Performance Investigation of the Modified Costas Carrier Recovery Loop for a QPSK Modulated Signal in the Presence of Co-Channel Interference

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***Abstract*—One of the important challenges in communication systems is drift in the carrier frequency of received signal due to the clock skew and Doppler effect. Every communication receiver requires an automatic frequency control (AFC) loop to estimate the carrier drift in the received signal. One of the well-known AFC loops in communication systems is the Costas loop. In this paper a modified version of Costas loop for binary phase-shift keying (BPSK) modulation is presented which is capable to estimate the carrier frequency of the quadratic phase-shift keying (QPSK) modulation and therefore significantly simplifies practical implementation. The performance of the modified Costas loop is investigated for the carrier recovery of a QPSK signal in the presence of a co-channel interference. Both simulations and analytical results illustrate that the modified Costas loop may estimate the carrier frequency drift of received signal when the desired signal is stronger than interference.**

***Index Terms*—Carrier recovery, Co-channel interference, AFC loops, Costas loop, QPSK modulation**

I. Introduction

In every digital communication system, the carrier frequency should be determined in order to demodulate the received signal. The performance of the carrier recovery loop directly affects the performance of the receiver [1]. Phased-lock loops (PLL) are widely used in coherent demodulator to control the carrier frequency [2]. Although coherent demodulators overcome the presence of noise in the received signal, they have weak performance in the presence of frequency drift in the received signal carrier [2]. Consequently, automatic frequency control (AFC) loops are required which have robust performance in the presence of link disturbances [3]. In coherent demodulators, an AFC loop is used in conjunction with PLL to perform the carrier recovery in the received signal [3], [4].

Three blocks are used to configure an AFC loop. The first block is a phase detector generating an error voltage in the cases that there is a phase/frequency mismatch in the receiver local oscillator and received signal. The error voltage generated by the phase detector is filtered by a loop filter. The output voltage of the loop filter drives a voltage controlled oscillator (VCO). Whenever the error voltage becomes zero, the VCO is locked on the carrier frequency of received signal [4], [5]. Squaring loop and Costas loop are the most well-known AFC loops [6]. In the squaring loop, the received signal should be squared in order to recover the carrier frequency [6]. Implementation of the squaring loop is a difficult task from circuit point of view [7]. The Costas loop avoids this problem and recovers the carrier frequency of binary phase-shift keying (BPSK) signals and quadratures phase-shift keying (QPSK) signals without requiring a squaring loop [7]. Hence, Costas loop has many application in wireless communication and Global Positioning System [8].

In this paper, the performance of modified Costas loop in the presence of co-channel interference for QPSK signal is analyzed. A squaring block is added to conventional Costas loop. This squaring block works at low frequency, and so its implementation is not difficult task. With this modification, conventional Costas loop presented for BPSK signal [7], can be used for carrier recovery of QPSK signal. However, it only can be used for offline signals. In comparison with Costas loop presented for QPSK signal in [6] and [7], modified Costas loop is much simpler to implement. Due to the Doppler effect and drift in the frequency of the oscillators in transmitter, the carrier of the desired signal has unknown drift. We investigate the performance of the modified Costas loop in the presence of interference. Simulation results are further presented to verify our analytical analysis.

This paper is organized as follows. Section II presents the system model of the received signal in the presence of interference. In section III structures of the Costas loop and modified Costas loop are described. Section IV describes the interference effect on the modified Costas loop performance. Simulation results are provided in section V. Finally, section VI is allocated to conclusions.

II. System Model

In our system we assume that both the desired signal and interference use QPSK modulation. For simplicity, we obtain the error voltage $e(t)$ in Fig. 1, in the absence of additive noise. The received signal contains both the desired signal and interference and is given as

$$\begin{aligned} r(t) &= S(t) + I(t) \\ &= A_s \sum_k b_k p\left(t - kT_B\right) \cos\left(2\pi\left(f_c + f_{d_s}\right)t\right) \\ &+ A_s \sum_k c_k p\left(t - kT_B\right) \sin\left(2\pi\left(f_c + f_{d_s}\right)t\right) \\ &+ A_i \sum_k d_k p\left(t - kT_B\right) \cos\left(2\pi\left(f_c + f_{d_i}\right)t\right) \\ &+ A_i \sum_k e_k p\left(t - kT_B\right) \sin\left(2\pi\left(f_c + f_{d_i}\right)t\right) \end{aligned} \quad (1)$$

where A_s and A_i denote the amplitudes of desired signal and interference, respectively. b_k and c_k determine the inphase and quadrature bits of desired signal. Furthermore, d_k denotes the inphase bit of the interference and e_k denotes quadrature bits of interference. $p(t)$ stands for the pulse shaping of the signal which is assumed as a raised cosine pulse to ensure the zero intersymbol interference conditions. The carrier frequency is assumed equal to f_c. Furthermore, f_{d_s} denotes the drift in the carrier frequency of the desired signal and f_{d_i} denotes the drift in the carrier frequency of the interference, both with respect to nominal values. In the coherent demodulation schemes, the exact carrier frequency is determined from the received signal and the frequency of the sinusoid at the output of VCO. Therefore, the output voltage of VCO can be expressed as

$$2\cos\left(2\pi\left(f_c + f_{VCO}\right)t\right) \quad (2)$$

where f_{VCO} denotes the drift from VCO free running frequency.

Performance of Costas loop depends on A_i and f_{d_i}, when an interference exists in the system. If the amplitude of interference, i.e., A_i, is greater than the desired signal amplitude, i.e., A_s, then the Costas loop locks on the carrier frequency of interference signal. However, due to interference there is an error depending on the relative amplitudes of the desired signal and interference. Inphase and quadrature bits are shaped by raised cosine pulse and then modulating by $\cos(2\pi f_c)$ and $\sin(2\pi f_c)$ and then transmitted through wideband channel. It is assumed that the channel noise is an additive white Gaussian noise (AWGN) to the signal $w(t)$ with variance σ^2 and mean μ.

III. Costas loop

The Costas loop, similar to other AFC loops, consists of three blocks; namely a VCO, multipliers and loop filters, see Fig. 1. VCO generates the sinusoidal frequency whose frequency depends on its input voltage. Multipliers are used to generate the error voltage due to the difference between the carrier frequency and VCO running frequency. Loop filter suppresses high frequency terms in output voltage of multipliers. Generated error voltage, $e(t)$, controls the running frequency of the VCO till it locks on the carrier frequency.

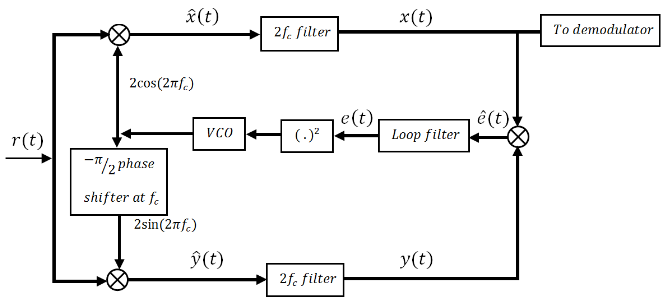

Fig. 1. Modified Costas Loop

In the usual conditions there is no interference in the system, and the received signal $S(t)$ is defined as follows

$$\begin{aligned} r(t) = S(t) &= A_s \sum_k b_k p\left(t - kT_B\right) \cos\left(2\pi\left(f_c + f_{d_s}\right)t\right) \\ &+ A_s \sum_k c_k p\left(t - kT_B\right) \sin\left(2\pi\left(f_c + f_{d_s}\right)t\right) \end{aligned} \quad (3)$$

In every communication system, the carrier frequency f_c is known, whereas the drift frequency f_{d_s} must be determined. The inphase and quadrature branches in the output of the Costas loop multipliers are defined as

$$\begin{aligned} \widehat{x}(t) &= r(t) \times 2\cos\left(2\pi\left(f_c + f_{VCO}\right)t\right) \\ \widehat{y}(t) &= r(t) \times 2\sin\left(2\pi\left(f_c + f_{VCO}\right)t\right) \end{aligned} \quad (4)$$

When the VCO locks on the carrier frequency of received signal, f_{VCO} is equal to f_{d_s}.

The output voltage of multipliers passes through $2f_c$ filters in order to suppress the high frequency terms. The output voltage of the $2f_c$ filters are obtained as

$$\begin{aligned} x(t) &= A_s \sum_k b_k p\left(t - kT_B\right) \cos\left(2\pi\left(f_{VCO} - f_{d_s}\right)t\right) \\ &- A_s \sum_k c_k p\left(t - kT_B\right) \sin\left(2\pi\left(f_{VCO} - f_{d_s}\right)t\right) \end{aligned} \quad (5)$$

$$\begin{aligned} y(t) &= A_s \sum_k b_k p\left(t - kT_B\right) \sin\left(2\pi\left(f_{VCO} - f_{d_s}\right)t\right) \\ &+ A_s \sum_k c_k p\left(t - kT_B\right) \cos\left(2\pi\left(f_{VCO} - f_{d_s}\right)t\right) \end{aligned} \quad (6)$$

In order to estimate the carrier frequency $x(t)$ and $y(t)$ must be multiplied

$$\widehat{e}(t) = x(t).y(t) \quad (7)$$

The loop filter suppresses the high frequency terms in the $\widehat{e}(t)$ and also determines the loop dynamic response. Then the error voltage $e(t)$ in (8) drives the VCO in order to determine the carrier frequency.

$$\begin{aligned} e(t) &= \frac{1}{2}\left(\alpha_1^2 - \alpha_2^2\right) \sin\left(2\pi\left(2f_{VCO} - 2f_{d_s}\right)t\right) \\ &+ \alpha_1\alpha_2 \cos\left(2\pi\left(2f_{VCO} - 2f_{d_s}\right)t\right) \end{aligned} \quad (8)$$

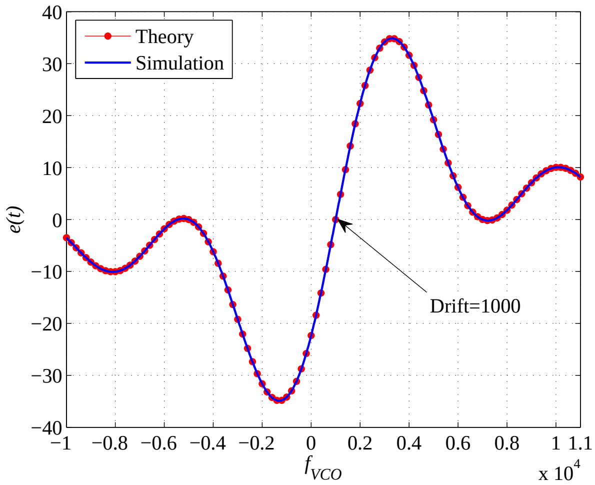

Fig. 2. Performance of Costas loop in ideal condition for BPSK modulation

where the following definition apply

$$\alpha_1 = \left(A_s \sum_k b_k p\left(t - kT_B\right) \right)$$
$$\alpha_2 = \left(A_s \sum_k c_k p\left(t - kT_B\right) \right) \tag{9}$$

To modify the Costas loop, a squaring block is added to the conventional Costas loop, see Fig. 1. This modification makes the Costas loop designed for carrier recovery of BPSK signals, applicable for carrier recovery of QPSK signals. While in BPSK modulation, the VCO locks on the carrier frequency when the error voltage reaches zero [7], in QPSK modulation the VCO locks on desired frequency when the squared error voltage reaches its maximum. Consequently, the modified Costas loop only can be used for offline signals. It is illustrated in Fig. 2. For BPSK signal, the error voltage $e(t)$ of the conventional Costas loop, reaches zero when running frequency of the VCO is equal to carrier frequency. However, in Fig. 3, for QPSK signal, the error voltage of modified Costas loop reaches its maximum when the VCO locks on carrier frequency. As it can be deduced from (8), when the drift in the VCO frequency, f_{VCO}, equals to drift in the carrier frequency, f_{d_s}, the squared error reaches its maximum value and the VCO locks on the carrier frequency. Compared to the AFC loop presented for QPSK modulation in [7], the modified AFC loop uses less blocks and is much simpler for practical implementation.

IV. Performance of Modified Costas Loop in the Presence of an Interference

In this section, an equation for the error voltage in the presence of the interference is derived. The received signal model is given in (1). The inphase and quadrature branches of Costas loop are defined in (4). The output voltage of multipliers is filtered by the filters to suppress the high frequency terms as we can see in following equations

$$\begin{aligned} x(t) = {} & A_s \sum_k b_k p\left(t - kT_B\right) \cos\left(2\pi \left(f_{VCO} - f_{d_s}\right) t\right) \\ & - A_s \sum_k c_k p\left(t - kT_B\right) \sin\left(2\pi \left(f_{VCO} - f_{d_s}\right) t\right) \\ & + A_i \sum_k d_k p\left(t - kT_B\right) \cos\left(2\pi \left(f_{VCO} - f_{d_i}\right) t\right) \\ & - A_i \sum_k e_k p\left(t - kT_B\right) \sin\left(2\pi \left(f_{VCO} - f_{d_i}\right) t\right) \end{aligned} \tag{10}$$

$$\begin{aligned} y(t) = {} & A_s \sum_k b_k p\left(t - kT_B\right) \sin\left(2\pi \left(f_{VCO} - f_{d_s}\right) t\right) \\ & + A_s \sum_k c_k p\left(t - kT_B\right) \cos\left(2\pi \left(f_{VCO} - f_{d_s}\right) t\right) \\ & + A_i \sum_k b_k p\left(t - kT_B\right) \sin\left(2\pi \left(f_{VCO} - f_{d_i}\right) t\right) \\ & + A_i \sum_k c_k p\left(t - kT_B\right) \cos\left(2\pi \left(f_{VCO} - f_{d_i}\right) t\right) \end{aligned} \tag{11}$$

After obtaining $\widehat{e}(t)$ using (7), $\widehat{e}(t)$ passes through the loop filter to generate the error voltage $e(t)$, which is as follows

$$\begin{aligned} e(t) = {} & \frac{1}{2}\left(\alpha_1^2 - \alpha_2^2\right)\left(\sin\left(2\pi\left(2f_{VCO} - 2f_{d_s}\right)\right)\right) \\ & + \alpha_1\alpha_2 \cos\left(2\pi\left(2f_{VCO} - 2f_{d_s}\right)\right) \\ & + \frac{1}{2}\left(\alpha_3^2 - \alpha_4^2\right)\left(\sin\left(2\pi\left(2f_{VCO} - 2fd_i\right)\right)\right) \\ & + \alpha_3\alpha_4 \cos\left(2\pi\left(2f_{VCO} - 2f_{d_i}\right)\right) \\ & + \alpha_1\alpha_3 \sin\left(2\pi\left(2f_{VCO} - f_{d_s} - f_{d_i}\right)\right) \\ & - \alpha_2\alpha_4 \sin\left(2\pi\left(2f_{VCO} - f_{d_s} - f_{d_i}\right)\right) \\ & + \alpha_1\alpha_4 \cos\left(2\pi\left(2f_{VCO} - f_{d_s} - f_{d_i}\right)\right) \\ & + \alpha_2\alpha_3 \cos\left(2\pi\left(2f_{VCO} - f_{d_s} - f_{d_i}\right)\right) \end{aligned} \tag{12}$$

where the following definitions apply

$$\alpha_3 = \left(A_i \sum_k d_k p\left(t - kT_B\right) \right)$$
$$\alpha_4 = \left(A_i \sum_k e_k p\left(t - kT_B\right) \right) \tag{13}$$

V. Simulation Results

A. Ideal Codition

We compare the obtained equation (8) for error voltage with simulation results. In the performed simulations, a QPSK signal is generated which has a drift frequency equal to $1\ KHz$. The amplitude of desired signal A_s is set to 1. The desired signal bits, b_k and c_k are generated randomly and then are shaped with raised Cosine pulse. In this case, 93 symbols are generated for a QPSK signal. The AWGN noise is zero mean and $\sigma = \frac{10}{3}\ \mu V$ which is added to the transmitted signal. To compare the obtained expression for $e(t)$ with simulation results, the squared form of error voltage, $e^2(t)$, is plotted versus the drift in VCO frequency, f_{VCO} in Fig. 3. As one can deduce from Fig. 3, the simulation and analytical curves

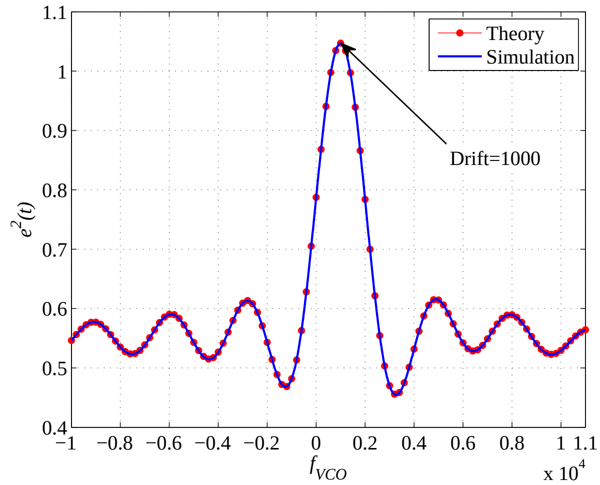

Fig. 3. Performance of modified Costas loop in ideal condition for QPSK modulation

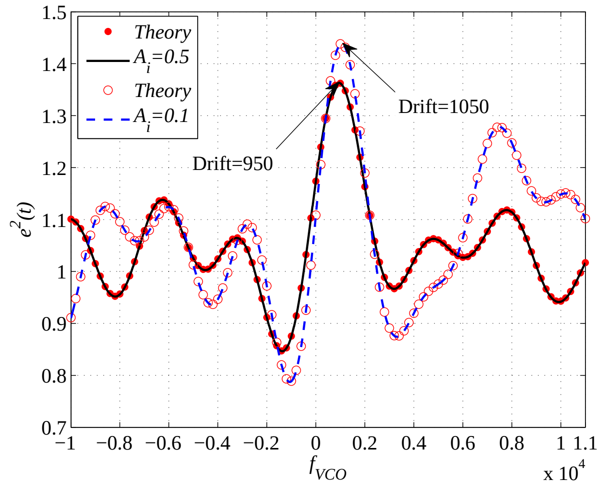

Fig. 4. Performance of modified Costas loop in the presence of an interference for two values of A_i

are well matched and $e^2(t)$ reaches its maximum value when f_{VCO} equals to f_{d_s}.

B. Effect of the interference in the Costas Loop Performance

Now, we compare the analytical equation and simulation results in the presence of interference. In the performed simulations, we generate QPSK signal with the desired signal amplitude $A_s = 1$. In order to evaluate the performance of modified Costas loop, an interference is added to the desired signal. b_k, c_k, d_k and e_k are generated randomly and are shaped with the raised cosine pulse. In this case, 93 symbols are generated for QPSK signals. The channel is AWGN with $\sigma = \frac{10}{3}\ \mu V$ and $\mu = 0$. As one can deduce from (12), A_i and f_{d_i} influence the performance of modified Costas loop.

To investigate the effect of A_i in the Costas loop performance, the square error $e^2(t)$ is plotted for two values of A_i in Fig. 4. Meanwhile, f_{d_i} is equal to $-100\ Hz$ for both curves. For two values of A_i, the squared form of error voltage, $e^2(t)$, is plotted versus the drift in VCO running frequency, f_{VCO}, in Fig. 4. As depicted in Fig. 4 simulation results and derived equation are well matched although the VCO locks on wrong frequency. In Fig. 4, the modified Costas loop locks on 20.00095 MHz and 20.00105 MHz instead of 20.001 MHz when $A_i = 0.1$ and $A_i = 0.5$, respectively.

To address the problem of f_{d_i}, $e^2(t)$ is plotted for two values of f_{d_i} while A_i is equal to 0.1 for both cases. The squared form of error voltage, $e^2(t)$, is plotted versus f_{VCO} in Fig. 5. As illustrated in Fig. 5, the modified Costas loop locks on 20.00098 MHz and 20.00098 MHz instead of 20.001 MHz when $f_{d_i} = 100\ Hz$ and $f_{d_i} = 500\ Hz$, respectively.

C. Effect of received bits on the modified Costas loop

The performed simulations are sensitive to inphase and quadrature bits. In cases that the carrier frequency changes quickly, the above simulations may be applied. It should be considered that if one runs simulations again, the PLL

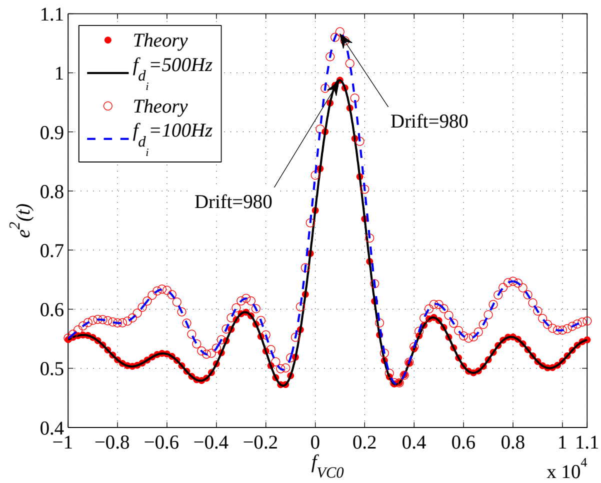

Fig. 5. Performance of modified Costas loop in the presence of an interference for two values of f_{d_s}

may locks on a different frequency. It is due to the random generation of b_k, c_k, d_k and e_k and limitation of number of symbols. In order to overcome the slow changes of carrier frequency, number of bits should be increased. To address this issue, number of symbols is tenfold. In MATLAB simulation parameters are chosen as follows: $A_s = 1$, $f_{d_s} = 1\ KHz$, $f_{d_i} = 100\ Hz$. The amplitude of interference, A_{d_i}, changes from 0 to 1. As depicted in Fig. 6 when the amplitude of interference increases from 0.85 to 0.95, a peak near the drift of the interference signal is revealed. In Fig. 6 the normalized squared error is plotted versus f_{VCO}. In Fig. 7 the running frequency of the VCO is plotted with respect to the signal to interference ratio (SIR). As apparent in Fig. 7, the performance of Costas loop remains constant till the amplitude of interference signal reaches the amplitude of the desired signal.

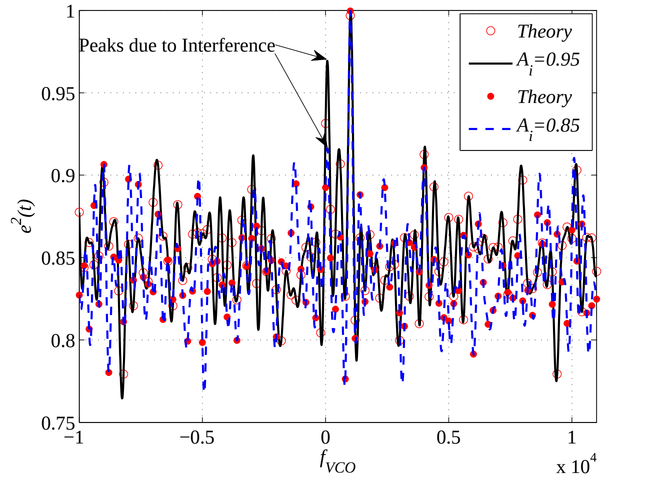

Fig. 6. Performance of modified Costas loop in the presence of interference for QPSK modulation and two values of A_i

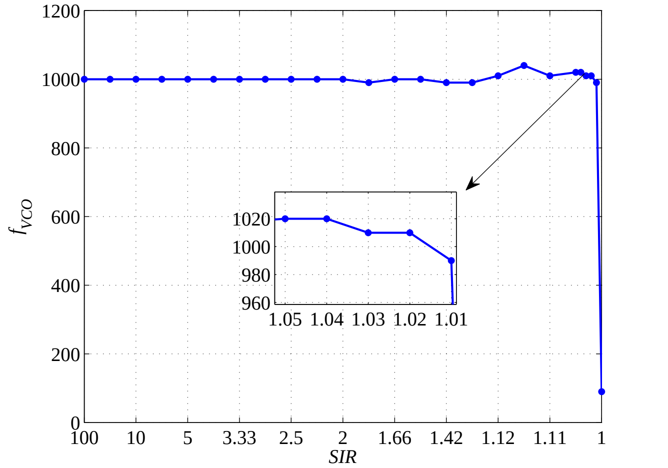

Fig. 7. The locked frequency of the VCO

VI. Conclusion

In this paper, the performance of the Costas loop is investigated. The modified version of Costas loop is proposed for QPSK modulation which is more efficient in terms of circuit implementation comparing to the well-known squaring loop and Costas loop. Two different conditions are studied. In the first scenario, Costas loop performance is examined without the co-channel interference using both theoretical and computer simulations. The modified version of Costas loop is proposed in this paper uses the square value of error voltage $e^2(t)$ which reaches its maximum value when the frequency of the VCO is equal to the drift of the desired signal. In the second scenario, the co-channel interference is considered to be added to the signal, both in simulation and theoretical analysis. It is shown that the performance of the Costas loop remains constant until the amplitude of interference reaches the amplitude of desired signal.

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