relay filter design and sources power allocation based on Algorithm 2 and compare the results with that obtained in the previous case. It can be seen that the joint optimization significantly outperforms the case that only design the relay filter. The sufficient length of FF filter to compensate the effects of frequency selective channels completely, is $L_r \geq L_A + L_B + L_g - 1$ which with exponential power delay profile the ISI term can be cancelled even with $L_r \geq L_A + L_B + L_g - 1$ [4]. According to Fig. 2, in our relay filter design, the filter length $L_r = 4$ is sufficient.

V. Conclusion

In this paper, we have studied a multiple access relay channel system based on OFDM transmission and with frequency selective channels. The relay uses FF strategy to reduce the complexity of OFDM processes at the relay. With the aim of obtaining the FF relay filter and sources power allocation, we have investigated an optimization criterion which is maximizing the worst subcarrier SNR with transmit power constraints at sources and the relay. An alternating algorithm has been proposed to achieve the optimal solution, which includes two sub-problems. We have shown that joint optimization has much more gain. Simulation results have shown that the FF strategy outperforms AF relay significantly and is more practical than AF scheme for OFDM transmission.

References

[1] H. Chen, A. B. Gershman, and S. Shahbazpanahi, “Filter-and-forward distributed beamforming for two-way relay networks with frequency selective channels,” IEEE Transactions on Signal Processing, vol. 60, no. 4, pp. 1927-1941, 2012.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: efficient protocols and outage behavior,” IEEE Transactions on Information Theory, vol. 50, no. 12, pp. 3062 – 3080, Dec. 2004.

[3] A. Sedonaris, E. Erkip, and B. Aazhang, “User cooperation diversity — Part I. System description,” IEEE Transactions on Communication, vol. 51, pp. 1927-1938, Nov. 2003.

[4] H. Chen, A. B. Gershman, and S. Shahbazpanahi, “Filter-and-forward distributed beamforming in relay networks with frequency selective fading,” IEEE Transactions on Signal Processing, vol. 58, pp. 1251-1262, Mar. , 2010.

[5] D. Kim, J. Seo and Y. Sung, “Filter-and-forward transparent relay design for OFDM systems,” IEEE Transactions on Vehicular Technology , vol. 62, no. 9, pp. 1-16, Nov. 2013.

[6] M. Maleki and V. Tabataba Vakili, “Filter-and-forward transceiver design for cognitive two-way relay networks,” IET Communications, 2015.

[7] L. Sankaranarayanan, G. Kramer and N. B. Mandayam, “Capacity theorems for the multiple-access relay channel,” Proc. 42nd Annu. Allerton Conference. Communications, Control, and Computing. 2004.

[8] M. El Soussi, A. Zaidi, and L. Vandendorpe, “Amplify-and-forward on a multiaccess relay channel with computation at the receiver,” Proc. IEEE International Conference on Network Games, Control and Optimization (NetGCooP), pp. 74-79, 2012.

[9] M. El Soussi, A. Zaidi, and L. Vandendorpe, “DF-based sum-rate optimization for multicarrier multiple access relay channel,” EURASIP Journal on Wireless Communications and Networking 2015.

[10] B. Han, Z. Zhao, M Peng, Y. Li, W. Wang, “Resource allocation for OFDM multiple-access relay channels with network coding,” Proc. Global Communications Conference (GLOBECOM), 2012 IEEE, pp. 4701-4706, Des. 2012.

[11] J. G. Proakis, Digital Communications, New York: McGraw-Hill, Inc., Fifth ed. 2008.

[12] S. Boyd and L. Vandenberghe, Convex Optimization, Cambridge, U.K.: Cambridge Univ. Press, 2004.

[13] N. D. Sidiropoulos, T. Davidson, and Z.-Q. Luo, “Transmit beamforming for physical-layer multicasting,” IEEE Transactions on Signal Processing, vol. 54, no. 6, pp. 2239–2251, Jun. 2006.