Impedance Control of Non-Passive Bilateral Teleoperation Systems With Uncertain Dynamics

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Abstract— In this paper, synchronization of nonlinear teleoperation systems in the presence of non-passive and uncertain conditions in both the operator and the environment is studied. Despite existing approaches in the literature which are based on linear impedance models, the proposed approach in this paper has considered uncertain nonlinearities in the impedance model which results in better tracking performance in both sides. Simulation results for a pair of 3-DOF master/slave robot manipulators verifies the accuracy of the proposed strategy.

Index Terms—Teleoperation; rehabilitation; position tracking.

I. INTRODUCTION

The number of stroked patients has increased in the world, deprived the patients from activities of daily living and imposed many disabilities. As a solution to recovery after this disease, rehabilitation robots have come into account, which make the treatment faster and more accurate. The process also needs the supervision of an expert therapist to consider the performance of the patient under the assigned tasks. All of these goals can be achieved together by the help of a master/slave robot manipulators connected through communication network. The latter is called teleoperatory robotic systems, enabling the therapist to send the rehabilitation services from a long distance [1].

Generally, in teleoperation systems, the human operator transmits the commands through the master manipulator, the information and data is then submitted through communication channel (usually internet) from master to slave robot [2]. The slave robot follows the master and performs an action on the environment. A teleoperation system is designed to control the system transparency while preserving stability.

To make the system stable while the task of rehabilitation is performed accurately, a control strategy must get employed, providing both patient and therapist a deep sense of touch. As a result, both force and velocity are important to be tracked in such systems. In many systems, the subspace of velocity control is orthogonal to the subspace of force. For instance, consider a window washing robot. In such systems, one can independently design velocity and force controllers, as they have no adverse effect on each other (Fig. 1. b). However, in rehabilitation robots, velocity and force vectors

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³A. Doust Mohammadi is with the Faculty of Electrical Engineering, Amirkabir University of Technology, Tehran, Iran dad@aut.ac.ir are aligned. Therefore, force and velocity cannot be controlled independently (Fig. 1. a). A well-known approach to deal with such situations is the impedance control, providing ideal dynamic impedance behavior between the robot and environment.



Fig. 1. Force and Velocity Control

Therefore, by choosing a desired impedance, one can control the dynamic characteristic between the robot and human operator [3],[4].

The literature for design and implementation of impedance control strategy for a single robot manipulator is very rich (since 1989) [4], [5], [6]. However, impedance control received less attention in the concept of teleoperation[7],[8].In [7] a control algorithm is proposed to adapt the desired impedance in hard contact. In [8] variable damping and stiffness is used to enhance tracking performance to reduce the impact forces. It is worthy to mention that [7],[8] failed to include operator and environment in their stability analysis and did not consider their forces as an important parameter. In [9], position-force is employed to realize the impedance control strategy in teleoperation networks. However, there were some weak points in this work, as model of the manipulators are assumed to be linear and terminals are considered to be passive. This is not true in real world systems [10] as the robot dynamics are nonlinear and coupled. Also, passivity condition is very restrictive.

Subsequently, many researchers focused on applying impedance control scheme in teleoperation systems. Among them, [11],[12] are the most outstanding ones and will be discussed here in more details. In [11] the robot dynamics are nonlinear and three channel architecture is used to get a better performance. However, again the passivity condition

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Fig. 2. Telerehabilitation system

is not relaxed yet. The authors in [12] released the passivity conditions in both terminals of human and environment and introduced a novel force observes. However, in [11], [12] no method is involved for uncertainties in manipulators which makes the controller highly model based.

In this work, we consider a full nonlinear dynamics for master and slave robot manipulators having uncertainties in the manipulator model. As an approach we used a modified impedance control method to overcome this problem.

Position is sent towards the slave side and force and position data are sent back to master, providing a better tracking performance in both sides. Also, the controllers are designed such that they deal with model uncertainties.

Another contribution of this paper is that, the passivity condition is also relaxed by the help of a Lyapunov candidate function to show the stability of the closed loop teleoperation system in contact motion. It was shown that the states will be uniformly ultimately bounded even though the terminals of the teleoperation system is not necessarily passive. Finally, Simulation is performed on a pair of 3-DOF master/slave robot manipulators, to show the efficiency of the proposed strategy.

The rest of this paper is organized in the following. In Section II the model of teleoperation system is given. The proposed nonlinear controller strategy is given in Section III. The associated stability analysis and the behavior of the position coordination error are analyzed in Section IV. In Section V simulation results are given and finally conclusion and future works are discussed in Section VI.

II. DYNAMICS OF TELE OPERATION SYSTEMS.

This section includes the dynamical model of the nonlinear teleoperator in addition to some properties

A. Dynamic model of Master and Slave Robots

In this paper, we consider the dynamic model of the master and slave robot, defined in the cartesian space.

$$\begin{array}{l}
 M_m(x_m)\ddot{x}_m + C_m(x_m, \dot{x}_m)\dot{x}_m = f_m + f_h \\
 M_s(x_s)\ddot{x}_s + C_s(x_s, \dot{x}_s)\dot{x}_s = f_s - f_e
 \tag{1}$$

Here $x_m, x_s \in \mathbb{R}^{n \times 1}$ are the vectors of end effector positions, $\dot{x}_m, \dot{x}_s \in \mathbb{R}^{n \times 1}$ are the vectors of joint velocities, $\ddot{x}_m, \ddot{x}_s \in \mathbb{R}^{n \times 1}$ are the vectors of joint acceleration of the master and slave robots, $f_m, f_s \in \mathbb{R}^{n \times 1}$ are the vector of applied torques, $M_m(x_m), M_s(x_s) \in \mathbb{R}^{n \times n}$ are the positive definite inertia matrices and $C_m(x_m, \dot{x}_m), C_s(x_s, \dot{x}_s) \in \mathbb{R}^{n \times n}$ are the matrices of Centripetal and Coriolis torques. $f_h \in \mathbb{R}^{n \times 1}$ is the force vector applied to the master robot by the human operator and $f_e \in \mathbb{R}^{n \times 1}$ is the environmental

force vector applied to the environment by the slave robot. Some properties for nonlinear robot are listed as follows.

Property 1: The inertia matrix of a robot is a symmetric, positive definite and bounded as:

$$\gamma_{s1}I \le M_s(x_s) \le \gamma_{s2}I$$
$$\gamma_{m1}I \le M_m(x_m) \le \gamma_{m2}I$$

where $\gamma_{s1}, \gamma_{s2}, \gamma_{m1}, \gamma_{m2} > 0$.

Property 2: The inertia and Coriolis matrix are skewsymmetric describes as:

$$\dot{x}_{i}^{T}(M_{i}(x_{i}) - 2C_{i}(x_{i}, \dot{x}_{i}))\dot{x}_{i} = 0$$

B. Dynamic Model of Intraction Forces

In many works the interaction forces between the teleoperator and the forces that applied to the end-effector, are considered as a passive model, but this assumption is not true in reality. In this work, we assume that operator's force is modeled as a non-passive system and the slave contact is modeled a passive environment. The operator and the environment dynamics describes as:

$$f_e = \alpha_s r_s$$

$$f_h = \alpha_0 - \alpha_m r_m$$

where $\alpha_0, \alpha_m, \alpha_s$ are bounded positive constants, α_0 is a constant non-passive force that is applied by human operator. f_h is the human hand force that can be disported into the active and the passive parts. Also, dynamic model of environment can be active if described by non-positive real impedance.

and $r_s, r_m \in \mathbb{R}^{n \times 1}$ are new parameters that describes as:

$$r_s = \dot{x}_s + \lambda x_s$$
$$r_m = \dot{x}_m + \lambda x_m$$

where λ is a constant positive definite matrix.

III. CONTROLLER STRUCTURE

In this section, an impedance controller and a slidingmode-based impedance controller designed for the master and the slave, respectively. Using sliding mode controller helps when system is not perfectly known.

These controllers can be designed for nonlinear system with uncertain dynamic. Using impedance controller with terms that arise from the teleoperation helps to have a better tracking performance in presence of exerted force.

Moreover, an adaptive controller is utilized to compensate for model uncertainty of the master robot.

A. Impedance control for master

With an impedance control, a mass-spring-damper system is modelled by maintaining a dynamic relationship between force and position, velocity and acceleration. Mass and spring are energy storing elements and damper is an energy dissipating element. Suppose that the desired impedance for the master is following by

$$M_{dm}\ddot{x}_m + B_{dm}\dot{x}_m + K_{dm}(x_m - x_s) = f_h - f_e \qquad (2)$$

where M_{dm} , B_{dm} , K_{dm} , are desired inertia, damping, and stiffness, respectively. Impedance control approaches guarantee to control the energy exchange during interaction. From (2) can be calculated the acceleration as follows:

$$\ddot{x}_m = M_{dm}^{-1}(f_h - f_e - B_{dm}\dot{x}_m - K_{dm}(x_m - x_s))$$
(3)

The torque input of the master robot by using (3) are as follows:

$$f_m = \hat{M}_m [M_{dm}^{-1} (-B_{dm} \dot{x}_m - K_{dm} (x_m - x_s) + f_h - f_e)] + \hat{C}_{1m} \dot{x}_m - f_h \quad (4)$$

The master robot dynamic with impedance controller is obtained by substituting (4) into (1):

$$M_{dm}\ddot{x}_m + B_{dm}\dot{x}_m + K_{dm}(x_m - x_s) = f_h - f_e + \eta$$
 (5)

where $\eta = Y_m \dot{\theta}_m$, is the uncertainty in nonlinear dynamic of master robot. In stability analyze section we find an adaptation law to estimate this term in each moment.

B. Sliding mode control for slave

The main advantage of sliding mode control is its robustness, because the control can be switched between two states, it is not sensitive to parameter variation that enter to the controller. Therefore, using sliding mode controller is suitable for this aim.

In order to present the slave controller, define the masterslave position tracking error as $e_s = x_s - x_m \in \mathbb{R}^{n \times 1}$. A desired impedance equation for the slave specifies such that:

$$M_{ds}\ddot{e}_s + B_{ds}\dot{e}_s + K_{ds}e_s = -f_e \tag{6}$$

where M_{ds} , B_{ds} , K_{ds} are desired inertia, damping and stiffness, respectively. In order to ensure a desired closed-loop impedance and tracking of the master trajectory, a sliding surface for the slave controller is defined as,

$$S = \int M_{ds}^{-1} (M_{ds} \ddot{e}_s + B_{ds} \dot{e}_s + K_{ds} e_s + f_e)$$

= $\dot{e}_s + \int M_{ds}^{-1} (B_{ds} \dot{e}_s + K_{ds} e_s + f_e) = 0$ (7)

Slave controller drives the system trajectories to (7), then, (6) satisfies and the slave robot have the desired closed loop behavior at the end effector. The slave sliding mode controller is given as,

$$f_{s} = -\hat{M}_{s}[K_{g}sign(S) + M_{ds}^{-1}B_{ds}(\dot{x}_{s} - \dot{x}_{m}) + M_{ds}^{-1}K_{ds}(x_{s} - x_{m}) + M_{ds}^{-1}f_{e} - a_{x} - \hat{M}_{m}^{-1}\hat{C}_{1m}\dot{x}_{m}] + \hat{C}_{s}\dot{x}_{s} + f_{e}$$

where $K_g = k_g I \in {}^{n \times n}$. This control law comes out of a lyapunov stability analysis of the sliding mode dynamic and ensure that the sliding mode dynamic are stable.

Fig.3 shows a block diagram of the control teleoperation system with impedance and sliding mode control in force contact.

IV. STABILITY ANALYSIS

This section deals with the stability of the teleoperation system that includes master and slave interaction with the human operator and the environment.

First, we should show the stability of sliding mode controller by considering the lyapunov function as follows:

$$V_s = \frac{1}{2}S^T S$$

The time derivative of lyapunov along the trajectories of the slave :

$$\dot{V}_s = S^T \dot{S} = -S^T k_g sign(S) \le -k_g \left\| S \right\| \le 0$$

Therefore, stability of the slave ensure and the closed loop equation of the slave in (6). Stability of the slave is not related to passivity of the environment. Consider a positive semi-definite storage functional V as follows to show the stability of teleoperation system, according to (5), (6) that show closed loop equations:

$$V = r_m^T M_{dm} r_m + r_e^T M_{de} r_e + e_s^T (k\lambda + 2(B_{ds} - \lambda M_{ds})) e_s + x_m^T \lambda (2\alpha_m + B_{dm} - \lambda M_{dm}) x_m + 2\alpha_s x_s^T \lambda x_s + \tilde{\theta}_m^T \Gamma_m \tilde{\theta}_m$$

where $B_{ds} - \lambda M_{ds}, B_{dm} - \lambda M_{dm}$ define to be positive definite and $r_e \in {}^{n \times 1}$ are new parameters that describes based on e_s as:

$$r_e = \dot{e}_s + \lambda e_s = r_s - r_m$$

The time derivative of V is:

$$V = 2r_m^T M_{dm} \dot{r}_m + 2r_e^T M_{de} \dot{r}_e + 2e_s^T (k\lambda + 2(B_{ds} - \lambda M_{ds}))\dot{e}_s + 2x_m^T \lambda (2\alpha_m + B_{dm} - \lambda M_{dm})\dot{x}_m + 4\alpha_s x_s^T \lambda \dot{x}_s + 2\tilde{\theta}_m^T \Gamma_m \tilde{\theta}_m$$

$$= 2r_m^T (f_h - f_e + Y_m \tilde{\theta}_m + K_{dm} e_s - (B_{dm} - \lambda M_{dm})r_m + \lambda (B_{dm} - \lambda M_{dm})x_m) + 2r_e^T (-f_e - (B_{ds} - \lambda M_{ds})r_e + (\lambda B_{ds} - \lambda^2 M_{ds} - k_{ds})e_s) + 2ke_s^T \lambda \dot{e}_s + 2x_m^T \lambda (2\alpha_m + B_{dm} - \lambda M_{dm})\dot{x}_m + 4\alpha_s x_s^T \lambda \dot{x}_s + 2\tilde{\theta}_m^T \Gamma_m \tilde{\theta}_m$$

By using adaptation law that is described as:

$$\dot{\tilde{\theta}}_m = -Y_m^T r_m \Gamma_m^{-T}$$

Along the trajectories of system (5), (6) \dot{V} is given as:

$$\dot{V} = 2(r_m^T f_h - r_e^T f_e) + 2x_m^T \lambda (2\alpha_m + B_{dm} - \lambda M_{dm})\dot{x}_m + 4\alpha_s x_s^T \lambda \dot{x}_s - 2r_m^T (B_{dm} - \lambda M_{dm})r_m - 2r_e^T (B_{ds} - \lambda M_{ds})r_e + 2r_m^T K_{dm} e_s + 2r_m^T \lambda (B_{dm} - \lambda M_{dm})x_m - 2\lambda e_s^T (-\lambda B_{ds} + \lambda^2 M_{ds} + k_{ds})e_s - 2\dot{e}_s^T (-\lambda B_{ds} + \lambda^2 M_{ds} + k_{ds})e_s + 2e_s^T (k\lambda + 2(B_{ds} - \lambda M_{ds}))\dot{e}_s$$



Fig. 3. control structure

choosing $k\lambda = -\lambda B_{ds} + \lambda^2 M_{ds} + k_{ds}$, and after some manipulation, we get:

 $\dot{V} \leq 2\alpha_0(\dot{x}_m + \lambda x_m) - 2\alpha_m \|\dot{x}_m\|^2 - 2\alpha_m \lambda_{\min}^2(\lambda) \|x_m\|^2 -$

by choosing $K_{min} = \kappa - \kappa_m$ in which $\kappa_m = 2\alpha_s(1 + \alpha_m)$ $\lambda_{min}(\lambda)$) we have:

$$\dot{V} \le 2\alpha_0 \|\bar{x}\| - K_{\min} \|\bar{x}\|^2$$
$$\dot{V} \le 2\alpha_0 \|\bar{x}\|^2 - K_{\min} \delta \|\bar{x}\|^2 - K_{\min} (1-\delta) \|\bar{x}\|^2$$

the smallest and largest eigenvalues of the enclosed matrix.

We use Youngs quadratic inequality with $|a^Tb| \leq$ $(\varepsilon/2)|a|^2 + (1/2\varepsilon)|b|^2$ that holds for all $\varepsilon > 0$. Therefore we can obtain the following relationship:

 $-2\lambda_{\min}(k\lambda^2)\|e_s\|^2+2\dot{x}_m^T K_{dm}e_s+2\lambda x_m^T K_{dm}e_s$

$$2\dot{x}_m^T K_{dm} e_s \leq \lambda_{\max}(K_{dm}) \|\dot{x}_m\|^2 + \|e_s\|^2$$
$$2x_m^T \lambda K_{dm} e_s \leq \lambda_{\max}(\lambda K_{dm}) \|x_m\|^2 + \|e_s\|^2$$

Then we get,

$$\dot{V} \leq 2\alpha_{0}(\dot{x}_{m} + \lambda x_{m}) - 2\alpha_{m} \|\dot{x}_{m}\|^{2} - 2\alpha_{m}\lambda_{\min}^{2}(\lambda)\|x_{m}\|^{2}$$
$$-2\alpha_{s}\|\dot{x}_{s}\|^{2} - 2\alpha_{s}\lambda_{\min}^{2}(\lambda)\|x_{s}\|^{2} - 2\lambda_{\min}(B_{dm} - \lambda M_{dm})\|\dot{x}_{m}\|^{2}$$
$$-2\lambda_{\min}(B_{ds} - \lambda M_{ds})\|\dot{e}_{s}\|^{2} - 2\lambda_{\min}(\lambda(B_{ds} - \lambda M_{ds}))\|e_{s}\|^{2}$$
$$-2\lambda_{\min}(k\lambda^{2})\|e_{s}\|^{2} + \lambda_{\max}(K_{dm})\|\dot{x}_{m}\|^{2}$$
$$+\lambda_{\max}(\lambda K_{dm})\|x_{m}\|^{2} + 2\|e_{s}\|^{2}$$

A. Passive model

if $\alpha_0 = 0$ then the human environment is modeled as a passive mapping form force to velocity. By choosing following condition V is negative semi-definite and $\lim_{t\to 0} V(t)$ is existes and also is limited.

$$\alpha_m + \lambda_{\min}(B_{dm} - \lambda M_{dm}) \ge \lambda_{\max}(K_{dm})$$
$$\alpha_m \lambda_{\min}^2(\lambda) \ge \lambda_{\max}(\lambda K_{dm})$$
$$\lambda_{\min}(\lambda(B_{ds} - \lambda M_{ds})) + \lambda_{\min}(k\lambda^2) \ge 1$$

B. Non-passive model

When the human operator and the environment is modeled as a non-passive system it means that in this situation, by defined $\bar{x} = \begin{vmatrix} x_m & x_s & \dot{x}_m & \dot{x}_s \end{vmatrix}$ and choosing following condition

$$\lambda_{\max}(K_{dm}) = 2\alpha_m$$
$$\lambda_{\max}(\lambda K_{dm}) = 2\alpha_m \lambda_{\min}^2(\lambda)$$
$$\lambda_{\min}(k\lambda^2) = 1$$

$$\dot{V} \le 2\alpha_0 \|\bar{x}\|^2 - K_{\min}(1-\delta) \|\bar{x}\|^2 \ \forall \|\bar{x}\| \ge \frac{2\alpha_0}{K_{\min}\delta}$$

Since α_0 is assumed to be bounded, for large values of the norm of \bar{x} , $\dot{V} < 0$, $\forall \bar{x} \neq 0$.

Consequently, the master/slave trajectories are ultimately bounded and ensure boundedness of the force tracking error between the human force and the environmental force. Lower bound of norm \bar{x} can be controlled by changing the impedance parameters.

V. SIMULATION RESULTS

In this section, we test the proposed controller structure on a pair of 3DOF master/slave robot manipulators. The model contains dynamic uncertainties. The $m_1 = m_2 = m_3 = 1kg$, $l_1 = l_2 = 0.8m$, $l_3 = 0.3m$. The environment is modelled as a spring-damper system and the non-passive operator modelled as a spring-damper system with an additional force term that is intended sin(t). Effect of the force feedback from the environment and operator are important in achieving these results.

Simulation results are presented in fig. 4-7. We see that these simulations demonstrate stability as well as good tracking properties of the proposed algorithm. Fig. 4 shows that the slave tracks the master in each direction in presence of non-passivity in constract to the operator and the environment.

Fig. 5 compares the tracking error in this algorithm with [12]. It can be seen that the proposed algorithm reduces the error and this will help to achieve better results in the implementation.

Fig. 7 shows that by the help of this algorithm and in presence of non-passivity in both side by choosing negative impedance for environment and it shows that this algorithm responds well for each conditions.



Fig. 4. Position tracking with non-passive operator



Fig. 5. Tracking error



Fig. 6. Velocity tracking



Fig. 7. Position tracking with non-passivity in both side

VI. CONCLUSION

In this paper, a control law for bilateral nonlinear teleoperation systems was developed which guaranteed ultimate boundedness of master/slave trajectories in the presence of non-passive terminal and uncertain nonlinearities in both sides. Under the proposed strategy, the master position information was sent to the slave and the position and force information of the slave were sent back to the master. Hence, an adaptation law was utilized to deal with uncertainties in the impedances models and to achieve a good tracking performance. However, the assosiated chattering phenomenon in first order sliding mode control, degrades the system performance. To overcame this phenomenon, future research will focus on modifying the control law by substituting the controller at the slave side with a second-order sliding mode controller and implementing the results on an experimental teleoperation system.

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