Unknown input estimation by applying extended kalman filter based on unknown but bounded uncertainties

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***Abstract*—In this paper a new input estimation method is proposed for a class of nonlinear stochastic systems in the presence of time dependent unknown inputs, when the system states and process noises are unknown but bounded. In this study, a new augmented state vector is constructed by adding unknown inputs as a new state to the original state vector. Then a recursive algorithm based on unknown but bounded (UBB) uncertainty is developed , that unlike the Bayesian models which consider the state estimate as a single vector, produces a time-varying set of state estimates that contains the system's true state. The particular sets to be discussed, are ellipsoids. The proposed method doesn't need any unknown input detection stage procedure and covariance resetting that are necessary in previous works. At last, efficiency of the proposed method is shown in a numerical simulation for a nonlinear system.**

Keywords. extended Kalman filter; unknown input estimation; unknown but bounded uncertainty

I. Introduction

State estimation play an important role in process control, performance monitoring, traffic control, robotics and defense [1].

The well known approach to solve state estimation problem is Kalman filter (KF) [2], it is an optimal filter for linear systems, but most of the systems in practical applications are nonlinear. Thus many alternative filtering schemes, such as extended Kalman filter (EKF), unscented Kalman filter (UKF) [3], cubature Kalman filter [4], interactive multiple model Kalman filter [5] and particle filter (PF) [6] have been developed to improve capability of handling nonlinearities and uncertainties.

However, in most of the practical situations, systems are subject to unknown constant or unknown random biases, modeling errors or system uncertainties.

The key point in this problem is input detection and estimation (IDE). Most familiar filters which have been dealt with this problem are introduced as follows. Input estimation (IE) method for detecting and obtaining the unknown inputs of a system has been proposed in [7]. It detects the existence of unknown input by calculating the measurement residual over a finite detection window, then estimates the unknown input by least squares algorithm, and finally estimates the system states using the estimated input. So, delay phenomenon and large state estimation error is inevitable in this approach. In [8],[9] modified IE (MIE) and enhanced IE (EIE) were developed respectively to overcome the deficiencies of the original IE. The idea of state augmentation has been proposed In [10]. This approach estimates observation bias along with system states by including the bias parameters as additional state variables. In [11] unknown input is considered as an additive state in the original state vector, and then a Kalman filter is developed for the model. However, not only this method is just applicable for linear systems, but also it needs covariance resetting whenever a change in unknown input happens. Recently, two stage kalman filter (TKF) has been proposed for linear systems with unknown constant biases [12] or random biases [13]. In [14] and [15] general two stage EKF (GTEKF) and two stage UKF (TUKF) have been proposed respectively, for nonlinear systems in the presence of unknown inputs.

However, the basic idea for the estimation procedure in these mentioned algorithms and all traditional approaches is to combine knowledge of the system dynamics with the noisy observations to calculate a time-varying single vector, the actual estimate is a set in state space which contains the true state of the system rather than a single vector. In other words, determination of the smallest estimate set is theoretically straightforward but computationally infeasible for most practical problems [16]. Also it should be noted that, all mentioned algorithms have assumed that we know probability density function (PDF) of system process and measurement noises, but there is no precise knowledge about them, because of inherent time dependency of uncertainties.

Motivated by the above discussion, we want to develop an augmented model to omit unknown input detection stage procedure, in order to overcome the delay phenomenon in input estimation methods and design recursive augmented UBB-EKF (AUBB-EKF) that yields a bounding ellipsoid which contains the true state, in real time.

The paper is organized as follows. In section II, we review a class of nonlinear stochastic systems in discrete time and present the unknown input estimation problem. An augmented model will be proposed by adding the unknown input term as a new state to the original state vector, in section III. AUBB-EKF is developed for nonlinear systems with unknown inputs in Section IV. The effectiveness of the introduced method is shown in section V. Finally some conclusions are given in section VII.

II. Statement of the problem

Consider the general class of the nonlinear systems with unknown inputs. The plant under consideration

$$\begin{aligned} X(K+1) &= f\big(X(K), u(k)\big) + w(k) \\ Z(k) &= h\big(X(k), u(k)\big) + v(k) \end{aligned} \tag{1}$$

Where $X(k) \in R^{n_1}$, $Z(k) \in R^{n_2}$ and $u(k) \in R^{n_3}$ are the system states, measured output and unknown input, and the dimension of X, Z and u are $n_1 \times 1$, $n_2 \times 1$ and $n_3 \times 1$ respectively.

Assumption 1. $w(k)$ and $v(k)$ denote the process and measurement additive noises, respectively. Where both of them are white unknown but bounded processes and $X(0)$ is an unknown-but-bounded initial state vector such that

$$X(0) \in \Omega_X(0), \quad v(k) \in \Omega_v(k), \quad w(k) \in \Omega_w(k) \tag{2}$$

where

$$\begin{aligned} \Omega_{X(0)} &= \{X(0) : X(0)^T \psi^{-1} X(0) \leq 1\} \\ \Omega_w(k) &= \{w : w^T Q^{-1} w \leq 1\} \\ \Omega_v(k) &= \{v : v^T R^{-1} v \leq 1\} \end{aligned} \tag{3}$$

That ψ, Q and R determine the size and shape (shaping matrix) of initial condition, process and measurement noise ellipsoids, respectively.

Assumption 2. $\Omega_X(0)$, $\Omega_v(k)$ and $\Omega_w(k)$ are sets whose size and shape can change with time. The sets are usually considered as ellipsoids. It should be noted that $X(0), v(k), w(k)$ are uncorrelated. To be more precise

$$\begin{gathered} E\{X(0)w^T(.)\} = 0, \quad E\{X(0)v^T(.)\} = 0, \\ E\{w(.)v^T(.)\} = 0 \end{gathered} \tag{4}$$

III. An augmented model for nonlinear systems with unknown inputs

In this section a new model for nonlinear system (1) is proposed. Using this model, we do not need any unknown input detection stage procedure.

$$X_{Aug}(k+1) = \Phi_{Aug} X_{Aug}(k) + F\big(X_{Aug}(k)\big) + w_{Aug}(k) \tag{5}$$

Where :

$$\begin{gathered} X_{Aug}(k) = \begin{bmatrix} X(k) \\ u(k) \end{bmatrix}, \Phi_{Aug} = \begin{bmatrix} 0_{n_1 \times n_1} & 0_{n_1 \times n_3} \\ 0_{n_3 \times n_1} & I_{n_3 \times n_3} \end{bmatrix}, \\ F\big(X_{Aug}(k)\big) = \begin{bmatrix} f(X_{Aug}(k)) \\ 0_{n_1 \times 1} \end{bmatrix} \end{gathered} \tag{6}$$

The new augmented system process noise is

$$w_{Aug}(k) = G_{Aug} w(k) \tag{7}$$

where

$$G_{Aug} = \begin{bmatrix} I & 0 \end{bmatrix}^T \tag{8}$$

Remark 1. Regarding to bounding elements of G_{Aug}, W_{Aug} remains an UBB process.
According to the assumptions 1 and 2, it is obvious that

$$E\{w_{Aug}(k)\} = 0, \quad E\{w_{Aug}(k) X_{Aug}^T(k)\} = 0 \tag{9}$$

Thus system (1) can be rewritten as follows

$$\begin{aligned} X_{Aug}(k+1) &= \Phi_{Aug} X_{Aug}(k) + F\big(X_{Aug}(k)\big) + w_{Aug}(k) \\ Z_{Aug}(k) &= h\big(X_{Aug}(k)\big) + v(k) \end{aligned} \tag{10}$$

Remark 2. Unlike the most unknown input estimation methods which detection delay causes large estimating errors, the proposed model omits input detection process. So, the concurrent estimation of states and unknown inputs eliminates the delay phenomenon.

IV. Main results

In this section we aim to develop AUBB-EKF algorithm using the augmented model (10). Since there is nonlinear terms in the system process and measurement model, we should compute the Jacobian matrices to facilitate the later developments.
Compute the process model Jacobians as

$$F(k) = \Big[\nabla_{X_{Aug}(k)} \Big(F^T\big(X_{Aug}(k)\big)\Big)\Big]\Big|_{X_{Aug}(k) = \hat{X}_{Aug}(k|k-1)} \tag{11}$$

Compute observation model Jacobian as

$$H(k) = \Big[\nabla_{X_{Aug}(k)} \Big(h^T\big(X_{Aug}(k)\big)\Big)\Big]\Big|_{X_{Aug}(k) = \hat{X}_{Aug}(k|k-1)} \tag{12}$$

The shaping matrix of the augmented system process noise is obtained as

$$\begin{aligned} Q_{Aug}(k) &= cov\big\{w_{Aug}(k) w_{Aug}^T(k)\big\} \\ &= G_{Aug} Q(k) G_{Aug}^T \end{aligned} \tag{13}$$

The linearized model of the system is

$$\begin{aligned} X_{Aug}(k+1) &= F^*(k) X_{Aug}(k) + w_{Aug}(k) \\ Z_{Aug}(k) &= H(k) X_{Aug}(k) + v(k) \end{aligned} \tag{14}$$

where

$$F^*(k) = \Phi_{Aug} + F(k) \tag{15}$$

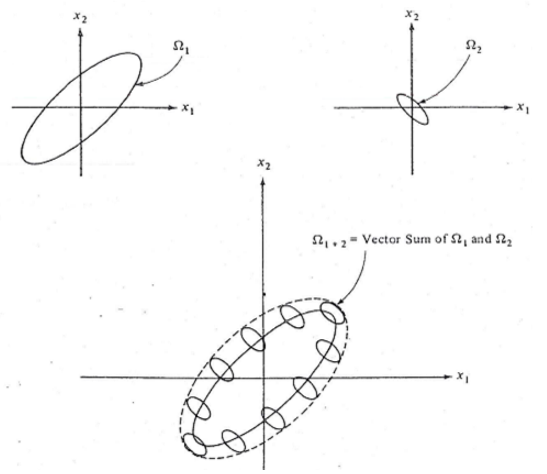

Fig. 1: Vector sum of two convex sets [16]

A. **Augmented unknown but bounded EKF**

As mentioned before, noise processes and initial condition are in ellipsoid form. According to set theory [16], an ellipsoid can be represented by a support function. Support function of a closed convex set is defined by

$$s(\eta) = max\{x^T\eta\}, s.t. \eta\eta^T = 1 \tag{16}$$

The set can be expressed as

$$\Omega = \{x | x^T\eta \leq s(\eta) \quad (\forall\eta), s.t. \eta^T\eta = 1\} \tag{17}$$

If Ω is an ellipsoid, it can be expressed as

$$\Omega = \{x | x^T\Gamma^{-1}x \leq 1\} \tag{18}$$

Where Γ determines the shape and size of it.
The vector sum (Minkoski sum) of two convex sets, is defined by

$$\Omega_{1+2} = \{x | x = x_1 + x_2, \ \forall x_1 \in \Omega_1 \ \forall x_2 \in \Omega_2\} \tag{19}$$

The vector sum operation is shown in Fig.1 According to definition of support function (16), support function of Ω_{1+2} is

$$s_{1+2}(\eta) = s_1(\eta) + s_2(\eta) \tag{20}$$

The support function of the ellipsoid Ω (18) is represented by

$$s(\eta) = \sqrt{\eta^T\Gamma\eta} \tag{21}$$

The vector sum of two ellipsoids is

$$s_{1+2}(\eta) = \sqrt{\eta^T\Gamma_1\eta} + \sqrt{\eta^T\Gamma_2\eta} \tag{22}$$

It is obvious that the vector sum of two ellipsoids may be not an ellipsoid. In order to come over this problem, we can use a bounding ellipsoid which contains the Ω_{1+2}.
A special case of Holder's inequality is

$$(1-\beta)^{-1}b_1^2 + \rho^{-1}b_2^2 \geq (b_1 + b_2)^2, \quad 0 \leq \rho \leq 1, \ 0 \leq \beta \leq 1 \tag{23}$$

Substituting $s_1(\eta)$ and $s_2(\eta)$ into (23) implies that

$$s_b(\eta) = \sqrt{\eta^T\big((1-\beta)^{-1}\Gamma_1 + \rho^{-1}\Gamma_2\big)\eta} \tag{24}$$

Regarding to (14) and (24) , support function and bounding ellipsoid of Z_{Aug} will be

$$\begin{aligned} s_{Z_{Aug},b}(\eta) &= \sqrt{\eta^T\big((1-\beta)^{-1}R + \rho^{-1}H\Sigma H^T\big)\eta} \\ \Omega_{Z_{Aug},b} &= \{Z_{Aug} | Z_{Aug}^T(H\tilde{\Sigma}H^T + \tilde{R})^{-1}Z_{Aug} \leq 1\} \end{aligned} \tag{25}$$

Similarly, bounding ellipsoid of $X_{Aug}(k+1)$ will be

$$\begin{aligned} \Omega_{x,b}(k) &= \{x | x^T\Gamma^{-1}(k)x \leq 1\} \\ \Gamma(k+1) &= \frac{1}{1-\beta(k)}F^*(k)\Gamma(k)F^{*T}(k) + \tilde{Q}_{Aug}(k) \end{aligned} \tag{26}$$

Solution of AUUB filtering problem, consists two phases
1) Calculate ellipsoid $\Omega_{\hat{X}_{Aug}}(k+1|k)$ such that

$$vectorsum\big[\Omega_{F^*\hat{X}_{Aug}}(k|k), \Omega_{w_{Aug}}(k)\big] \subset \Omega_{\hat{X}_{Aug}}(k+1|k) \tag{27}$$

2) Calculate ellipsoid $\Omega_{\hat{X}_{Aug}}(k+1|k+1)$ such that

$$\Omega_{\hat{X}_{Aug}}(k+1|k+1) \subset \big[\Omega_{\hat{X}_{Aug}} \cap \Omega_{X_{Aug}|Z_{Aug}}(k+1)\big] \tag{28}$$

Regarding to the above statements, AUBB-EKF can be written in form

$$\begin{aligned} &\Omega_{\hat{X}_{Aug}}(k+1|k+1) \\ &\quad = \Big\{X_{Aug} | \big[X_{Aug} - \hat{X}_{Aug}(k+1|k+1)\big]^T \\ &\quad \times \Sigma^{-1}(k+1|k+1)\big[X_{Aug} - \hat{X}_{Aug}(k+1|k+1)\big] \leq 1\Big\} \end{aligned} \tag{29}$$

$$\begin{aligned} &\hat{X}_{Aug}(k+1|k+1) \\ &\quad = F^*(k)\hat{X}_{Aug}(k|k) + K(k+1) \\ &\quad \times \big(Z_{Aug}(k+1) - H(k+1)F^*(k)\hat{X}_{Aug}(k|k)\big) \end{aligned} \tag{30}$$

$$K(k+1) = \Sigma_b(k+1|k+1)H^T(k+1)\tilde{R}^{-1}(k+1) \tag{31}$$

$$\begin{aligned} &\Sigma_b(k+1|k+1) \\ &\quad = \tilde{\Sigma}(k+1|k) - \tilde{\Sigma}(k+1|k)H^T(k+1) \\ &\quad \times \big[\tilde{R}(k+1) + H(k+1)\tilde{\Sigma}(k+1|k)H^T(k+1)\big]^{-1} \\ &\quad \times H(k+1)\tilde{\Sigma}(k+1|k) \end{aligned} \tag{32}$$

$$\Sigma(k+1|k+1) = \big(1 - \delta^2(k+1)\big)\Sigma_b(k+1|k+1) \tag{33}$$

$$\Sigma(k+1|k) = \frac{1}{1-\beta(k)}F^*(k)\Sigma(k+1|k)F^*T(k) + \tilde{Q}_{Aug} \tag{34}$$

$$\tilde{\Sigma}(k+1|k) = \frac{1}{1-\rho(k+1)}\Sigma(k+1|k) \tag{35}$$

$$\begin{aligned} \delta^2(k+1) &= \big[Z_{Aug}(k+1) - H(k+1)F^*(k)\hat{X}_{Aug}(k|k)\big]^T \\ &\quad \times \big\{H(k+1)\tilde{\Sigma}(k+1|k)H^T(k+1) + \tilde{R}(k+1)\big\}^{-1} \\ &\quad \times \big[Z_{Aug}(k+1) - H(k+1)F^*(k)\hat{X}_{Aug}(k|k)\big] \end{aligned} \tag{36}$$

Where

$$\begin{aligned}&\tilde{R}(k)=\frac{1}{\rho(k)}R(k),\quad \tilde{Q}_{Aug}(k)=\frac{1}{\beta(k)}Q_{Aug}(k)\\&0\le\rho(k)\le1,\quad 0\le\beta(k)\le1\\&\hat{x}(0|0)=0,\quad \Sigma(0|0)=\psi\end{aligned}\tag{37}$$

Remark 3. $\hat{X}_{Aug}$ is the center of bounding ellipsoid estimate set $\Omega_{\hat{X}_{Aug}}$, and Σ determines the size and shape of it. So, AUUB filtering model yields an estimate set. If we consider $\hat{X}_{Aug}$ as the best estimate of system state, and the Σ as covariance of error, it will be similar to Bayesian uncertainty based estimation.

Remark 4. Equations (29)-(34) denote estimated set, estimated system states, Kalman gain, maximum estimation error, estimation error and one step ahead prediction error respectively.

Remark 5. AUBB-EKF is similar to Bayesian based EKF, except $\beta(k),\rho(k)$ and $\delta^2(k+1)$ terms. Equation (33) shows that $\delta^2(k+1)$ makes the shaping matrix, a function of observations. So, unlike the Bayesian approach we can't calculate estimation error offline, but the maximum error estimation can be calculated independent of observations from (32).

V. Simulation results

In this section, the performance of the filtering algorithm developed in this paper is demonstrated. For nonlinear filtering with unknown inputs, to show the efficiency of our proposed algorithm in comparison with two existing algorithm, MIE and two stage EKF (TEKF), we consider the target scenario considered in [17].

Consider an object which is launched from one point on Earth to another point along a ballistic flight. The kinematics of the ballistic object in the reentry phase is derived under the following hypotheses. The forces acting on the target are gravity, and drag. The effects of centrifugal acceleration, Coriolis acceleration, wind, lift force, and spinning motion are ignored, due to their small effect on the trajectory.

The target motion and measurement equation is described by the following nonlinear discrete-time model with unknown inputs and multiplicative noises.

$$\begin{gathered}X(k+1)=\Psi\big(X(k)\big)+C(k)u(k)+G(k)w(k)\\Z(k)=H(k)X(k)+v(k)\end{gathered}\tag{38}$$

where

$$\Psi\big(X(k)\big)=F(k)X(k)+G(k)f\big(X(k)\tag{39}$$

$F(k),G(k),H(k),C(k)$ are defined as follows

$$\begin{gathered}F=\begin{bmatrix}1&T&0&0\\0&1&0&0\\0&0&1&T\\0&0&0&1\end{bmatrix},G=\begin{bmatrix}\frac{T^2}{2}&T&0&0\\0&0&\frac{T^2}{2}&T\end{bmatrix}^T\\H=\begin{bmatrix}1&0&0&0\\0&0&1&0\end{bmatrix},C=\begin{bmatrix}\frac{T^2}{2}&T&0&0\\0&0&\frac{T^2}{2}&T\end{bmatrix}^T\end{gathered}\tag{40}$$

In the above, T is the sampling time between two successive measurements.

The drag is a force directed in opposition to the target speed and with an intensity equal to $\frac{1}{2}(\frac{g}{\beta})\rho v^2$ being: g the gravity acceleration, β the ballistic coefficient, ρ the air density (typically it is an exponentially decaying function of height, $\rho=c_1e^{-c_2y}$ where $c_1=1.227, c_2=1.093\times10^{-4}$, for $y<9144m$, and $c_1=1.754, c_2=1.49\times10^{-4}$ for $y\ge9144m$ and v the module of target velocity. In terms of state vector components, the drag is

$$\begin{aligned}f\big(X(k)\big)=&-\frac{1}{2}\frac{g}{\beta}\rho\big(y(k)\big)\big(\dot{x}^2(k)+\dot{y}^2(k)\big)\\&\times\begin{bmatrix}\cos\Big(\arctan\big(\frac{y(k)}{x(k)}\big)\Big)\\\sin\Big(\arctan\big(\frac{y(k)}{x(k)}\big)\Big)\end{bmatrix}\end{aligned}\tag{41}$$

We could simplify the above formula by exploiting the following equalities

$$\begin{cases}\cos\Bigg(\arctan\Big(\frac{y(k)}{x(k)}\Big)\Bigg)=\frac{x}{\sqrt{x^2+y^2}}\\\sin\Bigg(\arctan\Big(\frac{y(k)}{x(k)}\Big)\Bigg)=\frac{y}{\sqrt{x^2+y^2}}\end{cases}\tag{42}$$

Then

$$f\big(X(k)\big)=-\frac{1}{2}\frac{g}{\beta}\rho(y(k))\sqrt{\dot{x}^2(k)+\dot{y}^2(k)}\begin{bmatrix}\dot{x}(k)\\\dot{y}(k)\end{bmatrix}\tag{43}$$

At first we have to calculate Jacobian matrixes.

By using the Taylor series expansion around $\hat{X}(k|k)$

$$F_j(k)=\Big[\nabla_{X_{Aug}(k)}\Big(f^T\big(X_{Aug}(k)\big)\Big)\Big]\tag{44}$$

$$\begin{aligned}&F[1,1]=0,F[2,1]=0\\&F[1,2]=-0.5\frac{g}{\beta}\rho\big(y(k)\big)\frac{2\dot{x}^2(k)+\dot{y}^2(k)}{\sqrt{\dot{x}^2(k)+\dot{y}^2(k)}}\\&F[1,3]=0.5\frac{g}{\beta}c_2\rho\big(y(k)\big)\dot{x}(k)\sqrt{\dot{x}^2(k)+\dot{y}^2(k)}\\&F[1,4]=F[2,2]=-0.5\frac{g}{\beta}\rho\big(y(k)\big)\frac{2\dot{x}^2(k)\dot{y}^2(k)}{\sqrt{\dot{x}^2(k)+\dot{y}^2(k)}}\\&F[2,3]=0.5\frac{g}{\beta}c_2\rho\big(y(k)\big)\dot{y}(k)\sqrt{\dot{x}^2(k)+\dot{y}^2(k)}\\&F[2,4]=-0.5\frac{g}{\beta}\rho\big(y(k)\big)\frac{\dot{x}^2(k)+2\dot{y}^2(k)}{\sqrt{\dot{x}^2(k)+\dot{y}^2(k)}}\end{aligned}\tag{45}$$

In the example we consider the unknown inputs as $[0g\quad 0g]^T$ for $t\le99s$ and for $t>99s$ target begins to maneuver. Acceleration in x direction is considered as a harmonic type unknown input with frequency of 50 Hz and magnitude of $7g$, between 200-300 seconds. Acceleration in y direction is considered as a pulse type unknown input with magnitude $9g$ between 60-200 seconds. Where $g=9.8\ m/s^2$, $T=1s$ and the covariance matrix of the system process and measurement noises are $R=10e{-}3I_{2\times2}, Q=10e{-}3I_{2\times2}$. The initial state and estimate vector and the corresponding covariance matrix are chosen to be $X(0)=[500\quad -50\quad 200\quad 100]^T, \hat{X}(0)=[1\quad 1\quad 1\quad 1]^T, P(0|0)=10I_{4\times4}$.

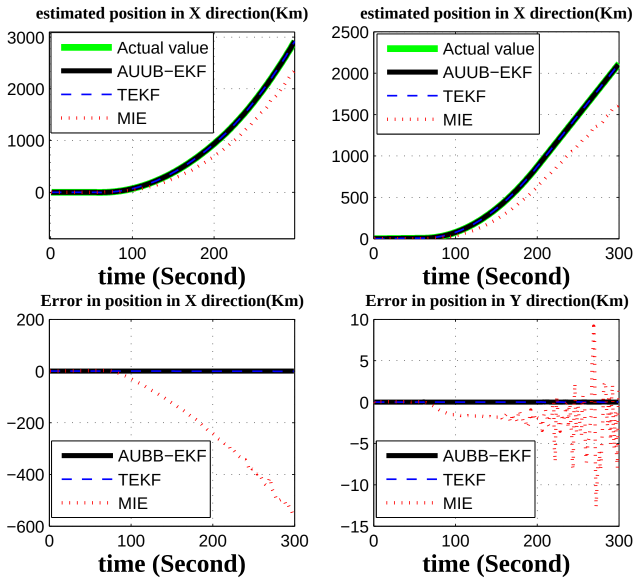

Fig. 2: The actual and estimated positions and their relevant estimation errors

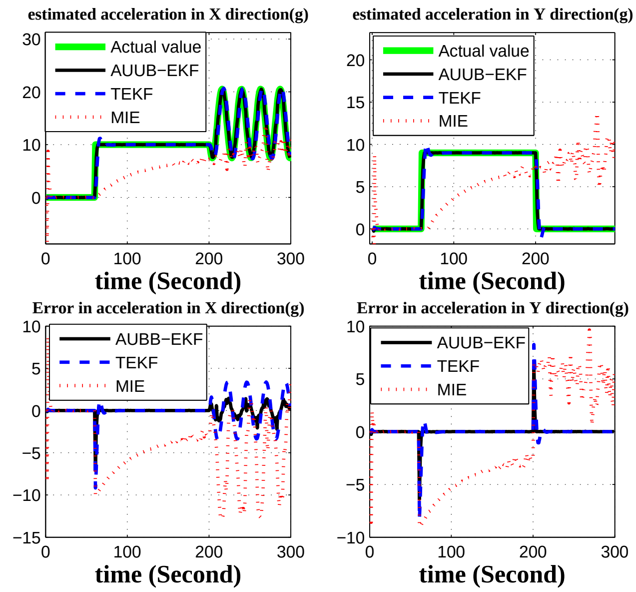

Fig. 4: The actual and estimated accelerations and their relevant estimation errors

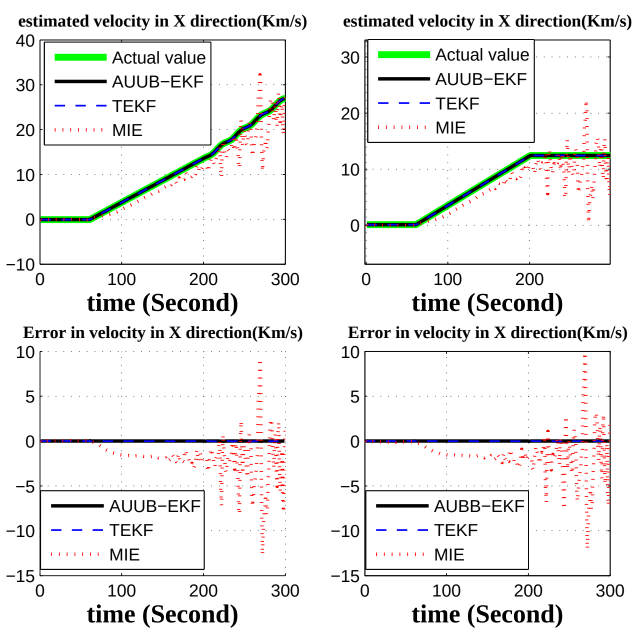

Fig. 3: The actual and estimated velocities and their relevant estimation errors

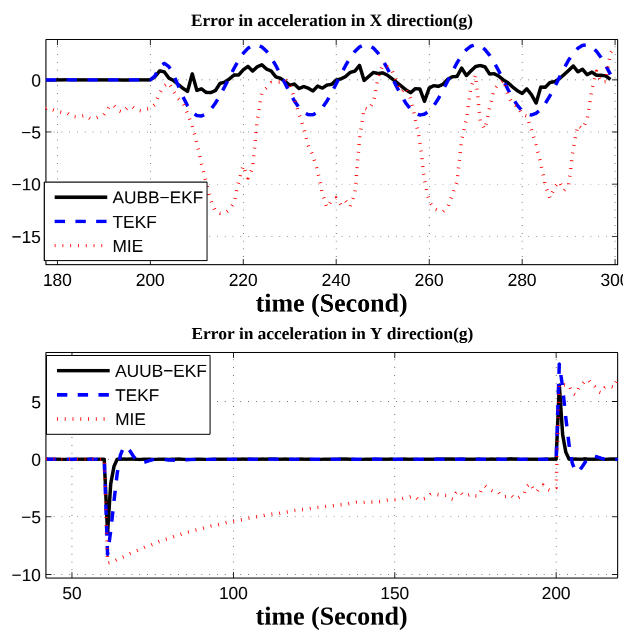

Fig. 5: The actual and estimated accelerations and their relevant estimation errors (magnified)

VI. Discussions

Fig. 2, Fig. 3, Fig. 4 and Fig. 5 show the performance comparisons of MIE, TEKF and the proposed AUBB-EKF method. As we discussed in the introduction, the MIE method calculate the measurement residual by using the technology of sliding window to detect the abrupt occurrence of unknown input term (acceleration). After that, estimates the unknown input and system states. It is clear from the results that this approach can't detect the onset time of maneuver real-time, and has large state estimation errors. Unlike MIE, TEKF can detect the change of unknown input efficiently. This approach estimate non-harmonic inputs with an overshoot, also it is obvious that, TEKF can't estimate harmonic ones. This defect can be observed from Fig. 5, which indicates that TEKF track the unknown input with a bias. On the contrary, AUBB-EKF

TABLE I: Monte-Carlo simulation results for 200 run at t=100*T (ignoring initial transitions)

Error	AUBB-EKF	TEKF	MIE
X-position (km)	3.08e-05	3.66e-05	1.12
Y-position (km)	1.58e-04	2.71e-04	1.11
X-velocity (km/s)	2.16e-07	5.15e-07	0.1
Y-velocity (km/s)	4.80e-08	1.91e-07	0.046
X-acceleration (harmonic) (g)	0.0225	0.0484	4.14
Y-acceleration (pulse) (g)	5.74e-04	0.003	0.41

method, doesn't have any limitation on the type of the unknown input and it can estimate both harmonic and non-harmonic inputs with minimum steady state error and without overshoot. Moreover due to the fact that we have eliminated the detection stage process. So, it detects the occurrence of unknown input in real time and produces much less position and velocity tracking errors.

Table 1, shows the performance of MIE, TEKF and the proposed AUBB-EKF method based on the indexes of position, velocity and acceleration root mean squared errors $(RMSE)$ which computed by 200 Monte-Carlo runs. The results indicate that, AUBB-EKF method has the best performance in all of the scenarios and can be more applicable to the practical systems. Bayesian model considers a little chance for states to become unbounded. Thus, TEKF has lower performance respect to AUUB-EKF. On the other hand, the MIE method has the worst results, because of delay phenomenon.

VII. Conclusion

In this paper, we have made one of the first few attempts to design the augmented unknown but bounded EKF for nonlinear stochastic systems with time dependent unknown inputs. Unlike the most existing approaches which estimate a single estimate vector of system states, the proposed method calculates a bounding ellipsoid estimate set which contains the true state. Moreover, delay phenomenon in input estimation approaches has been solved by augmenting unknown inputs with the original state vector. Finally the effectiveness of the developed method has been shown in a numerical simulation example.

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