Exploiting Vector Extremum Seeking Control for Simultaneous Optimization of Net Power and Lifespan Enhancement of PEM Fuel Cells

Mohammad Reza Hayati
M. Sc. Student
Department of Power and Control Engineering
Shiraz University
Shiraz, Iran, 7134851154
Email: mrhayati@shirazu.ac.ir

Maryam Dehghani
Assistant Professor
Department of Power and Control Engineering
Shiraz University
Shiraz, Iran, 7134851154
Email: mdehghani@shirazu.ac.ir

Alireza Khayatian
Associate Professor
Department of Power and Control Engineering
Shiraz University
Shiraz, Iran, 7134851154
Email: khayatia@shirazu.ac.ir

Abstract— **In many real world problems, it is needed to solve an online optimization problem to achieve a certain criteria. One of the practical tools for online optimization is extremum seeking control. The main disadvantage of this method is its weakness in dealing with multi variable optimization problems. The vector extremum seeking control technique overcomes this weakness and is able to determine several variables for online optimization of a performance function.**

This paper uses the vector gradient based extremum seeking control technique to optimize the net power (the produced power minus the used power) in the Proton Exchange Membrane Fuel Cell (PEMFC) while the PEMFC lifespan is enhanced. The net power optimization is achieved by controlling the oxygen flow and the lifespan enhancement is obtained by controlling the inlet hydrogen flow which keeps the pressure difference of cathode and anode close to zero.

The controller performance is investigated by simulating it on a sample PEMFC model. The simulation results indicate that the partial pressure difference of cathode and anode is close to zero and the PEMFC net power is optimized at the same time. The controller increases the net power up to 20 percent.

Keywords- Lifespan; Net power; PEMFC; Vector gradient based extremum seeking control.

I. INTRODUCTION

Fuel cell is one of the human being inventions which produce electrical energy by electrochemical reactions. Low emission, low noise and being a clean electrical energy source are the important features of fuel cell [1], [2].

Among the fuel cell types, PEMFC is very famous in industry. PEMFC features such as High power density, solid electrolyte, low operating temperature, low corrosion, long stack life, favorable power-to-weight ratio and being used in hybrid vehicles makes it valuable and useful [2], [3].

Online optimization of PEMFC net power is one of the PEMFC problems. In this optimization, the manipulated variable is the air compressor voltage. The main uncertainty is stack current which is changed by load [4], [5].

Another problem in PEMFC is enhancing the PEMFC lifespan by protecting the membrane. Large deviation between gases pressures at cathode and anode will harm the thin and sensitive membrane of PEMFC. Therefore, the pressure difference of gases at cathode and anode should be close to zero via controlling the inlet hydrogen flow.

In literature, the net power of PEMFC has been maximized in [4], [5]. The perturbation based extremum seeking control has been used for online optimization of net power in [4]. The main disadvantages of that method is its long time (about 2 minutes) to find the optimum point and unacceptable oscillations in the system. Reference [5] has used the sliding mode control to maximize the PEMFC net power. In [5], first the optimum values are calculated in different stack currents and then a second order sliding mode control makes the system follow the optimum points which means that the optimization is not completely online.

References [6]-[8], have defined identical predefined set points for the partial pressures of anode and cathode to protect the PEMFC membrane. In [6]-[8] both the inlet hydrogen and oxygen has been controlled to follow the predefined values. Forcing the partial pressures to achieve predefined values is the disadvantage of the method used in [6]-[8] while adaptive set points are more suitable.

In this paper, the vector gradient based extremum seeking control maximizes the PEMFC net power and enhances the PEMFC lifespan, simultaneously. The previous researches only solve one of the aforementioned challenges. This paper uses the vector extremum seeking control to solve both problems at the same time. The uncertainties in this paper are stack current and stack temperature. Keeping the pressures of cathode and anode as close as possible is a constraint in the optimization problem. Speeding up the system response by using the gradient extremum seeking technique is the advantage of this optimization method over the previous ones. Another advantage of this paper compared to the previous methods is that the inlet oxygen flow is controlled such that the net power is maximized and the method controls the inlet hydrogen flow such that the gases pressure at cathode is identical to the gases pressure at anode.

The paper method has some pros and cons which are stated as follows:

The advantages of the paper method are fastness, simplicity, and being independent of state observer. The disadvantage of the paper is that the humidity of inlet gases is not controlled.

The rest of the paper is organized as follows. The system model is presented at section II. An explanation about the vector gradient based extremum seeking control is in Section III. Section IV represents the control problems. The control strategy is given in Section V. Section VI shows the simulation results. Finally, conclusion is presented in section VII.

II. SYSTEM MODEL

The system model consists of a compressor model with DC motor and a PEMFC model. The compressor model is based on the model used in [5] and the PEMFC model is based on the model presented in [2], [9]. The system model has eight states. It consists of two states of compressor model and six states of PEMFC model. Here, the equations of the system model are presented.

PEMFC output voltage is derived by subtracting voltage losses from the open-circuit output voltage [9]-[13].

$$V_{fc} = V_{O,FC} - n_s(V^{Act} + V^O + V^{Conc}) - E_{d,cell} \tag{1}$$

$$V^O = IR^O \tag{2}$$

$$R^O = R_C^O + K_I I + K_T T \tag{3}$$

$$V^{Act} = a_0 + T[a + b\ln(I)] \tag{4}$$

$$V^{Conc} = -\frac{RT}{eF}\ln(1 - \frac{I}{I_L}) \tag{5}$$

$$E_{d,cell}(s) = \lambda_e I(s)\frac{\tau_e s}{\tau_e s + 1} \tag{6}$$

V_{fc} is output voltage of PEMFC, $V_{O,FC}$, PEMFC open circuit output voltage, n_s, number of cells in the stack, V^{Act}, activation voltage loss, V^O, ohmic voltage loss, V^{Conc}, concentration voltage loss, $E_{d,cell}$,total delay in PEMFC, I, stack current, R^O, ohmic resistance, R_C^O, K_I and K_T, constants in computation of R^O. T is stack temperature, a_0, a and b are constants in computation of V^{Act}. R is universal gas constant, e, number of participating electrons, F, Faraday's constant, I_L, limiting current, λ_e, constant factor in calculating $E_{d,cell}$ and τ_e is overall flow delay.

By modeling the charge double layer capacitor, output voltage is stated as follows:

$$V_{fc} = n_s[E_0^{cell} + \frac{RT}{2F}\ln(\frac{P_{H_2}P_{O_2}^{0.5}}{P_{H_2O}}) - V_C - R^O I] - E_{d,cell}(s) \tag{7}$$

E_0^{cell} is reference potential at standard operating condition, P_x, partial pressure of x and V_C is voltage of charge double layer capacitor.

As it is mentioned, the system model has eight states. x_1 is armature current of DC motor, x_2, shaft angular speed of DC motor, x_3, stack temperature, x_4, x_5, x_6 and x_7 are respectively partial pressure of hydrogen, oxygen, nitrogen and water. x_8 is voltage of charge double layer capacitor.

The system model has two inputs. u_1 is voltage of DC motor and u_2 is channel pressure of hydrogen. The state equations are as follows [5], [10], [14]-[18]:

$$\dot{x}_1 = -\frac{R_s}{L}x_1 - \frac{k\phi}{L}x_2 + u_1 \tag{8}$$

$$\dot{x}_2 = \frac{k\phi}{J}x_1 - A_{00} - A_{10}P_{cp} - A_{20}P_{cp}^2 - A_{01}x_2 - A_{11}x_2P_{cp} - A_{02}x_2^2 - B_0 - B_1x_2 \tag{9}$$

$$\dot{x}_3 = [\frac{-h_s n_s A_s}{M_{fc}C_{fc}}]x_3 - \theta_1(x_3, I) + [\frac{h_s n_s A_s}{M_{fc}C_{fc}}]u_{TR} \tag{10}$$

$$\dot{x}_4 = 2\theta_2(x_3)u_2 - 2\theta_2(x_3)x_4 - \theta_3(x_3)I \tag{11}$$

$$\dot{x}_5 = 2\theta_4(x_3)0.21u_{PC} - 2\theta_4(x_3)x_5 - \theta_5(x_3)I \tag{12}$$

$$\dot{x}_6 = 2\theta_4(x_3)0.79u_{PC} - 2\theta_4(x_3)x_6 \tag{13}$$

$$\dot{x}_7 = 2\theta_6(x_7)x_3 + 2\theta_5(x_3)I \tag{14}$$

$$\dot{x}_8 = [\frac{-1}{C(R^{Act} + R^{Conc})}]x_8 + [\frac{1}{C}]I \tag{15}$$

R_s is electrical resistance of stator winding, L, electrical inductance of stator winding, $k\phi$, motor constant, J, motor inertia, A_{ij} and B_i are constants in compressor model. h_s is convective heat transfer coefficient, A_s, area of a single cell, M_{fc}, total mass of PEMFC stack, C_{fc}, specific heat capacity of PEMFC stack, u_{TR}, room temperature and u_{PC} is channel pressure of air.

In (8)-(15), θ_i and u_{PC} are as follows:

$$\theta_1(x_3, I) = -n_s[V^{Act} + V^{Conc} + V^O] \quad (16)$$

$$\theta_2(x_3) = [(R(m_{H_2O})_{in}^a x_3)/(V_a(P_{H_2O})_{in}^a)] \quad (17)$$

$$\theta_3(x_3) = [Rx_3 / 2V_aF] \quad (18)$$

$$\theta_4(x_3) = [(R(m_{H_2O})_{in}^c x_3)/(V_c(P_{H_2O})_{in}^c)] \quad (19)$$

$$\theta_5(x_3) = [Rx_3 / 4V_cF] \quad (20)$$

$$\theta_6(x_7) = [\frac{R(m_{H_2O})_{in}^c(2 - x_7)}{V_c(P_{H_2O})_{in}^c}] \quad (21)$$

$$u_{pc} = \frac{(P_{H_2O})_{in}^c}{(m_{H_2O})_{in}^c}[B_{00} + B_{10}P_{cp} + B_{20}P_{cp}^2 + B_{01}x_2 + B_{11}x_2P_{cp} + B_{02}x_2^2] \quad (22)$$

$(m_{H_2O})_{in}^a$ is mole flow rate of water at anode, V_a, volume of anode, $(P_{H_2O})_{in}^a$, partial pressure of water at anode, $(m_{H_2O})_{in}^c$, mole flow rate of water at cathode, V_c, volume of cathode, $(P_{H_2O})_{in}^c$, partial pressure of water at cathode and B_{ij} are constants in compressor model.

III. Vector Gradient Based Extremum Seeking Control

In this section, the vector gradient based extremum seeking control is introduced briefly.

Vector gradient based extremum seeking is one of the methods of online optimization which needs to know the objective function exactly.

Suppose that the objective function $g(x, y)$ is concave and only has a maximum point. If $(x, y) \in \mathrm{R}$ and (x^*, y^*) is the maximum point, a simple choice for Lyapunov function would be as follows [19]:

$$V = g(x^*, y^*) - g(x, y) \geq 0 \quad (23)$$

If the Lyapunov function converges to zero, $g(x, y)$ will converge to $g(x^*, y^*)$. Therefore, the time derivative of Lyapunov function should be negative to achieve the maximum point of the objective function [20].

$$\dot{V} = -\frac{dg}{dx}(x, y)\dot{x} - \frac{dg}{dy}(x, y)\dot{y} \quad (24)$$

According to (24), by designing $\dot{x}$ and $\dot{y}$, derivative of Lyapunov function could become negative. In vector gradient based extremum seeking control, $\dot{x}$ and $\dot{y}$ are defined as follows:

$$\dot{x} = h_1\frac{dg}{dx}(x, y) \quad (25)$$

$$\dot{y} = h_2\frac{dg}{dy}(x, y) \quad (26)$$

Where $h_1 > 0$ and $h_2 > 0$. By increasing h_1 and h_2 convergence rate would get faster. According to (25) and (26), $\frac{dg}{dx}(x, y)$ and $\frac{dg}{dy}(x, y)$ should be known for the vector gradient based extremum seeking control.

IV. The Control Problems

The control problems consist of two problems. The first problem is Enhancing the PEMFC lifespan and the second one is maximization of the PEMFC net power. In this section, these problems are investigated.

A. Reducing the Pressure Differnece of Cathode and Anode

In this subsection, the control objective is to keep the pressure difference of cathode and anode close to zero to enhance the PEMFC lifespan. Therefore, the difference between the partial pressure of hydrogen at anode and sum of the partial pressures of gases at cathode (i.e. nitrogen and oxygen) should be close to zero. Equation (27) defines the control objective:

$$x_4 - x_5 - x_6 = 0 \quad (27)$$

In this paper, the control objective in (27) would be a constraint which is considered in the vector extremum seeking controller.

B. Maximization of the PEMFC Net Power

In this subsection, optimization of the net power will be investigated. It is assumed that the pressure of gases at cathode and anode are identical by controlling u_2. The net power is given below:

$$P_{net} = V_{fc}I - u_1x_1 \quad (28)$$

P_{net} is the net power. The uncertain variable is the stack current which changes the maximum point. By using online optimization, the maximum point would be followed in different stack currents.

Substituting V_{fc} and x_1 in (28), P_{net} would be a function of stack current, stack temperature, compressor voltage and channel pressure of hydrogen.

In this paper, vector gradient based extremum seeking control will be used for maximization of net power. The control variables are compressor voltage and channel pressure of hydrogen (u_1 and u_2). u_2 should be controlled such that the pressure of gases at cathode and anode are identical. Therefore, the net power must be a concave function of u_1. Fig. 1 shows the curve of net power versus u_1 which indicates that the net power is a concave function of u_1.

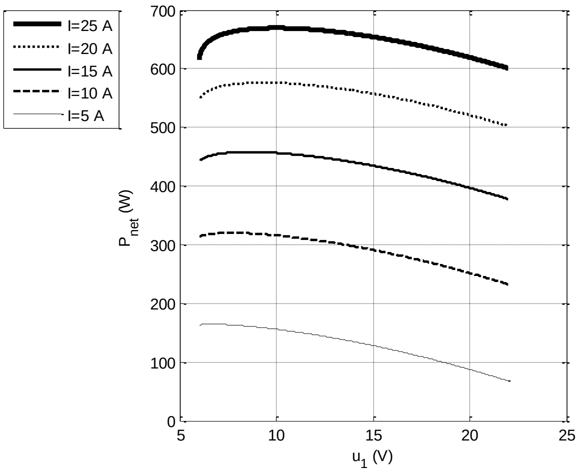

Figure 1. Net power versus u_1

As it is stated in section III, the derivative of P_{net} due to u_1 and u_2 should be known. By differentiating the P_{net}, these two derivatives would be known and the vector gradient based extremum seeking control could be used for online optimization.

V. The Control Strategy

The control strategy is such that the assumption of equality of cathode and anode gases pressure is considered as a constraint in the optimization of PEMFC net power. This control strategy maximizes the PEMFC net power and enhances the PEMFC lifespan simultaneously.

The aforementioned constraint could enter the performance function using Lagrange multipliers. Here, the Lagrange multiplier is chosen 1000 to keep the pressure difference at cathode and anode almost zero. Therefore, the performance function is as follows:

$$f = V_{fc} I - V_{cp} i_a - 1000(x_4 - x_5 - x_6)^2 \quad (29)$$

The detailed formula for f as a function of stack current, stack temperature, compressors voltage and hydrogen channel pressure in the equilibrium point is given in the appendix.

The control laws are obtained as follows:

$$u_1(t) = \int k_1 \frac{dJ(t)}{dV_{cp}(t)} dt \quad (30)$$

$$u_2(t) = \int k_2 \frac{dJ(t)}{du_{PA}(t)} dt \quad (31)$$

Where $k_1, k_2 > 0$.

The control laws depend on the compressor voltage (u_1), the stack current (I), stack temperature (T) and the hydrogen channel pressure (u_2) which are available. Therefore, the control signals can be implemented in practice easily.

The control strategy consists of the vector gradient based extremum seeking controller which controls the air channel pressure and the hydrogen channel pressure simultaneously. Fig. 2 shows the PEMFC and its controller. In the followings, the control strategy is described in details.

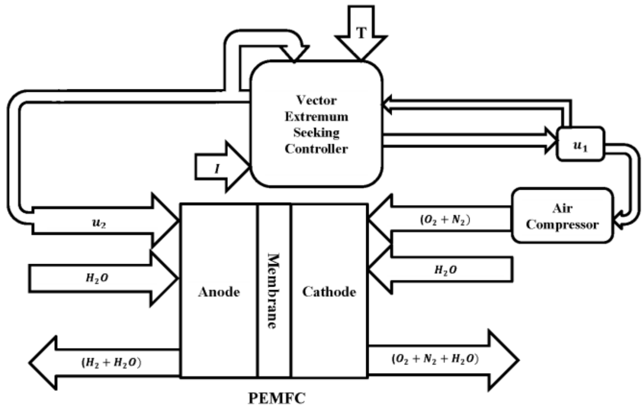

Figure 2. PEMFC and its controller

VI. Simulation Results

To investigate the performance of the vector gradient based extremum seeking controller, two simulation cases are considered:

Case 1: Simulating the system model considering the compressor voltage and the hydrogen channel pressure are kept at a constant value.

TABLE I shows the simulation assumptions in case 1. The values of u_1 and u_2 are chosen such that enough oxygen and hydrogen are provided for the system.

TABLE I. Simulation Assumptions in Case 1

Quantity	Value	Quantity	Value
u_{TR}	308 k	u_1	7.5 Volt
P_{cp}	2 atm	u_2	2 atm

Case 2: Simulating the system model using the vector gradient based extremum seeking control for controlling the compressor voltage and the hydrogen flow, simultaneously.

Fig. 2 shows the control strategy in case 2. TABLE II shows the simulation assumptions in case 2. The constant values of k_1 and k_1 is chosen such that the extremum seeking controller has a suitable convergence rate.

TABLE II. SIMULATION ASSUMPTIONS IN CASE 2

Quantity	Value	Quantity	Value
u_{TR}	308 k	k_1	1000
P_{cp}	2 atm	k_2	1000

The stack current in both cases changes between 1 to 25 Amperes step by step. Fig. 3 shows the stack current in both cases.

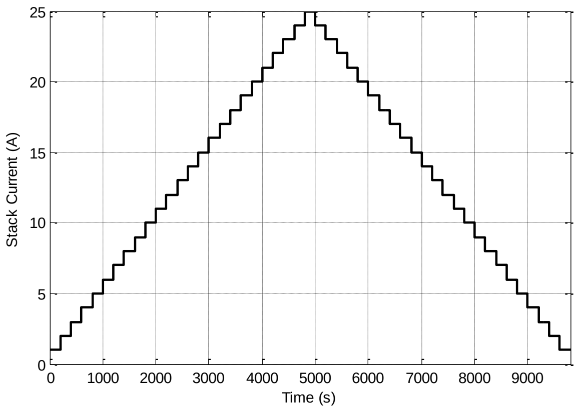

Figure 3. The stack current in both cases

A. *Enhancing the PEMFC Lifespan*

To investigate the performance of extremum seeking controller in enhancing the PEMFC lifespan, Fig. 4 shows the pressure difference of gases at cathode and anode in case 2.

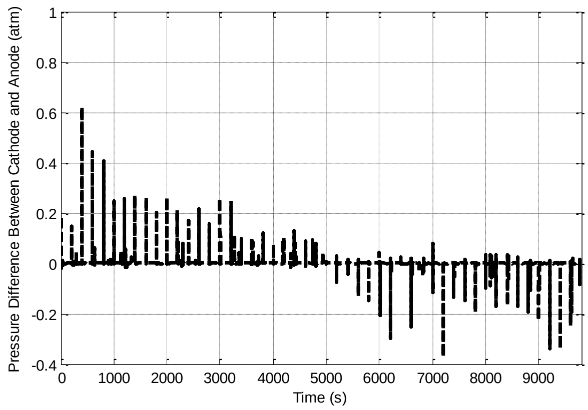

Figure 4. Pressure difference between cathode and anode

The pressure difference is almost zero and it shows that the vector extremum seeking controller enhanced the PEMFC lifespan. The main reason of the spikes in the Figure is changes of stack current. These spikes can be ignored since they last less than 0.5 seconds and the maximum value of them is 0.63 atm.

The hydrogen channel pressure in case 2 which is one of the control signals of the vector extremum seeking controller is shown in Fig. 5. Fig. 5 indicates that the hydrogen channel pressure is increased along with increasing the stack current, and it is decreased along with decreasing the stack current. This is because the optimum value of oxygen at the cathode is increased by increasing the stack current and the channel pressure of hydrogen should be controlled to keep the pressure difference between cathode and anode close to zero.

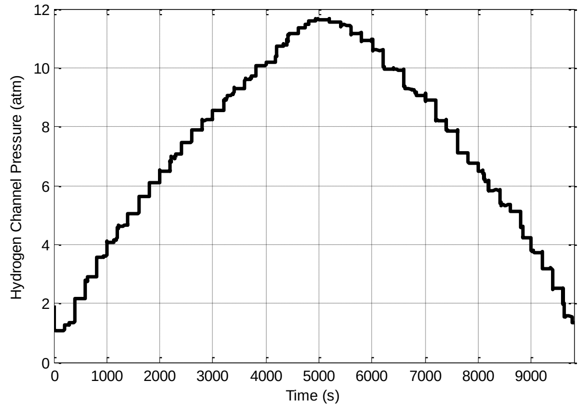

Figure 5. Hydrogen channel pressure

B. *Improvement of the Net Power*

The difference between the net power in case 1 and 2 is the improvement in the net power. Fig. 6 shows the improvement of the net power.

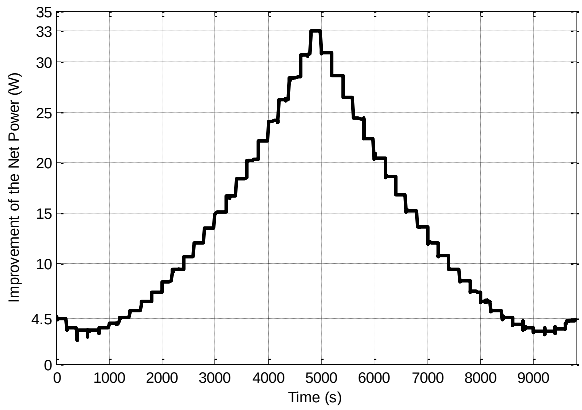

Figure 6. Improvement of the net power

The figure indicates that the maximum value of the improvement is about 33 watts that occurs when the stack current is 25 Amperes. However, the maximum percentage of net power improvement is for the stack current of 1 Ampere and is about 20 percent (i.e. $\frac{4.5}{22.3}$ percent).

The compressor voltage (which is one of the control signals of the vector extremum seeking controller) is shown in Fig. 7. Fig. 7 shows that the compressor voltage in case 1 is 7.25 Volts. The figure shows that in case 2, when the stack current is increased, the controller raises the compressor voltage to achieve the online optimization.

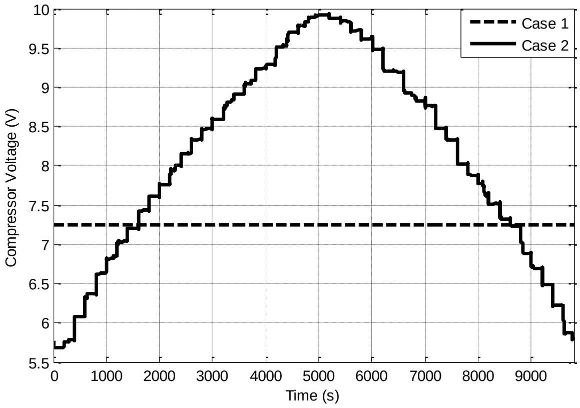

Figure 7. Compressor voltage

VII. CONCLUSION

In this paper, the vector gradient based extremum seeking control is used for online optimization of PEMFC net power and enhancement of PEMFC lifespan. The advantage of the paper method is that it solves both problems simultaneously by using the vector extremum seeking control. The enhancement of PEMFC lifespan is considered as a constraint in the extremum seeking problem and this constraint is entered the performance function by using Lagrange multipliers. Simulation results indicate that the net power is improved up to 20 percent and the pressure difference between cathode and anode is kept close to zero at the same time. In feature works, the system model could be improved and the humidity of inlet gases could be controlled to improve the PEMFC performance.

APPENDIX

J = I *((462* T)/3125 - (49313* I)/50000 + (237* T * I)/100000 + (4986* T *log(1 - I /30))/2412175 + (4986* T *log(-(1843185^(1/2)*((215* I)/7158 - u_2)*(- 2.0094e+65* I - 1.5870e+40*pi^2*((2^(1/2)*(1.4224e+33- 1.6839e+30* u_1)^(1/2))/32 - 1.6650e+15)^2 - 2.7759e+54*pi*((2^(1/2)*(1.4224e+33 - 1.6839e+30* u_1)^(1/2))/32 - 1.6650e+15) - 2.8356e+67)^(1/2))/(6.9378e+32*(215* I + 14316))))/2412175 - (2.1525e+16* T *log(I))/ 4.6117e+18 - (187* I ^2)/100000 - 4191/625) - (125*(5.0175e+67* u_2 - 7.5353e+65* I + 2.8340e+41*pi^2*((2^(1/2)*(1.4224e+33 - 1.6839e+30* u_1)^(1/2))/32 - 1.6650e+15)^2 + 4.9569e+55*pi*((2^(1/2)*(1.4224e+33 - 1.6839e+30* u_1)^(1/2))/32 - 1.6650e+15) + 5.0636e+68)^2)/ 3.1469e+134 - (u_1 *(2.4615e+49* u_1 + 4.1795e+35*2^(1/2)*(1.4224e+33- 1.6839e+30* u_1)^(1/2) - 2.2268e+52))/ 4.9968e+49

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