Model Predictive Control of a class of Uncertain Nonlinear Discrete Time Systems: The LMI approach

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Abstract— **Model predictive control (MPC) of linear systems is well stablished in the literature. However, The MPC design of nonlinear systems has many challenges and complex calculations due to the inclusion of non-convex optimization problem. This paper, proposes a new approach based on linear matrix inequality to solve the MPC problem with finite horizon for nonlinear uncertain systems which contain additive uncertainty. The proposed algorithm can be applied to a wide class of nonlinear systems and guarantees their stability. The performance and effectiveness of the proposed controller is illustrated with numerical examples.**

Keywords- Finite Horizon, Linear Matrix Inequalities, Model Predictive Control, Nonlinear Cystems, Uncertainty.

I. INTRODUCTION

MPC is a wide range of control approaches, where one uses a model for calculating a control signal explicitly, while via minimizing cost function (see e.g., [1]) Nowadays, MPC is applied on many industrial processes [2]. Since practical systems often have a nonlinear behavior, to improve the control performance and increase the quality of prediction, nonlinear model predictive control (NMPC) should be used for these systems [3-4]which changes the optimization problem from convex programming to non- convex nonlinear problem [5]. In dealing with nonlinear systems, linearizing can change the problem to an easier problem [6]

In practice, the actual system's model contains some uncertainties to be considered in the control design while the controller should guarantee the stability of the system.

In recent years, extensive works have been done to formulate the robust MPC problems as some LMIs which can be readily solved. The following researches are most relevant ones.

In [7-9] robust MPC problem for discrete linear systems with Polytopic uncertainty models are considered.

The main ideas in [7-9] are the use of infinite horizon cost functions and to formulate the necessary conditions for state-feedback control laws in the framework of LMIs. In [10] the problem of a robust MPC based on LMI for a linear system with bounded uncertainty is presented.

In [11-13] nonlinear MPC problems by considering infinite horizon cost function are solved through LMIs for some classes of nonlinear systems.

In [14] a dynamic output feedback model predictive control for nonlinear systems expressed by Hammerstein-Wiener model is considered.

In this paper, robust nonlinear MPC problem for a class of nonlinear systems is considered and the problem is formulated as LMIs. The approach is based on linear approximation of the nonlinear model in the vicinity of the current state. This approximation is updated iteratively in the MPC calculations.

The paper is structured as follows; in Section II, the general nonlinear model is introduced. In Section III, the proposed approach is given and the robust nonlinear MPC problem is defined. The simulation results are given in Section IV. Finally, the conclusion is given in Section V.

II. PROBLEM FORMULATION

Consider the following nonlinear discrete system:

$$\begin{aligned} x_i(k+1) &= f_i(x(k),u(k),\theta) \\ y(k) &= cx(k) \end{aligned} \tag{1}$$

where k is the discrete time index, $x(k)\in R^n$ is the state, $u(k)\in R^m$ is the input, $f(.,.)\in C^2$, and θ is the additive uncertainty such that:

$$\Theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_n \end{bmatrix} \in \varepsilon(Q_\theta, 0) \quad \& \quad \underline{\theta}_i \le \theta_i \le \overline{\theta}_i \tag{2}$$

In order to design MPC for system (1) as a state-feedback law, at first the necessary conditions for locating the states of the future time within an ellipse is obtained. Then, due to the ellipses calculated, through minimization of the cost function in the finite horizon, the control signal is obtained.

$$J(k)=\sum_{i=1}^{N}(y(k+i\mid k)-y_d)^T Q_{ii}(y(k+i\mid k)-y_d)$$
$$s.t$$
$$x(k+j\mid k)\in\varepsilon_j\left(\tilde{Q}_j, C_j\right) \& Q_{ii}>0 \qquad (3)$$

III. THE PROPOSED APPROACH

The problem considered in Section II is a robust nonlinear MPC problem. To solve the problem, we must follow four steps:

In the first step, the nonlinear system is linearized around the initial operating point and the system dynamics is written as a summation of two terms, a linear term and a second order nonlinear term (residue of linear system). In the second step, the upper bound for quadratic nonlinear term is calculated. In the third step, a condition is considered to guarantee the system states remain in an ellipse in each predictive step for the next predicting time. Finally in the last step, by establishing some conditions, these ellipses in each predictive step are forced to become small which will converge to around origin after some steps. Therefore the stability of the system is guaranteed. The proposed algorithm is extended to output tracking problem as well.

The First step (The linearization step):

Linearizing The system around the working points, according to the Taylor expansion in the vicinity of the current state i.e. $\bar{x}=x_i(k)$ and $\bar{u}=u_i(k)$, we have:

$$x_i(k+1)= f_i(\bar{x},\bar{u})+\frac{\partial f_i}{\partial x}(\bar{x},\bar{u})(x-\bar{x})$$
$$+\frac{\partial f_i}{\partial u}(\bar{x},\bar{u})(u-\bar{u})+\frac{1}{2!}[(x-\bar{x})^2\frac{\partial^2 f_i}{\partial x^2}+$$
$$2(x-\bar{x})(u-\bar{u})\frac{\partial^2 f_i}{\partial x\,\partial u}+(u-\bar{u})^2\frac{\partial^2 f_i}{\partial u^2}]+\ldots \qquad (4)$$

By considering the parameters in the form of equation (5) the infinite expansion (4) can be written in the form of a finite expansion [15],

$$X(k)=\left[x(k)\quad u(k)\right]$$
$$X_c(\mathrm{k})\in B(\bar{X}(k),\|X(k)-\bar{X}(k)\|) \qquad (5)$$

Then we have:

$$x_i(k+1)= f_i(\bar{x},\bar{u})+\frac{\partial f_i}{\partial x}(\bar{x},\bar{u})(x-\bar{x})$$
$$+\frac{\partial f_i}{\partial u}(\bar{x},\bar{u})(u-\bar{u})+\frac{1}{2}\left(X-\bar{X}\right)^T\frac{\partial^2 f_i}{\partial X^2}|_{X_c}\left(X-\bar{X}\right)$$
$$\rightarrow x_i(k+1)= f_i(\bar{x},\bar{u})+\frac{\partial f_i}{\partial x}(\bar{x},\bar{u})(x-\bar{x})$$
$$+\frac{\partial f_i}{\partial u}(\bar{x},\bar{u})(u-\bar{u})$$
$$+\frac{1}{2}\left(X-\bar{X}\right)^T\frac{\partial^2 f_i}{\partial X^2}|_{X_c}\left(X-\bar{X}\right)+\theta_i(k)$$
$$=\underbrace{f_i(\bar{x},\bar{u})-\frac{\partial f_i}{\partial x}(\bar{x},\bar{u})(\bar{x})-\frac{\partial f_i}{\partial u}(\bar{x},\bar{u})(\bar{u})}_{f_0(\bar{x},\bar{u})}$$
$$+\underbrace{\frac{\partial f_i}{\partial x}(\bar{x},\bar{u})}_{A}x_i(k)+\underbrace{\frac{\partial f_i}{\partial u}(\bar{x},\bar{u})}_{B}u_i(k)$$
$$\underbrace{+1/2(X(k)-\bar{X}(k))^T\underbrace{M_i(X_c(k))}_{M_i(X_c(\mathrm{k}))}(X(k)-\bar{X}(k))}_{v_i(k)} \qquad (6)$$

Finally the dynamic of the nonlinear system can be summarized in the form of equation (7):

$$x_i(k+1)=f_0(\bar{x},\bar{u})+Ax_i(k)+Bu_i(k)+v_i(k) \qquad (7)$$

Now we can consider the quadratic term $v_i(k)$ as the additive uncertainty where the upper bound of this term is calculated at the next step.

The Second step (obtaining the upper bound for nonlinear part of the system):

The upper bound of $v_i(k)$ is calculated as:

$$v_i(k)\le\lambda_{max}\left(M_i\left(X_c(\mathrm{k})\right)\right)\|X(k)-\bar{X}(k)\|^2 \qquad (8)$$

By solving the optimization problem (8) an upper bound for $\lambda_{max}\left(M_i\left(X_c(\mathrm{k})\right)\right)$ is calculated as:

$$\max\lambda_{max}\left(M_i(X)\right)$$
$$s.t:$$
$$X=\varepsilon\left(\tilde{Q}_j, C_j\right) \qquad (9)$$

Since, the state of the system must be within an ellipse, the condition $X=\varepsilon\left(\tilde{Q}_j, C_j\right)$ which is more conservative than $X_c(\mathrm{k})\in B(\bar{X}(k),\|X(k)-\bar{X}(k)\|)$ is used.

By solving optimization problem (9), an upper bound for each v_i (k), is calculated as:

$$v_i(\mathrm{k})\le\alpha\|X-\bar{X}\|^2=\alpha r^2 \qquad (10)$$

By calculating the upper bound for all

$v_i(\mathrm{k})\ \forall\ i=1,\dots,n\ \&\ k=0,1,\dots,N-1$ in n-dimensional space, a cube is created. In each step of prediction, the biggest ellipse which contains this cube in the form of $\varepsilon(Q_{V(k)},0)$ is considered as a bound for nonlinear part of the system:

$$\begin{cases} v(j-1)^T Q_{V(j-1)} v(j-1) < 1 \\ \vdots \\ v(1)^T Q_{V(1)} v(1) < 1 \end{cases}$$

$$\rightarrow \underbrace{\begin{bmatrix} v(j-1) \\ v(j-2) \\ \vdots \\ v(0) \end{bmatrix}^T}_{V^T} \underbrace{\begin{bmatrix} Q_{V(j-1)} & 0 & \dots & 0 \\ 0 & Q_{V(j-2)} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & Q_{V(0)} \end{bmatrix}}_{Q_V}$$

$$\times \underbrace{\begin{bmatrix} v(j-1) \\ v(j-2) \\ \vdots \\ v(0) \end{bmatrix}}_{V} < 1 \rightarrow V^T Q_V V < 1 \tag{11}$$

The Third step (Obtaining the necessary conditions for locating the states of the future time within ellipse):

After linearizing the system, the state-space equations of the nonlinear system can be written as:

$$\begin{cases} x(k+1) = f_0 + Ax(k) + Bu(k) + v(k) \\ y(k) = Cx(k) \end{cases} \tag{12}$$

Thus, for k=0 we have:

$$x(k+j|k) \cong x(j)$$

$$= A^j x(0) + \sum_{i=1}^{j} A^{(j-i)} Bu(i-1)$$

$$+ \sum_{i=1}^{j} A^{(j-i)} v(i-1) + \sum_{i=1}^{j} A^{(j-i)} f_0 \tag{13}$$

To obtain the control law, the necessary conditions for locating $x(j)$ within the ellipse is as follows:

$$x(j) \in \varepsilon_j(\tilde{Q}_j, C_j) \rightarrow (x(j) - C_j)^T \tilde{Q}_j (x(j) - C_j) < 1 \tag{14}$$

By substituting equation (13) in constraint (14), we obtain:

$$\{\underbrace{A^j x(0) + \sum_{i=1}^{j} A^{(j-i)} Bu(i-1) + \sum_{i=1}^{j} A^{(j-i)} f_0 - C_j}_{d}$$

$$+ \underbrace{\begin{bmatrix} I & A & \dots & A^{j-1} \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} v(j-1) \\ v(j-2) \\ \vdots \\ v(0) \end{bmatrix}}_{V} \}^T \times \tilde{Q}_j \times$$

$$\{\underbrace{A^j x(0) + \sum_{i=1}^{j} A^{(j-i)} Bu(i-1) + \sum_{i=1}^{j} A^{(j-i)} f_0 - C_j}_{d}$$

$$+ \underbrace{\begin{bmatrix} I & A & \dots & A^{j-1} \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} v(j-1) \\ v(j-2) \\ \vdots \\ v(0) \end{bmatrix}}_{V} \} < 1 \tag{15}$$

Therefore, inequality (14) can be decomposed to the following form

$$\begin{bmatrix} \mathrm{V} \\ \mathrm{I} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R^T \tilde{Q}_j R & R^T \tilde{Q}_j d \\ * & d^T \tilde{Q}_j d - 1 \end{bmatrix} \begin{bmatrix} \mathrm{V} \\ \mathrm{I} \end{bmatrix} \le 0 \tag{16}$$

According to equations (11) and (16) and S-procedure lemma [16], we have:

$$\begin{bmatrix} R^T \tilde{Q}_j R - \begin{bmatrix} t_1^j Q_{V(j-1)} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & t_j^j Q_{V(0)} \end{bmatrix} & R^T \tilde{Q}_j d \\ * & d^T \tilde{Q}_j d - 1 + \sum_{i=1}^{j} t_i^j \end{bmatrix}$$

$$\le 0$$

$$\rightarrow$$

$$\begin{bmatrix} -\tilde{Q}_j^{-1} & \begin{bmatrix} I & A & \dots & A^{j-1} \end{bmatrix} & A^j x(0) + \sum_{i=1}^{j} A^{(j-i)} Bu(i-1) + \sum_{i=1}^{j} A^{(j-i)} f_0 - C_j \\ \begin{bmatrix} * \\ * \\ \vdots \\ * \end{bmatrix} & -\begin{bmatrix} t_1^j Q_{V(j-1)} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & t_j^j Q_{V(0)} \end{bmatrix} & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ * & * \ \dots\ * & -1 + \sum_{i=1}^{j} t_i^j \end{bmatrix}$$

$$\le 0 \tag{17}$$

The Forth step (Minimizing ellipse in the last step):

Theorem1: The size of ellipse's diameter $\varepsilon(\tilde{Q}_j, 0)$ is equivalent to [17]: $\frac{1}{\sqrt{\lambda\left(\tilde{Q}_j\right)}}$ (18)

To obtain system stability, the predicated ellipse should be as small as possible and close to origin. For minimizing the calculated ellipses through LMI (17), according to theorem 1, it is enough to make matrix's eigenvalues to be larger, while minimizing the norm of the center of the ellipse.

A. The output tracking

Consider the outputs as $y(k) = cx(k)$, for tracking the desired output, the upper bound of function which contains the difference between the predicted output and the desired output must be minimized:

$$\min_{\gamma} \gamma$$
$$s.t:$$
$$\sum_{i=1}^{N}(Cx(k+i\mid k)-y_d)^T Q_{ii}(Cx(k+i\mid k)-y_d)<\gamma$$
$$=\begin{bmatrix} Cx(k+1|k)-y_d \\ \vdots \\ Cx(k+N|k)-y_d \end{bmatrix}^T Q_y \begin{bmatrix} Cx(k+1|k)-y_d \\ \vdots \\ Cx(k+N|k)-y_d \end{bmatrix} - \gamma < 0 \quad (19)$$

By considering the Equation (13), it can be sho-wn that:

$$\begin{bmatrix} Cx(k+1|k)-y_d \\ \vdots \\ Cx(k+N|k)-y_d \end{bmatrix} = \underbrace{\begin{bmatrix} CAx(0)+Cf_0 \\ \vdots \\ CA^N x(0)+C\sum_{i=1}^{j}A^{(j-i)}f_0 \end{bmatrix}}_{X}$$
$$+\underbrace{\begin{bmatrix} CB & 0 & 0 & 0 & \cdots & 0 \\ CAB & CB & 0 & 0 & \dots & 0 \\ CA^2B & CAB & CB & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{N-1}B & CA^{N-2}B & CA^{N-3}B & CA^{N-4}B & \dots & CB \end{bmatrix}}_{G}$$
$$\times \underbrace{\begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(N-1) \end{bmatrix}}_{U} + \underbrace{\begin{bmatrix} C & 0 & 0 & 0 & \cdots & 0 \\ CA & C & 0 & 0 & \dots & 0 \\ CA^2 & CA & C & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{N-1} & CA^{N-2} & CA^{N-3} & CA^{N-4} & \dots & C \end{bmatrix}}_{G_v=[G_{v0}\ \ G_{v1}\ \ \dots\ \ G_{vN-1}]}$$
$$\times \underbrace{\begin{bmatrix} v(0) \\ v(1) \\ \vdots \\ v(N-1) \end{bmatrix}}_{V} - \begin{bmatrix} y_d \\ \vdots \\ y_d \end{bmatrix} \quad (20)$$

Therefore, the optimization problem (19) can be displayed in the following form:

$$\left\|G_vV+X+GU-y_{dW}\right\|_Q<\gamma$$
$$\to V^TG_v^TQG_vV+V^TG_v^TQ\left(X+GU-y_d\right)+$$
$$\left(X+GU-y_d\right)^TQG_vV+$$
$$\left(X+GU-y_d\right)^TW\left(X+GU-y_d\right)<\gamma \quad (21)$$

According to equations (11) and (21) and S-procedure lemma, we have:

$$\to$$
$$\begin{bmatrix} -\tilde{Q}_j^{-1} & [G_{v0} \quad G_{v1} \quad \dots \quad G_{vN-1}] & X+GU-y_d \\ \begin{bmatrix} * \\ * \\ \vdots \\ * \end{bmatrix} & -\begin{bmatrix} t_0^jQ_{V(0)} & 0 & \dots & 0 \\ 0 & t_1^jQ_{V(1)} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & t_{N-1}^jQ_{V(N-1)} \end{bmatrix} & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ * & * \quad \dots \quad * & -\gamma+\sum_{i=0}^{N-1}t_i^j \end{bmatrix}$$
$$\le 0 \quad (22)$$

B. Considering constraint on the input and convert it to a form of linear matrix inequality:

$$u_{min} \le u_j \le u_{max} \to \begin{cases} \frac{1}{2}I_jU+\frac{1}{2}U^TI_j^T \le u_{max} \\ u_{min} \le \frac{1}{2}I_jU+\frac{1}{2}U^TI_j^T \end{cases} \quad (23)$$

In the above expression I_j is $1\times N$ vector the *j*th element is equal to one and the other elements are equal to zero.

$$I_j = \begin{bmatrix} 0 & \dots & 0 & 1 & 0 & \dots & 0 \end{bmatrix}_{1\times N} \quad (24)$$

C. Considering constraint on the rate of changes of the input and convert it to a form of linear matrix inequality:

$$du_{min} \le u_j - u_{j-1} \le du_{max} \rightarrow \begin{cases} \frac{1}{2}P_j U + \frac{1}{2}U^T P_j^T \le du_{max} \\ du_{min} \le \frac{1}{2}P_j U + \frac{1}{2}U^T I_j^T \end{cases} \quad (25)$$

Where:

$$P_j = I_j - I_{j-1} = \begin{bmatrix} 0 & \dots & 0 & -1 & 1 & 0 & \dots & 0 \end{bmatrix}_{1\times N} \quad (26)$$

IV. NUMERICAL EXAMPLES

In this section, to demonstrate the effectiveness of the proposed algorithm two numerical examples for stabilization and output tracking are considered.

A. The Stabilization example:

Consider a discrete nonlinear system as [13]:

$$\begin{cases} x_1(k+1) = x_2(k) + \theta_1(k) \\ x_2(k+1) = x_3(k) + \dfrac{x_2(k)}{10 + x_3^2(k)} + \theta_2(k) \\ x_3(k+1) = 2x_1(k) + .1x_2(k) + x_3(k) + \dfrac{x_2(k)}{10 + x_3^2(k)} + u(k) + \theta_3(k) \end{cases} \quad (27)$$

In this example, the only goal is to stabilize the states of system with finite MPC under these constraints on control signal:

$$-1.5 \le u \le 1.5$$

&

$$-0.01 \le u_j - u_{j-1} \le 0.01 \quad (28)$$

Besides, the additive uncertain systems are given by:

$$\Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \in \varepsilon(Q_\theta, 0) \,\&\, Q_\theta = .01 \times I \quad (29)$$

Fig. 1 shows the effect of this algorithm on the stability of states for the initial condition $x_0 = [-2:3:5]^T$ and the control signal is depicted in Fig. 2.

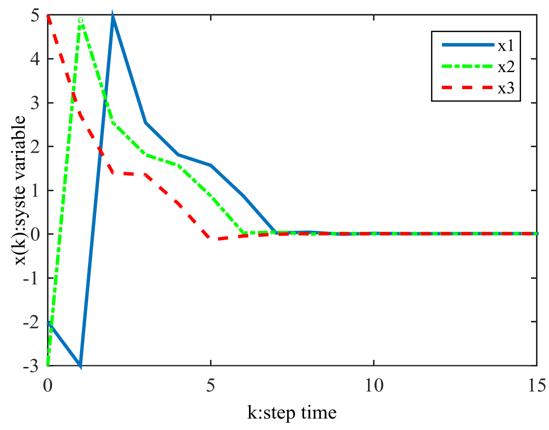

Figure 1. States responese of system (27)

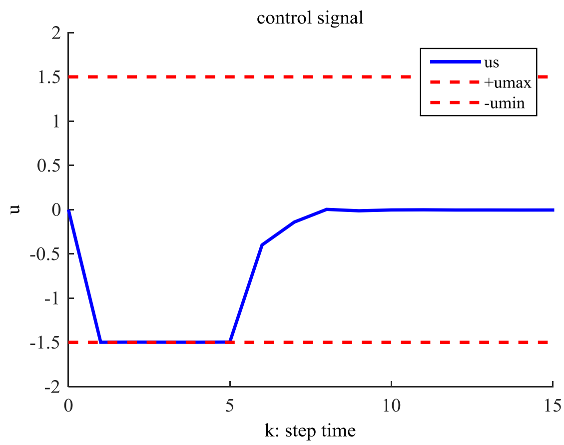

Figure 2. Control signal for system (27)

B. The Output tracking example

Consider the following DC/AC converter system [18] that shows the effectiveness of the proposed approach for output tracking.

The state space equations are given as:

$$\begin{cases} \dot{x}_1(t) = \dfrac{x_2^2(t)}{x_1(t)} - 5x_1(t) + 5u(t) \\ \dot{x}_2(t) = \dfrac{x_2^3(t)}{x_1^2(t)} - 7x_2(t) + \left(5\dfrac{x_2(t)}{x_1(t)} + 2x_1(t)\right)u(t) \\ y(t) = x_2(t) \end{cases} \quad (30)$$

The system model was discretized with a sampling time as t=0.01 min before the algorithm was applied. Moreover we consider the following constraint on the control signal:

$$-2 \le u \le 2 \tag{31}$$

The aim of control system is to track $y_d = 1$ that is shown in Fig. 3 and the control effort control for this purpose is given in Fig. 4:

V. CONCLUSION

In this paper, an efficient and robust algorithm for nonlinear systems using finite horizon model predictive control is utilized through linear matrix inequalities. The Predicted state of the system is placed in an ellipsoids and by satisfying suitable constraints, the mentioned ellipses become smaller and smaller at each time step. This procedure finally will converge to zero. This algorithm can be used for a wide range of nonlinear systems to track the various types of desired inputs. The stability of this algorithm has been proved. Besides, the upper and lower bounds for the control and its rate can be satisfied using the preplanned constraints.

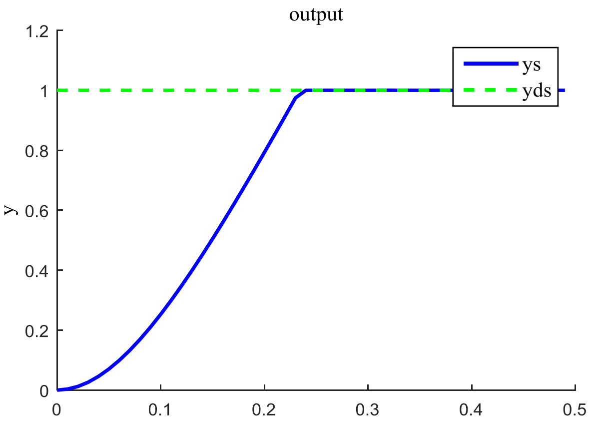

Figure 3. Output response for system (30)

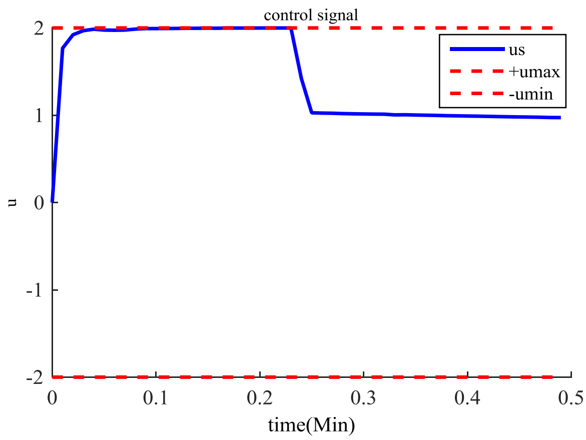

Figure 4. Control signal for system (30)

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